MA 112 - Calculus II
Worksheet #2
Professor Broughton

Name:_____ Box #:_____

The various approximation sums: Let $f(x)$ be defined for $a \leq x \leq b$. Subdivide the interval $[a, b]$ into n equal intervals of length $\Delta x = \frac{b-a}{n}$. The i'th interval is the set of x satisfying $a + (i-1)\Delta x \leq x \leq a + i\Delta x$ Let m_i, M_i be points in the i'th interval, i.e.,

$$a + (i-1)\Delta x \leq m_i \leq a + i\Delta x$$
$$a + (i-1)\Delta x \leq M_i \leq a + i\Delta x$$

and such that $f(m_i)$ and $f(M_i)$ are the minimum and maximum values of $f(x)$ on the i'th. The various approximation sums for the area under the curve, for $a \leq x \leq b$, are given by:

$$\text{left sum} = \sum_{i=1}^{n} f(a + (i-1)\Delta x)\Delta x$$
$$\text{right sum} = \sum_{i=1}^{n} f(a + i\Delta x)\Delta x$$
$$\text{middle sum} = \sum_{i=1}^{n} f(a + (i - \frac{1}{2})\Delta x)\Delta x$$
$$\text{lower sum} = \sum_{i=1}^{n} f(m_i)\Delta x$$
$$\text{upper sum} = \sum_{i=1}^{n} f(M_i)\Delta x$$

1. Complete the following table for the function $f(x) = 1 - x^2$ and $a = -1, b = 1$.

n	left	right	(left+right)/2	middle	lower	upper
20						
40						
80						
160						
1000						

2. The value for the area under the curve is 4/3. Which of the above seems to be the best approximation method.

3. Now divide $[-1, 1]$ into an even number of intervals $2n$. Compute the following as a function of n.

$$\begin{aligned} left &= \\ right &= \\ (left + right)/2 &= \\ middle &= \\ lower &= \\ upper &= \end{aligned}$$

4. Compute the limit of each to verify that you get 4/3.