Lab 8

EXPERIMENTALLY DETERMINING THE TRANSFER FUNCTION OF A SPRING-MASS SYSTEM

OBJECTIVES

At the conclusion of this experiment, students should be able to:

Experimentally determine the best fourth order transfer function model for the 2 DOF system.

Collect experimental frequency response data.

Understand the significance of the Bode plot for predicting system behavior, and determining non-parametric system models.

DELIVERABLES

The deliverables of this experiment are:

A memo style report including, but not limited to the following:

-introduction, results/discussion, conclusion, and appropriate appendices

-Show your best fit transfer function, and the location of its poles.

-A Bode magnitude plot showing the experimental data, and best fit. An example is shown on the last page of this handout, your results may vary significantly.

-List any suggestions for improving the lab.

THEORY

By frequency response, we mean the response of a system to a harmonic input. A linear system cannot change the frequency from input to output. Thus, as we have seen earlier, the output will be the a sinusoid of frequency identical to that of the input, only amplified or attenuated, and shifted in phase:

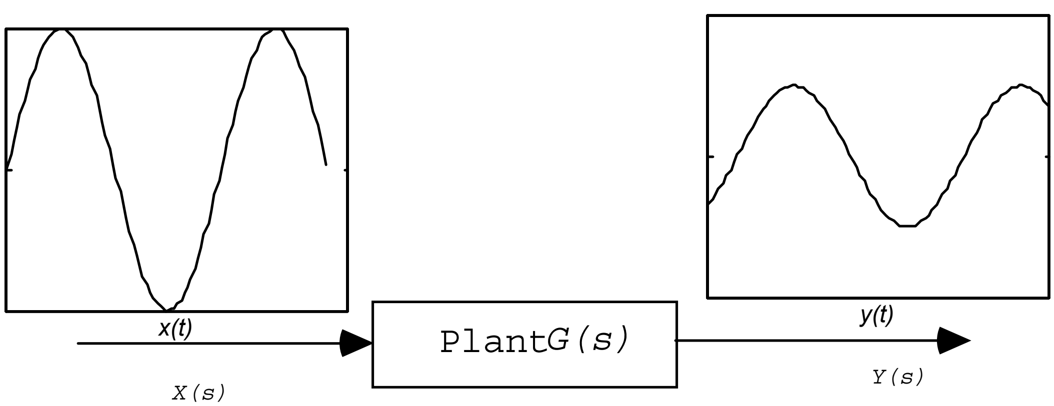


Figure 1, Physical Meaning of Frequency Response

$$x(t) = X \sin \omega t$$

$$y(t) = |G(j\omega)| X \sin(\omega t + \angle G(j\omega))$$

A Pole-Zero Map is the easiest way to visualize the magnitude and phase of a Transfer Function. The figure below shows the pole zero map of the transfer function:

$$G(s) = \frac{s^2 + 2.1s + 0.2}{s\left(s^3 + 3s^2 + 93.25s + 91.25\right)}$$

We have drawn the vectors from each pole and zero to the point $s = j9.5$, which is the 'resonant' frequency of the system.

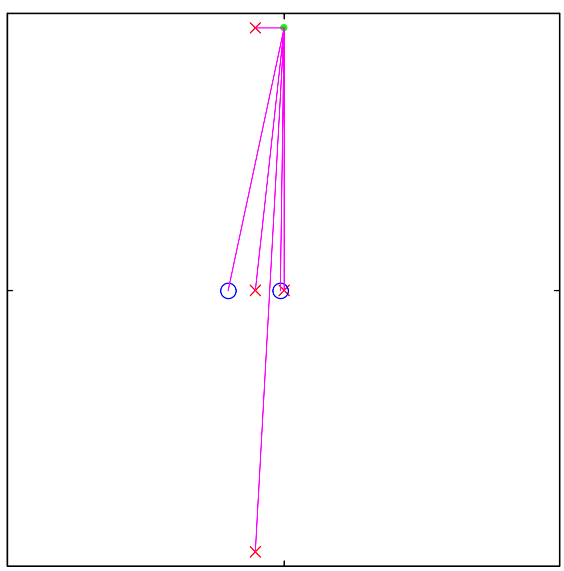


Figure 2, Graphical Evaluation of Frequency Response

Note that the magnitude of the transfer function at this point is given as the product of the lengths of the vectors from the zeros over the product of the lengths of the vectors from the poles:

$$|G(j\omega)| = \frac{\prod_i |z_i|}{\prod_j |p_j|} \tag{1}$$

The transfer function argument (angle) is the sum of the zero vector angles minus the sum of the pole vector angles. In each case the angles are defined as counter-clockwise from a segment parallel to the real axis to the vector.

$$\angle G(j\omega) = \sum_i \angle z_i - \sum_j \angle p_j$$

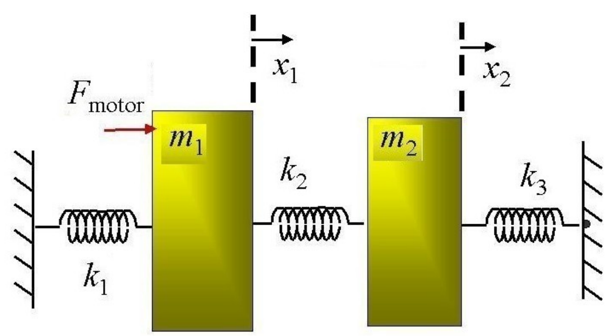
The closer a complex pole is to the $j\omega$ axis, (less system damping) the higher a resonant peak will be. Likewise, the closer a zero, the lower the 'notch'.

The Bode plot is a mapping of the entire $+j\omega$ axis (the entire positive frequency spectrum) through the system transfer function to Gain and Phase plots. We typically plot Gain and Phase together on a semilog axis. The abscissa (x-axis) for the Bode plot is log frequency. The distance between a frequencies 1 and 10, 0.1 and 1, 10 and 100, etc. is termed a 'decade'. Gain is plotted in 'decibels' (dB) where the conversion from magnitude to dB is given by

$$|G(j\omega)|\ (\text{dB}) = 20\log_{10}|G(j\omega)|$$

This week in class we are investigating how to determine the Bode plot knowing the system transfer function. In this lab we will explore the *inverse* problem, that is, knowing the frequency response, can we determine an appropriate transfer function? We will base our transfer function estimate on the experimental Bode magnitude plot only. As illustrated in Fig 2, and Eqn 1, without considering the Bode phase plot, the poles and zeros could lie on either side of the $j\omega$ axis (right or left half plane) and we would observe the same Bode magnitude plot. With this in mind, you may find it necessary to arbitrarily change unstable system poles to their stable equivalents after tuning your model.

The system we are trying to identify is a two mass, three spring system as shown to the right. The input is a voltage to a DC motor connected to a rack and pinion, then directly connected to the first mass. We will neglect the motor dynamics. Since we have some insight into the physical characteristics, this is a *gray box* problem, rather than a *black box* problem, where we have no inkling as to what connects input to output.



The output is the position of the second mass x_2. From first principles, we would expect the system transfer function to be fourth order. Thus, we begin with the following transfer function, and try to identify the unknown parameters.

$$G(s) = \frac{K\omega_1^2\omega_2^2}{\left(s^2 + 2\zeta_1\omega_1 s + \omega_1^2\right)\left(s^2 + 2\zeta_2\omega_2 s + \omega_2^2\right)} \qquad (2)$$

THE ADVENTURE BEGINS

During the adventure you will set out to accomplish the following:

1. Set up the environment.

a. Each station should be set up in 2 DOF mode with two 500g brass masses on each carriage. If your station is not configured properly, ask your instructor to fix it.

b. Log on to the computer (username=student, password=student) and start the ECP executive program under Programs/ECP. Select Setup/Control Algorithm... In the dialog box, select the State Feedback radio button, and press Implement Algorithm. Press OK. Push the black button of the ECP control box. You might hear the system rattle a bit due to sensor noise in the feedback loop.

2. Recording Frequency response data: For each frequency of interest, do the following steps:

a. Select Command/Trajectory... In the dialog select the Sinusoidal radio button and press Setup. Now select Open Loop Move. The Amplitude should be set to 0.5v. Select a frequency of interest. You will need to run the following set of frequencies: [1 2 3 4 5 6 7 8 9 10] Hz
Set the number of reps equal to 10 times the frequency (in Hz) and click OK. Click OK on the Trajectory Configuration dialog.

b. Under Utility, select Zero Position

c. Select Command/Execute and press Run.

d. Watch the response. After the Upload successful dialog completes, click OK.

e. (Optional) To look at an individual data set, select Plotting/Setup Plot. In the dialog, add Commanded Position and Encoder 2 Position to the Left Axis, the click Plot Data. You might want to do this for some of the higher frequencies to insure that the system has reached steady state.

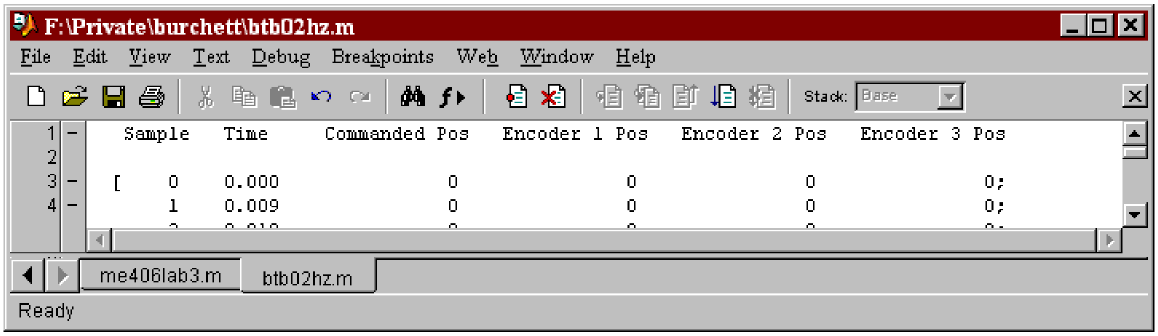
f. Export your data. Select Data/Export Raw Data... Browse to a convenient directory, floppy disks are recommended for storing this data. Save as type All files (*.*). Choose a name with meaning, like 'inixx.m' where ini is a group member's initials, xx is the frequency in Hz.

g. Repeat for the remaining frequencies of interest.

h. You can use Secure FX to move your data to your afs space. A shortcut is provided on the lab computer desktop.

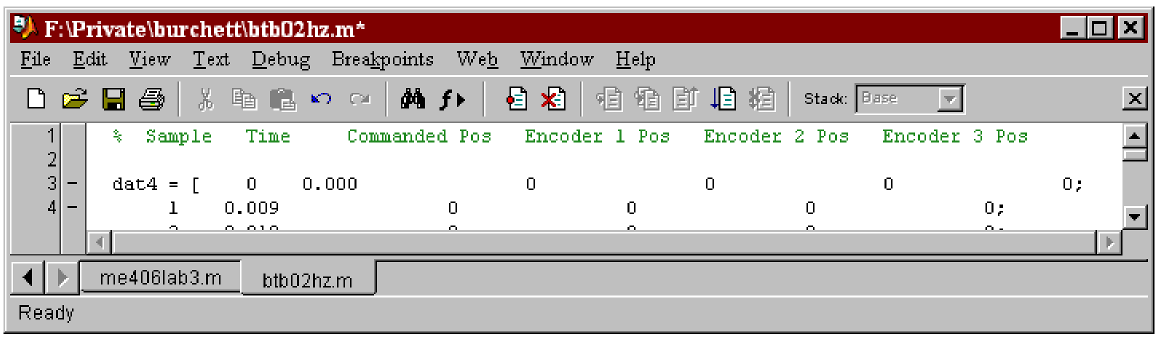
3. Analysis:

a. Each exported data file will have the following format.



Each row of the array is a sample and the columns are, respectively: Sample, Time, Commanded Pos, Encoder 1 Pos, Encoder 2 Pos, Encoder 3 Pos.

Change the file so that you have an array assignment.



Extract the Encoder 2 Position data and subtract the mean:

```
»enc2cm = dat4(:,5)-mean(dat4(:,5));
```

Convert Encoder 2 Postion to centimeters. The encoder sensitivity is 1604.1 counts/cm.

```
»enc2cm = enc2cm/1604.1;
```

Now determine the ratio of output to input amplitude. You might want to plot the data to determine where the oscillations have reached steady state. Output amplitude can then be determined by taking the maximum value from several steady state oscillations. Since the input amplitude was specified to be 0.5 volts, we divide by 0.5 to get the ratio of output magnitude over input magnitude. For example:

```
Magy(10)  = max(en2cm(500:1000))/0.5;
```

Note, the array indices in this statement need to be adjusted to that you consider only oscillations after the system has reached steady state. It is probably easiest to do these calculations inside each data file.

b. Write a top-level script that 1) executes each data file. 2) Converts the Magy vector to dB, and plots the experimental data on a semilog plot. 3) Uses fminsearch to find the fourth order transfer function that best fits the data. Use a transfer function form with no finite zeros. You should field an initial guess based on where the resonant peaks appear to be in the experimental data. The system is very lightly damped, so guess each damping ratio to be 0.1. Reduce the transfer function of Eqn (2) to the following form, and use the x_i s for your initial guess.

$$G(s) = \frac{x_1}{x_2 s^4 + x_3 s^3 + x_4 s^2 + x_5 s + x_6}$$

Then put the coefficients of your guess into a column vector to serve as the initial guess for **`fminsearch`**. Use the following syntax to run your optimization:

```
»options = optimset(@fminsearch)
»options = optimset(options,'Display','iter');
»coeffs = fminsearch(@lab8,x0,options)
```

You will need to write a function 'lab8' that computes the sum squared error between experimental magnitudes and theoretical magnitudes along the Bode plot. You are encouraged to use the features of the control toolbox, including, in particular, the bode command. Here are a few lines of code to get you started:

```
function J = lab8(x)
num = x(1);
den = x(2:6);
sys = tf(num,den);
ww = 2*pi*[1:10]; % be sure to convert freqs. to rad/sec
mag = [% put your experimental data here %]';
maggie = bode(sys,ww);
maggie = 20*log10(maggie(:));  % Comparison should be done in
mag = 20*log10(mag);           % dB for proper weighting
J = norm(mag - maggie);
```

Plot the magnitude of your best fit transfer function, and determine the resonant frequencies. (You will need to re-run the bode function with the 'best' transfer function coefficients, and use a fine frequency vector like `ww=logspace(0,2,100)`). Time permitting, try exciting the system at the resonant frequencies. You will probably need to reduce the input amplitude to 0.25 volts to avoid exceeding the travel limits of the device. Incorporate this data into your analysis, and re-do the numerical fit analysis.

Check the stability of your tuned transfer function. If any of the poles lie in the right half plane (have positive real parts) you should arbitrarily change the real parts to be negative, and re-calculate the transfer function. Be sure to report a *stable* transfer function as your final answer.

A sample plot of experimental and theoretical best fit Bode magnitude data is provided below, your results may be significantly different depending on which set-up you use for the analysis.

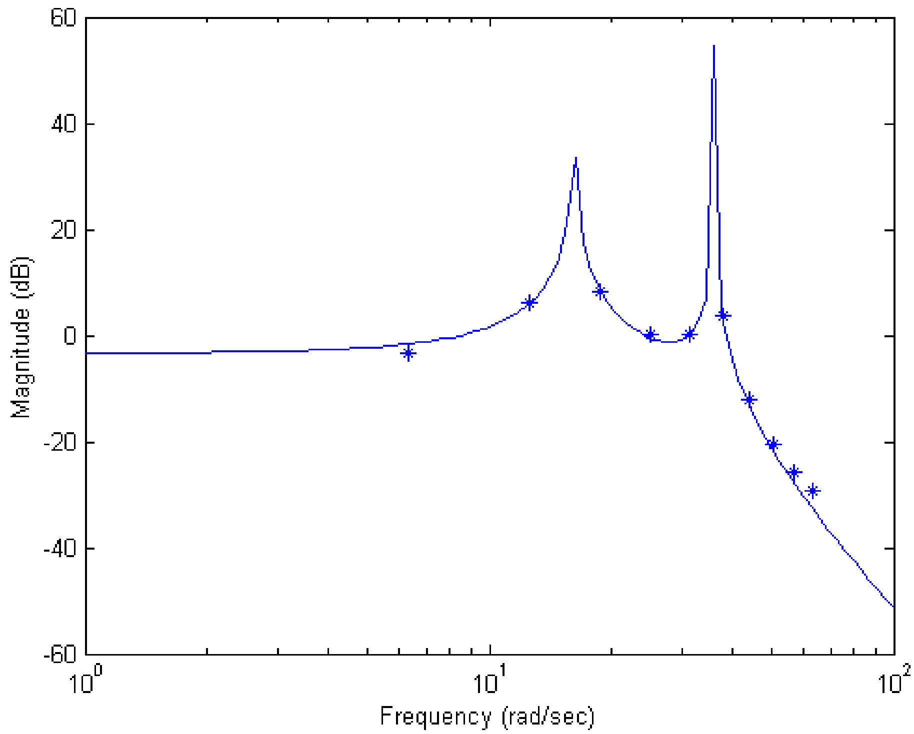


Figure 3, Experimental Bode Magnitude Data with best fit theoretical transfer function.