Calculus II - Test #2- Review

1. Need to know

1. Be able to compute areas
2. Be able to use these rules
   - substitution
   - integration by parts
   - partial fractions
3. polynomials, exponentials, trig functions,powers, hyperbolic function, $\frac{1}{\sqrt{a^2-x^2}} \cdot \frac{1}{a^2+x^2}$

2. Integrals

1 Compute the following definite integrals.

1. $\int_0^2 \frac{(x+5)dx}{\sqrt{4-x^2}} = \frac{-1}{2}\int_0^2 \frac{-2xdx}{\sqrt{4-x^2}} + 5\frac{1}{2}\int_0^2 \frac{dx}{\sqrt{4-x^2}} = \frac{-1}{2}\left[4-x^2\right]_0^2 + 5\left[\arcsin\left(\frac{x}{2}\right)\right]_0^2$
2. $\int_0^T t^3e^{-5t}dt = \frac{-1}{5}\int_0^T t^3de^{-5t} = \left[\frac{-1}{5}t^3e^{-5t}\right]_0^T + \frac{1}{5}\int_0^T 3t^3dt = \frac{-1}{5}T^3e^{-5T} + \frac{1}{5}\int_0^T 3t^3dt$, now continue
3. $\int \frac{x^2+1}{x^2+4x-12}dx = \int \frac{x^2+4x-12\ \ -4x+11}{x^2+4x-12}dx = \int dx + \int \frac{-4x+11}{x^2+4x-12}dx\backslash$ $\frac{-4x+11}{x^2+4x-12} = \frac{-4x+11}{(x+6)(x-4)} = \frac{A}{x+6} + \frac{B}{x-4}$, $-4x+11 = A(x-4)+B(x+6)$ use root substitution to find $A = \frac{13}{10}$, $B = \frac{-5}{2}$
4. $\int dx + \frac{13}{10}\int \frac{dx}{x-4} - \frac{5}{2}\int \frac{dx}{x+6}$
5. $\int_0^5 \frac{du}{25-u^2} = \frac{1}{2}\int_0^5 \frac{du}{5-u} + \frac{1}{2}\int_0^5 \frac{du}{5+u} = \frac{1}{2}\left[\ln|5-u|\right]_0^5 + \frac{1}{2}\left[\ln|5+u|\right]_0^5 = \frac{1}{2}\ln 0 - \ln 5 + \frac{1}{2}\ln 2$. this is undefined sin $\ln 0$ is undefined

3. Areas

1. Setup the integral to calculate the area enclosed by the x-axis and curve $y = x\cos x$ shown in the graph

$$\int_0^{\pi/2} x\cos x dx - \int_{\pi/2}^{3\pi/2} x\cos x dx$$

2. Compute the area. $\int_0^{\pi/2} x \cos x dx - \int_{\pi/2}^{3\pi/2} x \cos x dx = \frac{5}{2}\pi - 1$ (integration by parts)

4. Vibrational Energy

Let $f(t)$ be a function defined for $0 \leq t \leq 2\pi$. The strengths of the vibrational energies associated to frequency n are given by:

$$\begin{aligned} a_n &= \frac{1}{\pi} \int_0^{2\pi} f(t) \cos(nt)\, dt \\ b_n &= \frac{1}{\pi} \int_0^{2\pi} f(t) \sin(nt)\, dt \end{aligned}$$

Compute a_n and b_n for $f(t) = t^2 - 2\pi t,$. using integration by parts

$$\begin{aligned} a_n &= \frac{1}{\pi} \int_0^{2\pi} (t^2 - 2\pi t) \cos(nt)\, dt \\ &= \frac{1}{\pi} \left[\frac{1}{n} \left(\frac{1}{n^2} \left(n^2 t^2 \sin nt - 2 \sin nt + 2nt \cos nt \right) - 2\frac{\pi}{n} (\cos nt + nt \sin nt) \right) \right]_0^{2\pi} \\ & \frac{4}{\pi} (\cos \pi n) \frac{- \sin \pi n + \pi n \cos \pi n}{n^3} \\ a_n &= \frac{1}{\pi} \int_0^{2\pi} (t^2 - 2\pi t) \cos(nt)\, dt = \frac{1}{\pi} \int (t^2 - 2\pi t) \sin(nt)\, dt \\ & \frac{4}{\pi} (\cos \pi n) \frac{- \sin \pi n + \pi n \cos \pi n}{n^3} \end{aligned}$$