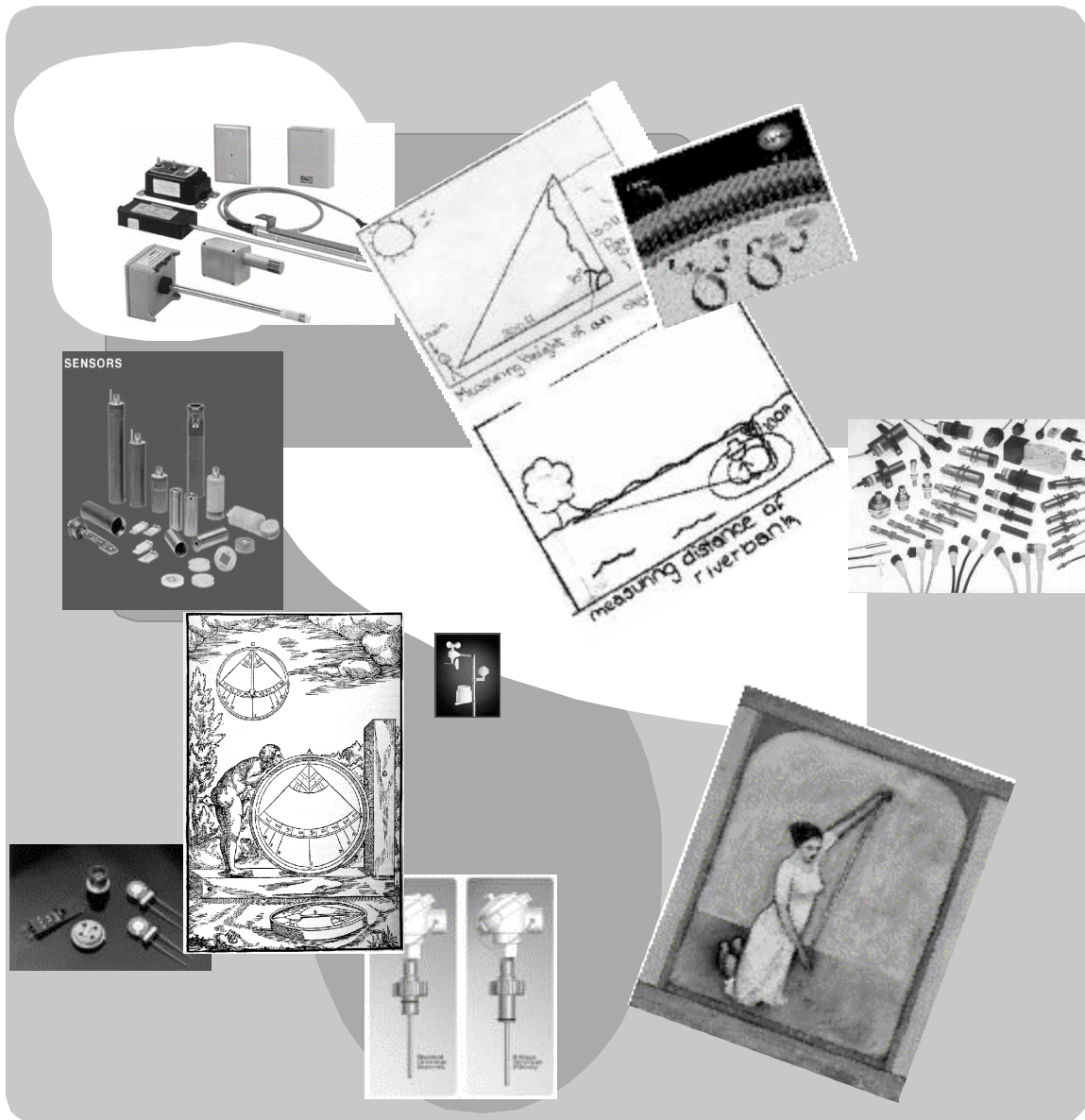# CHAPTER 8

SENSORS

"The power of accurate observation is commonly called cynicism by those who have not got it." – George Bernard Shaw

"All credibility, all good conscience, all evidence of truth come only from the senses." – Friedrich Nietzsche

"The only man who behaved sensibly was my tailor; he took my measurement anew every time he saw me, while all the rest went on with their old measurements and expected them to fit me." – George Bernard Shaw

8.1 Introduction

Measurement can be split into two types: 1) direct comparison with a standard measure and 2) measurement using a calibrated system, which is also based on standard measures.

Type of Standard	Definition
Primary Standard	A standard that is designated or widely acknowledged as having the highest metrological qualities and whose value is accepted without reference to other standards of the same quantity.
Secondary Standard	A standard whose value is assigned by comparison with a primary standard of the same quantity.
Reference Standard	A standard generally having the highest metrological quality available at a given location or in a given organization from which measurements made there are derived.
Working Standard	A standard that is used routinely to calibrate or check material measures, measuring instruments or reference materials.

Table 1: Types of Standards (from http://www.measurement.gov.au **)**

Direct comparison is used, for example, when a length is measured by comparing the length to a ruler, the ruler typically being the standard. The ruler is typically a secondary standard based on a primary standard.

Using a calibrated measurement system is the most common type—for example, voltage with a calibrated voltmeter, power with a calibrated power meter, and weight with a calibrated scale. A calibrated measurement system typically involves the use of one or more sensors and signal conditioning systems to "condition" the signal(s) into a desired form.
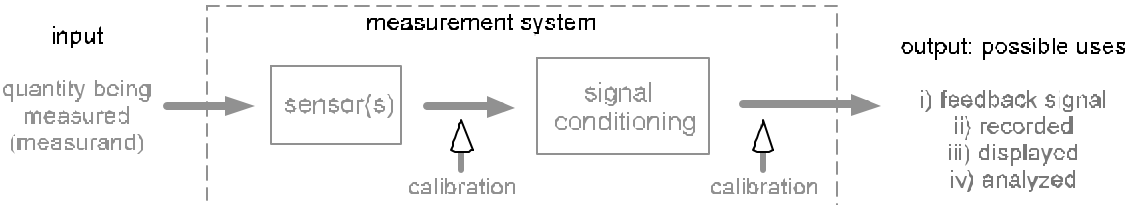

Figure 8.1: Overview of a Calibrated Measurement System

When measuring strain, for example, strain is the measurand. If strain is measured using a resistive strain gage, the signal from the strain gage is a change in the strain gage resistance. Signal conditioning might, in this case, include transforming the change in resistance into a change in voltage using a Wheatstone bridge and subsequent amplification.

Since measurement is central to all engineering and scientific work, special care must be taken to fully understand all aspects of measurement, from the physical

principles of the sensors to proper choices for signal conditioning to knowing how measurement errors affect system function.

Signal conditioning is treated in chapter 9 and can include amplification, buffering, or filtering. The characteristics and operating principles of sensors are treated in this chapter.

Physical Quantity (SI Unit)	Definition
time (second)	The second is equal to 9,192, 631, 770 periods of the radiation emitted from the transition between the two hyperfine levels of the ground state of the Cesium-133 atom.
length (meter)	The meter is the distance traveled by light in vacuum in 1/299 792 458 of a second.
	The meter is usually determined using a laser with a known and highly stable frequency. From the frequency ν and the speed of light, the wavelength λ of the stabilized laser can be calculated from the known relationship $\lambda = c/\nu$. This allows length to be determined by direct comparison.
mass (kilogram)	The kilogram is the unit of mass equal to that of international prototype kilogram.
	To date, attempts to base mass on a natural constant with sufficient accuracy have failed. The international prototype kilogram, made in 1889 from platinum and iridium, is still used today as the primary standard for mass. It is kept at the *Bureau International des Poids et Mesures* (BIPM) at Sèvres near Paris.
temperature (Kelvin)	The Kelvin (K), unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.
	Temperature is a particularly troublesome property for which to determine standards. It is a measure of kinetic energy, but an indirect one. The practical realization of the temperature scale is usually done by means of a series of highly stable fixed temperature points. Between these points, quartz glass-coated standard resistance thermometers, containing a wire spiral of high purity platinum are calibrated to these fixed temperature points. These are then used as interpolation instruments for calibrating between the fixed points.
current (Ampere)	The ampere is the current which, if in two straight parallel conductors—placed one meter apart, of negligible cross section and of infinite length—would result in a force of $2(10^{-7})$ N/m between them.

Table 2: Definition of Selected SI Units

Sensors comprise a broad area of study and application. Here, after discussing how sensors can be characterized, the focus will be on sensors in which an electrical property—such as resistance, capacitance, or inductance—changes as the physical property being sensed changes. In these sensors the electrical property serves as the analog to the property being measured.

Not all sensors are of the type treated here. For example, a mercury thermometer uses the expansion of mercury with temperature to create a temperature sensor, a

thermometer, where the height of the mercury in a glass tube serves as an analog, or the measure, of the temperature.

8.2 Sensor Characteristics

Sensors are characterized by the relation between their input (the measurand, the physical quantity being sensed) and their output (the signal from the sensor). From the sensor's transfer function parameters such as range, full-scale output, sensitivity, and linearity can be defined.

The *range*, also called the span or full-scale input, of a sensor refers to the range of inputs for which the sensor functions within specifications.

The *full-scale output* is the difference between the sensor's output when the sensor's input is maximum and when it is minimum.

Sensitivity refers to change of output per change of input. It is the magnitude of the slope of the sensor characteristic curve.

The input/output characteristic is *linear* to the extent in which its sensitivity or slope does not change as the input changes.

Example 8.1

Suppose, for a given application, that a temperature sensor's linearity and sensitivity are determined to be adequate between points **a** and **b** in Fig. 8.2. This defines the sensor's range. The sensor output is a voltage which, at full scale, is 50 mV.
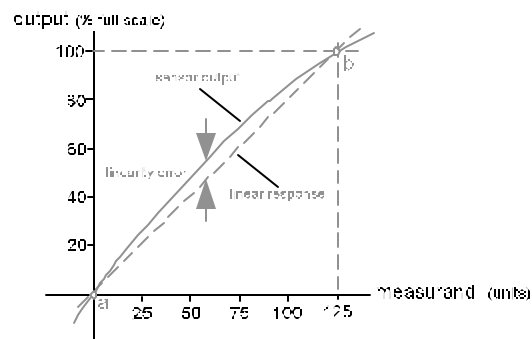

Figure 8.2: Sensor Characteristic Curve

Range $= R =$ input$_b$ - input$_a \cong 125$ °C

Average sensitivity over entire range $= S_{avg} = \dfrac{50 \text{ mV}}{125 \text{ °C}} = 0.4 \text{ mV}/\text{°C}$

The maximum linearity error is approximately 10% full scale or about 5 mV.

Often, sensors are used in measurement systems in which the concepts of accuracy and precision are useful terms by which to characterize the performance of the overall measurement system.

The *accuracy* of a sensor or measurement system is a measure of how far the measured quantity differs from that of the ideal. Implicit in this definition is the ability to compare the measurement in question to a standard measurement. The accuracy of an measurement is a measure of "how far off" it is from the true value.

Note, that when the accuracy of a measurement is given, what is usually specified is actually the measurement's inaccuracy. That is, if an instrument's accuracy is listed as 2% full scale, the meaning to be inferred is that the inaccuracy of any measurement over the instruments valid range will be less that 2% of the full-scale output.

The inaccuracy of a measurement can be split into two parts—those that are repeatable and those that are not. Errors that are repeatable lead to systematic error, also termed bias error. Errors that are not repeatable and are essentially random lead to random error, also called precision error.

Example 8.2

Suppose a force transducer is used to take ten measurements of the same force. Suppose the readings for the 10 trials are: 3.25 N, 3.05 N, 3.40 N, 2.90 N, 3.05 N, 2.95 N, 3.15 N, 3.40 N, 2.90 N, 3.25 N. Suppose the true value is known to be 3.0 N.

What is the systematic error (ΔF_s) in this set of measurements? What is the range of random error (ΔF_r)?

$\Delta F_s = \bar{F} - F_{true}$, where $\bar{F}$ is mean of the set of measurements.

$(\Delta F_r)_i = F_i - \bar{F}$, where $(\Delta F_r)_i$ is the random error associated with the ith measurement and F_i is the ith measurement.

For the above ten trials, the mean force measured, $\bar{F}$, is 3.13 N.

$\Delta F_s = 3.13 - 3.00 = 0.13$ N
The range of random error is from 0.27 (for F_i = 3.40 N) to -0.23 (for F_i = 2.90 N).

Example 8.3

Suppose the performance of two precision positioning instruments are compared. Each instrument measures the position of an object whose true position is (x,y) = (30, 20) μm.

Each instrument is used to measure this position in a trial of six individual measurements. The results are shown below. How can the accuracies of instrument a and instrument b be characterized?
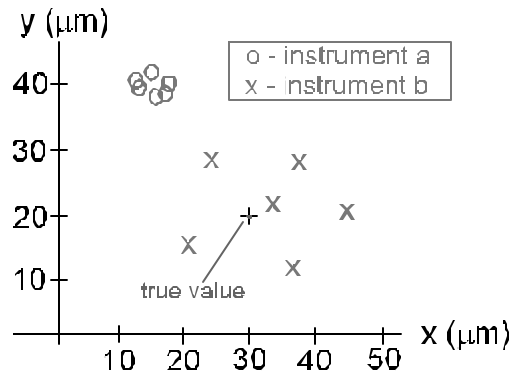

Figure 8.3: Random and Systematic Error

Instrument a has a large systematic error when compared to that of instrument b and a smaller range of random error. The average measurement obtained using instument b is more accurate than instrument a but less precise.

8.2 Resistive Sensors

The resistance of a resistor is determined by its geometry and its material properties. A simple and useful model for the resistance appears below.
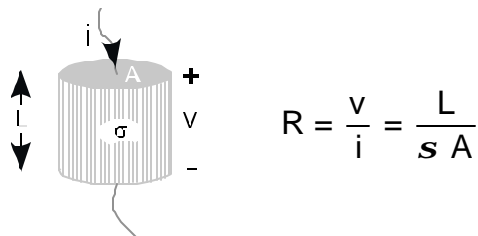

$$R = \frac{v}{i} = \frac{L}{s\,A}$$

Figure 8.4: Lumped Resistance

Note: A reminder concerning the parameters in Fig. 8.3

	R ~ resistance	(ohm)
	v ~ voltage across R	(volts)
$R = \dfrac{v}{i} = \dfrac{L}{s\,A}$	i ~ current through R	(amps)
	L ~ length	(meter)
	s ~ conductivity	(1/(ohm meter)
	A ~ cross sectional area	(meter^2)

In general, a resistive sensor can be any device in which its electrical resistance (R) changes in response to changes of a physical quantity to be measured.

Example 8.2

Metallic strain gages are widely used sensors to detect strain. To understand the principle of metallic strain gages, consider the simple bar under longitudinal stress as shown in Fig. 8.5.
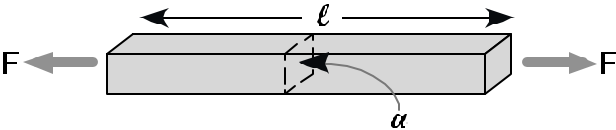

Figure 8.5: Simple Strain Gage

What happens as F increases? As F increases, two things happen: First, the bar increases in length. The *longitudinal strain* is a measure of this increase $e_L = \partial l / l$.

Second, the bar's cross sectional area decreases. The *transverse strain* is a measure of this increase $e_t = \partial a / a$.

If the stress is not too large, strain will vary linearly with stress (Young's modulus, E) and the ratio of the transverse strain to the longitudinal strain is a constant of the material (the Poisson ration, ν).   One can readily appreciate the fact that the bar's resistance, see Fig. 8.6, will increases as F increases (which causes ↑l and ↓a).This is a resistive sensor! It is a resistive strain sensor.
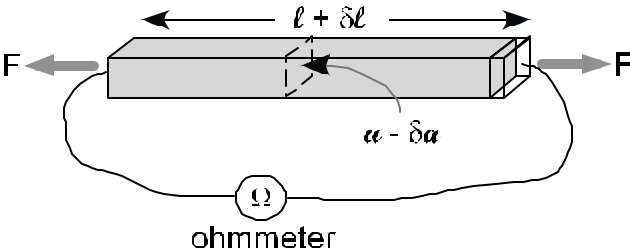

Figure 8.6: Simple Bar Under Stress

The most common practical metallic strain gage is a constantan (a Cu-Ni alloy) metal film to be deposited on polyimide (a high-temperature polymer).
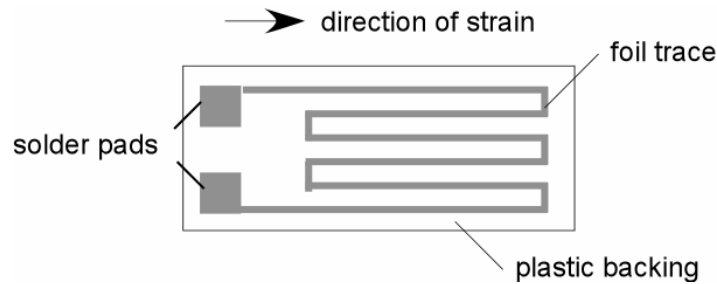
Figure 8.7: Metallic Film Strain Gage

Notice the direction of strain in Fig. 8.7. The fact that most of the films length lies in one direction (horizontal in Fig. 8.7 and Fig. 8.8) causes the sensor to be especially sensitive to strain in that direction and defines the "direction of strain."

The strain gage (the metal film and polyimide substrate) is bonded to some material to be strained so that the strain gage experiences *exactly* the same strain as that of the material. For an accurate sensor, it is simply vital that the stain gage adheres intimately to the material under strain.
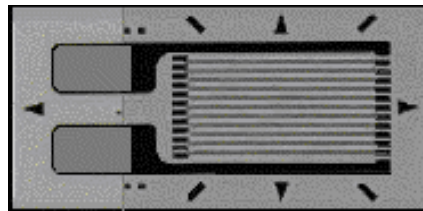

Figure 8.8: Strain Gage (*Vishnay Measurement Group*)

While the constantan-on-polyimide strain gage is one common type, metallic-foil strain gages are manufactured from a variety of different metals and alloys, with a range of resistance values, a range of sensitivity of resistance to strain, and varying resistance-to-temperature characteristics. Common nominal resistance values (the resistance of the unstrained strain gage) are 120 Ω, 350 Ω, and 1000 Ω. Metallic strain gages will be explored in greater depth in a design example.

Resistive sensors change resistance due to either changes in geometry (as with the metallic strain gage above) or/and due to changes in conductivity. The fact that either geometry or material properties (for resistive sensors, conductivity) can be used to for sensing is readily seen in the basic equation defining the resistance.

Resistive Sensors	Principle of Operation	Measurement Uses
Metallic Strain Gage	Geometry varies with strain.	Strain When used with a load cell or membrane, force, torque, or pressure can be

		detected. Metallic strain gages have gage factors, S, of around 2 while semiconductor-based strain gages can have gage factors in the tens to one hundred.
Resistance Temperature Detector (RTD)	Conductivity varies with temperature	Temperature Typical metals are Pt, Cu, or Ni. Platinum is the most popular metal due to its linearity and its stability.
		RTDs provide extremely stable and reproducible performance. The PT-RTD is used as the interpolation standard from the boiling point of oxygen (-182.96°C) to the melting point of antimony (630.74°C).
Potentiometer	Wiper varies resistor geometry	Displacement
Magneto Resistive Sensor	Conductivity varies with magnetic field	Detection of current, magnetic fields, detection of data recorded on magnetic media
Photoconductor	Conductivity varies with radiation	Detection of infrared, visible, ultraviolet radiation is possible by varying material used in photoconductor

Table 3: Resistive Sensors

A photoconductor is an example of a resistive sensor in which the resistance change is due to the material's conductivity being changed due to the presence of infrared radiation, visible light, or ultraviolet radiation. A photoconductor can be used as a component in light sensing instrumentation. One simple application of a photoconductor would be to allow lights to be turned on automatically at night.

By changing the materials used in the photoconductor, the sensor can be made to be sensitive from infrared to ultraviolet. The reason for this lies in the mechanism of photoconductivity. In a photoconductor, electrons are bound to nuclei with a characteristic strength. Since the electron is bound to the nuclei, it is not free to be affected by the application of a voltage.

On the other hand, if an electron were freed by something breaking its bonds, it then could contribute to conductivity and thereby increasing the material's conductivity. In photoconductivity, charge carriers are freed when a photo is absorbed. The energy required to free the charge carriers depends on the materials energy gap.
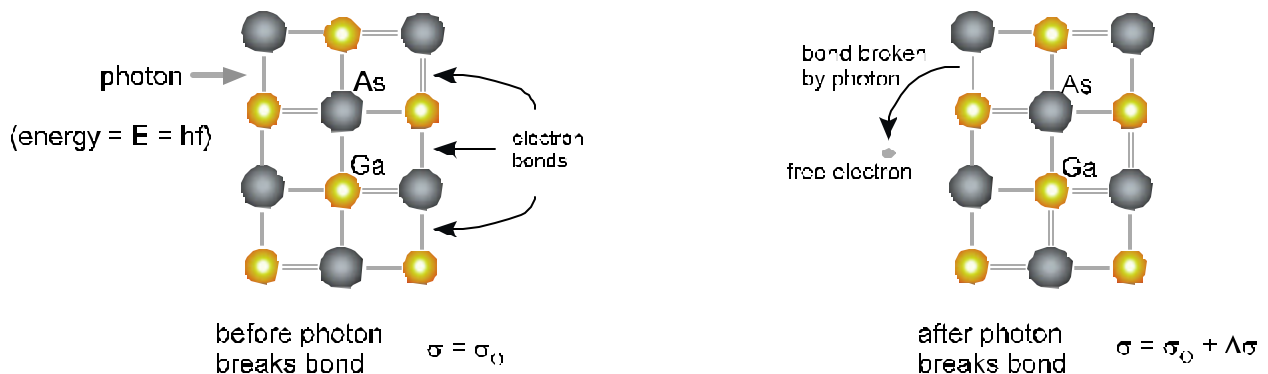

Figure 8.9: Charge Carrier Freed by Photon

For steady-state generation of charge carriers, the change in conductivity of a photoconductor is given by

$$s = e\, g_{op}\, t\, (m_h + m_p) \quad \text{where}$$

$e \qquad = 1.6(10^{-19})\,C \quad$ is the magnitude of the electronic charge

$g_{op} \qquad =$ rate that charge carriers are optically generated $\;(\dfrac{\#/sec}{m^3})$

$t \qquad =$ recombination time (sec)

$m_h \qquad =$ mobility of negative charge carriers (m/Vs)

$m_{pm/} \qquad =$ mobility of positive charge carrrier

The energy of a photon, E, can be calculated from De Brolie's relation, E = hf where h is Planck's constant and f is the linear frequency of the radiation in Hertz. Materials with larger energy gaps respond to higher energy (higher frequency) radiation. See appendix C for a more complete discussion of semiconductor physics.

8.3 Capacitive Sensors

A capacitor is formed when two or more conductors are separated by insulating material. The capacitance of the structure is determined by the geometry of the conductors and the permittivity of the insulation. A simple model is that of the parallel plate capacitor where the conductors are two parallel plates separated by an insulator.
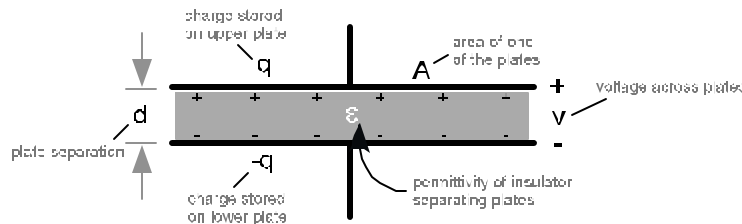

Figure 8.10: Parallel Plate Capacitor

The capacitance of a capacitor is defined as the ratio between the magnitude of the charge stored on the plates to the voltage difference between the plates. The term capacity refers to the "capacity of the structure to store charge.

$$C = q/v$$

Note that by taking a time derivative of both sides of this equation (assuming C does not vary with time), the familiar quasistatic approximation for the capacitance used in electrical circuits.

$$\frac{d}{dt}q = \frac{d}{dt}(C\,v) \quad \Rightarrow \quad i = C\,\frac{dv}{dt}$$

If the plate separation is small compared to their area, the capacitance is simply related to d, A, and ε.

$$C = e\,\frac{A}{d}$$

While this relation holds in detail only for the parallel plate capacitor, it does show the general trends that hold true for all capacitors. The capacitance increases as the insulator permittivity increases, increases as the conductor area increases, and decreases as the conductor separation increases.

$$\text{In summary} \quad \begin{cases} \uparrow e, \ \uparrow A, \ \downarrow d & \Rightarrow \quad \uparrow C \\ \downarrow e, \ \downarrow A, \ \uparrow d & \Rightarrow \quad \downarrow C \end{cases}$$

Capacitive sensors change their capacitance due to either changes in geometry (plate separation, plate area) or/and due to changes in permittivity.

Capacitive Sensors	Principle of Operation	Measurement Uses
Pressure	Geometry varies with pressure	Capacitive pressure sensors are typically formed by forming closed cavities which contain capacitor conductors—typically one located on a membrane exposed to pressure to be measured. Varying the pressure varies the plate separation and so varies the capacitance.
Displacement	Geometry varies with displacement	One plate is usually fixed with the other movable. Capacitance displacement sensors are made which have resolutions of less than 10 pm
Humidity	Permittivity varies with humidity	Water is absorbed by an oxide or polymer dielectric. The permittivity of the insulator between capacitor conductors depends on the amount of water absorbed which varies with humidity.
		Capacitive humidity sensors have become more commonly used than resistive sensors and can be used in many chemically harsh environments.
Accelerometer	Geometry varies with acceleration	Vibration control in hard-disk drives.
		Vibration detection in various consumer products.
		Avionic controls and safety systems.
		MEMS-based capacitive accelerometers are widely used to deploy automobile air bags.
Proximity	Geometry and/or permittivity varies with proximity	Detection of liquid level, products in inspection lines and in assembly lines.

Table 3: Capacitive Sensors

Example 8.3

Microelectromechanical Systems (MEMS) based accelerometers are standard sensors used in deploying air bags in automobiles.

The accelerometer can be modeled as a 2nd-order system consisting of a mass, a spring, and a damper.   The mass is primarily that of the proof mass. The spring constant is determined by the spring elements between the proof mass and the accelerometer substrate. Damping is mainly determined by the viscous damping of the accelerometer atmosphere—often "squeezed film damping" due to the small dimensions of the device.
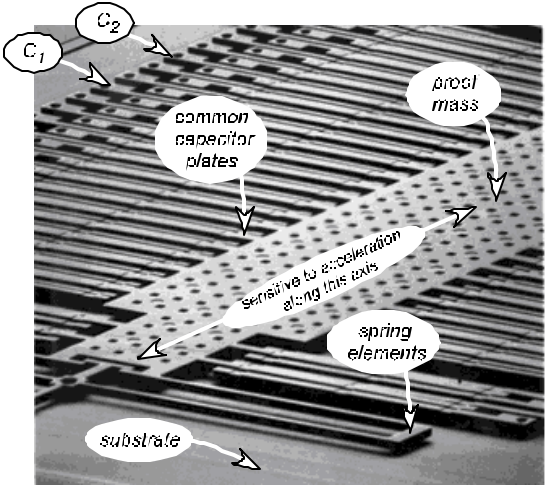
Figure 8.11: MEMS Accelerometer (from Analog Devices)

The sensor is comprised of two capacitors connected by a common plate as shown below in a simplified diagram.
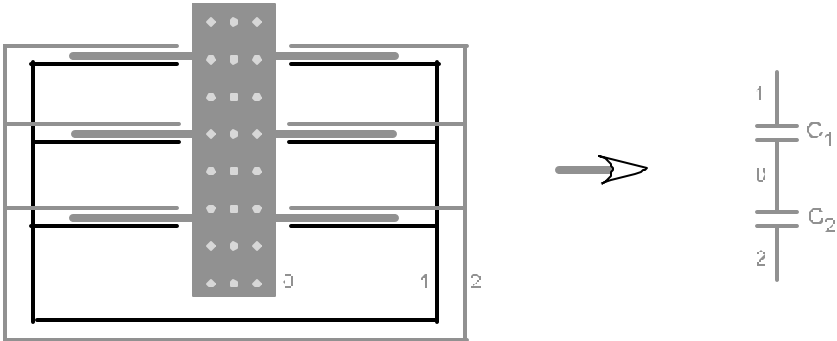
Figure 8.12: Simplified Functional Diagram of MEMS Accelerometer

The capacitors in a typical MEMS accelerometer at rest have capacitances of about 0.1 pF with a gap distance of approximately 1.3 μm.

$$C_{(1,2)} = \frac{0.1\ \text{pF}}{1 \pm \dfrac{x}{x_o}} \cong 0.1\ \text{pF}\left(1 \mp \frac{x}{x_o}\right) \qquad \text{(for } x \ll x_o\text{)}$$

MEMS accelerometers are often designed so that they remain very nearly in quasi-static steady-state during acceleration. That is, their time constants are much smaller than those involved in the acceleration.
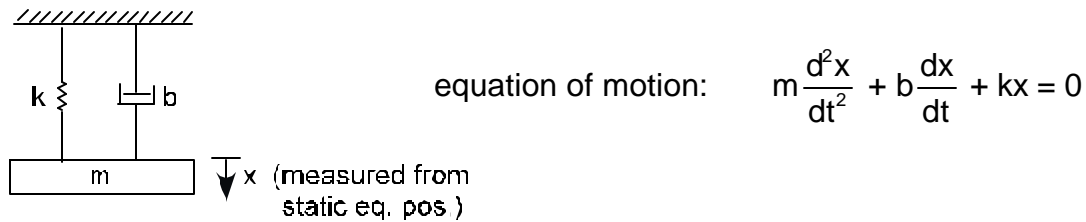

equation of motion: $\quad m\dfrac{d^2x}{dt^2} + b\dfrac{dx}{dt} + kx = 0$

Figure 8.13: Mechanics of MEMS Accelerometer

In this quasi-static case, the derivatives can be neglected.

$$kx = ma \qquad \Rightarrow \qquad x = \frac{ma}{k} = \frac{a}{k/m} = \frac{a}{w_n^2}$$

The basic trade-off can be seen to be one between sensitivity and speed. To make the device able to respond more quickly, the natural frequency should be increased—either by increasing the spring constant or by reducing the proof mass. Do so, however will cause x to be smaller for a given acceleration and thus cause the capacitance to deviate less from its nominal value.

Suppose the natural frequency, ω_n, is 50 kr/s. What is the displacement for a = 30g?

$$x = \frac{30\ \left(9.8\ \text{m/s}^2\right)}{\left(50000\ \text{r/s}\right)^2} = 0.118\ \textit{m}\text{m}$$

This would result in the two capacitances being

$$C_{(1,2)} = \frac{0.1\ \text{pF}}{1 \pm \dfrac{0.118}{1.3}} = \begin{cases} 0.092\ \text{pF} \\ 0.11\ \text{pF} \end{cases}$$

Example 8.4

Capacitive humidity sensors employ a polymer or oxide film as the dielectric between the capacitor plates. One of the plates is porous so that the film is exposed to

the atmosphere in which the humidity is being measured, and, as the humidity changes, the water absorbed by the film varies.

Figure 8.14: Capacitive Humidity Sensor (from Rotronic-USA)

Since the low frequency permittivity of water is approximately $80\varepsilon_o$ compared to around 3-10 for many polymers and oxides, it can readily be appreciated that the permittivity of the film will be dependent on the amount of water absorbed by the film, the result is that the sensor's capacitance responds to changes in humidity.

$$C = e\frac{A}{d} \quad \Rightarrow \quad \frac{dC}{dh} = \frac{d}{dh}\left(e\frac{A}{d}\right) = \frac{dC}{de}\frac{de}{dh} = \frac{A}{d}\frac{de}{dh}$$

What is required for this type of sensor is the relation between the permittivity, ε, and the humidity, h. Typical capacitances for existing sensors are between 100 and 500 pF at 50% relative humidity (RH) and 25 °C. Sensitivity is around 0.2 to 0.5 pF per 1% change in RH. Response times for capacitive humidity sensors are in the range of tens of seconds.

8.4 Inductive Sensors

To begin the discussion of inductive sensors, we must first discuss magnetic circuits, Faraday's law, and the idea of inductance.

8.4.1 Magnetic Circuits

A magnetic circuit is formed when an electric current produces a magnetic flux. The Fig. 8.13 shows an inductor that utilizes a high-permeability core. The magnetic circuit model is shown on the right.

The magnetomotive force (mmf), the product of the current and the number of turns ($F_s = Ni$), plays the role that a voltage source would in an electric circuit.

R is the reluctance of the magnetic core and is analogous to resistance in an electric circuit.

The mmf divided by **R** is the magnetic flux (ϕ). One can see that this relation is analogous to Ohm's law and so ϕ is analogous to electric current.

$$F_s = Ni \qquad\qquad F_R = Rf$$

The equations governing a magnetic circuit can be obtained using laws analogous to Kirchoff's Laws.

> *Magnetic KVL*: The sum of **F** about any loop is zero.
> *Magnetic KCL*: The sum of ϕ into any closed region equals sum of ϕ leaving.

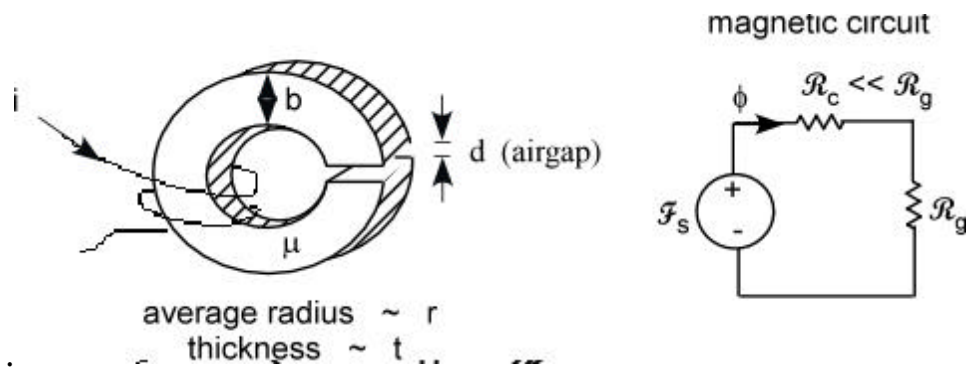$$R = \frac{l}{mA}$$ l~length of flux path, m~permeability, A~cross-sectional area of flux path

Figure 8.15: Magnetic Circuit

In the magnetic circuit above there are two reluctances—that of the core and that of the air gap. They are shown in series since the same magnetic flux flows through both. In the diagram, R_c is shown as much less than R_g since the relative permeability of the core is often much larger than that of air.

From this introduction to magnetic circuits, one should appreciate the fact that magnetic flux will increase as N or i increase (**R** constant) and will decrease as the reluctance of the flux path increases (Ni constant).

8.4.2   Faraday's Law

Faraday's law reads $V = -\frac{df}{dt}$. What does the negative sign mean?   It is Lenz' law and refers to the fact that the EMF produced by the changing magnetic field tends to produce a current, producing a magnetic field, which opposes the original magnetic field.
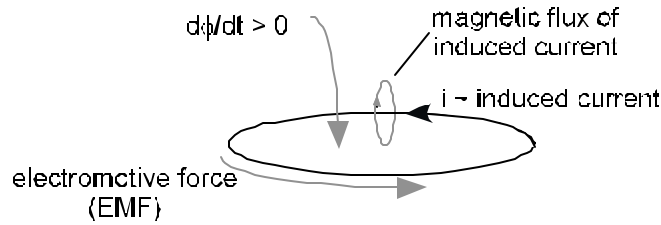
Figure 8.16: Faraday's Law

In Fig. 8.16, a changing magnetic field produces an electromotive force (EMF) in the loop. Positive charges in the loop experience a force in the direction of the EMF which results in a current in the direction indicated. Notice the direction of the induced current and that the magnetic flux produced by the induced current is directed to oppose the original magnetic flux.

Example 8.5

What if a resistance were placed in the above loop? As shown in Fig. 8.14, an induced current would flow. What would be the magnitude of the induced current? It would be the EMF divided by R. If the EMF were given in volts and R given in ohms, then i would be in ampere.
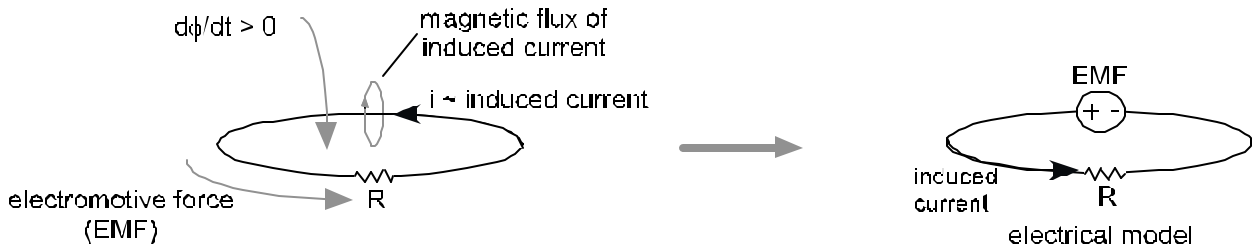
Figure 8.17: Induced Current in One Loop

Note: In order to produce an electromotive force, Faraday's law requires that the flux linking a loop change with time. Considering the loop stationary, this requires the magnetic flux change with time.   The result is that the EMF at each instant of time is proportional to the derivative of the magnetic flux.

If the changing magnetic flux, were produced by a sinusoidal current, the flux would also be sinusoidal (assuming a linear magnetic material). Since the flux is sinusoidal, its derivative would also be sinusoidal. Therefore, for a sinusoidal excitation current, the EMF will be sinusoidal as well.

What would happen if the loop were opened and a voltmeter were used to measure the induced voltage?
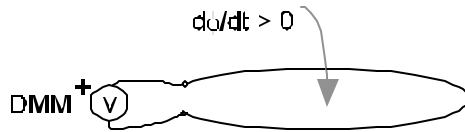
Figure 8.18: Induced Voltage

In Fig. 8.18, since $d\phi/dt > 0$, the voltage measured with the digital multimeter (DMM) will also be greater than zero.

Example 8.6

How would several loops in series with a resistance be modeled? The model will be similar to that shown in Fig. 8.17. There are two main differences. First, the overall EMF induced in the loops will be the product of the EMF induced in one loop and the number of loops.

$$EMF_{n\ loops} = n\ (EMF)_{one\ loop}$$

The second difference is that for n loops, it is more likely that the voltage induced by the induced current itself can no longer be neglected. The EMF produced by the induced current itself is modeled by a self inductance, L.

Note: Recall the causal sequence: $d\phi/dt$ produces an EMF in the coils which results in the induced current.

This current produces its own magnetic field opposing the original field. The magnetic field produced by the induced current results in an EMF which is opposed to the original EMF. This mechanism is modeled by a self inductance.
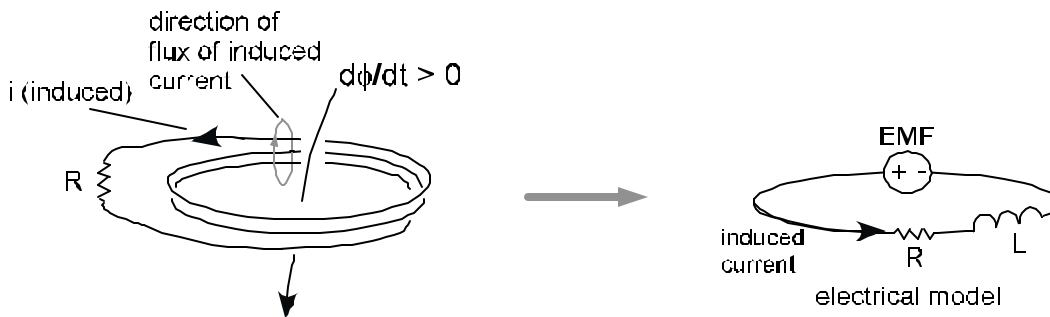

Figure 8.19: Model of Loops Linked by Magnetic Field

8.4.3   The Idea of Inductance

From Faraday's Law, the electromotive force or voltage induced about a loop is equal to the changing flux "linked" by the loop. The sense of the voltage induced is tends to produce current whose flux opposes the original flux.

Consider one loop. Inductance, more specifically self-inductance, refers to a current flowing in the loop which then produces a magnetic field which, in turn, induces an EMF or voltage opposing the original current.

From Ampere's law, the magnetic flux linking the loop is proportional to the current producing it.

$$f \propto i$$

The derivative of the magnetic flux density is therefore proportional to the derivative of the current.

$$\frac{df}{dt} \propto \frac{di}{dt}$$

Then, by Faraday's law, the voltage produced by this changing flux is also proportional to the time variation of the current producing it. The proportionality constant is the self inductance, here referred to as the one-turn self inductance. Lenz' law is accounted for by using the passive sign convention

$$V \propto \frac{di}{dt} \qquad \Rightarrow \qquad V = L_{1T}\frac{di}{dt} \quad \text{(V and i labeled PSC due to Lenz' law)}$$

Example 8.7

Consider an inductor with n turns. What relation will its inductance have to the one-turn inductance of a single loop?

For a given current flowing in the n-turn coil, the flux will be n times the flux produced by one coil working alone. Therefore, the voltage induced across *each turn* of the n-turn coil will be n times the voltage induced in the single turn coil.

$$V_{\text{each turn}} = nL_{1T}\frac{di}{dt}$$

Since there are n turns in the n-turn coil, the overall voltage across the entire n-turn coil will be n times $V_{\text{each turn}}$.

$$V_{\text{n-turn coil}} = n\,V_{\text{each turn}} = n^2 L_{1T}\frac{di}{dt} \qquad \Rightarrow \qquad L = n^2 L_{1T}$$

Example 8.8

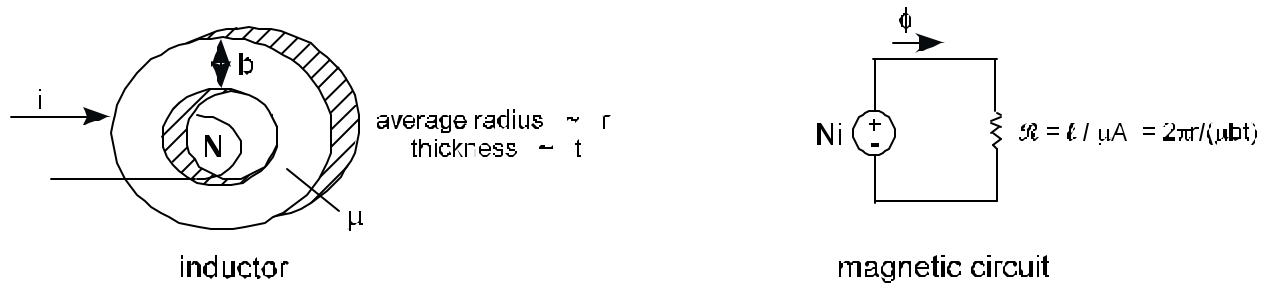Find the inductance of the inductor shown in Fig. 8.20.

Figure 8.20: Self-Inductance

The magnetic flux in the core can readily be determined from the magnetic circuit.

$$f = \frac{Ni}{R} = \frac{N\mu bt}{2pr} i$$

Now using Faraday's law, each turn of the N-turn coil will have a voltage induced (recall that the sign is accounted for in self-inductance by labeling v and i according to the PSC.

$$v_{each\ turn} = \frac{df}{dt} = \frac{N\mu bt}{2pr} \frac{di}{dt}$$

The overall voltage across the coil is just the product of the per-turn voltage and the number of turns.

$$v = N v_{each\ turn} = N\frac{df}{dt} = \frac{N^2 \mu bt}{2pr} \frac{di}{dt} \qquad \Rightarrow \qquad L = \frac{N^2 \mu bt}{2pr}$$

> ***Note:*** The self inductance can be written as $L = \dfrac{N^2}{R} = \dfrac{N^2}{2pr/\mu bt} = \dfrac{N^2 \mu bt}{2pr}$

8.4.4. Mutual Inductance

An EMF or voltage is induced whenever a loop is linked by a changing magnetic field. When this magnetic field is produced by a current in the same loop, the relation between voltage and current is via a self-inductance. When the magnetic field is caused by another current, the relation between induced voltage and this other current is via mutual inductance.
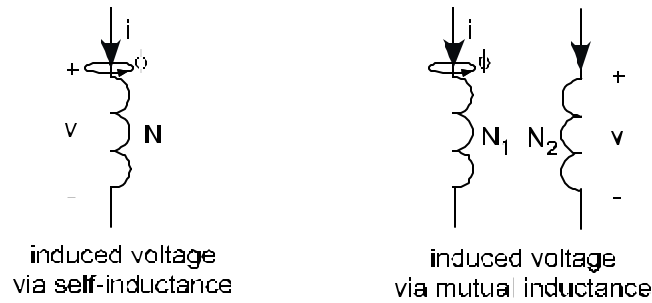
Figure 8.21: Induced Voltage Via Self and Mutual Inductance

The relation for mutually coupled voltages is

$$V = \pm M \, \frac{di}{dt} = \pm \frac{N_1 N_2}{R}\frac{di}{dt} \qquad \Rightarrow \qquad M = \frac{N_1 N_2}{R}$$

Where R is the reluctance of the flux path linking the coils. The sign of the voltage is determined by the relative orientation of the two coils. Faraday's law is used to determine whether the relation involves a positive or negative sign.

This is in contrast to the case of self-inductance, where one can simply rely on the passive sign convention as a guide, since the passive sign convention accounts for Lenz' law.

Mutual inductance can also be determined via the coupling coefficient, k, and the self inductances.

$$k = \frac{f_{21}}{f_1}$$

In this relationship, ϕ_1 is the flux due to a coil 1 carrying a current i_1. ϕ_{21} is the flux from current i_1 that links coil 2. That is ϕ_{21} is some part of ϕ_1.

From this definition, one can see that $k_{max.}=1$.

$$M = k\sqrt{L_1 L_2}$$

Example 8.9

Consider two mutually coupled coils. In this example, the object is to identify dependencies and trends, not to develop precise analytic models. Two coils are pictured—coil 1 and coil 2.
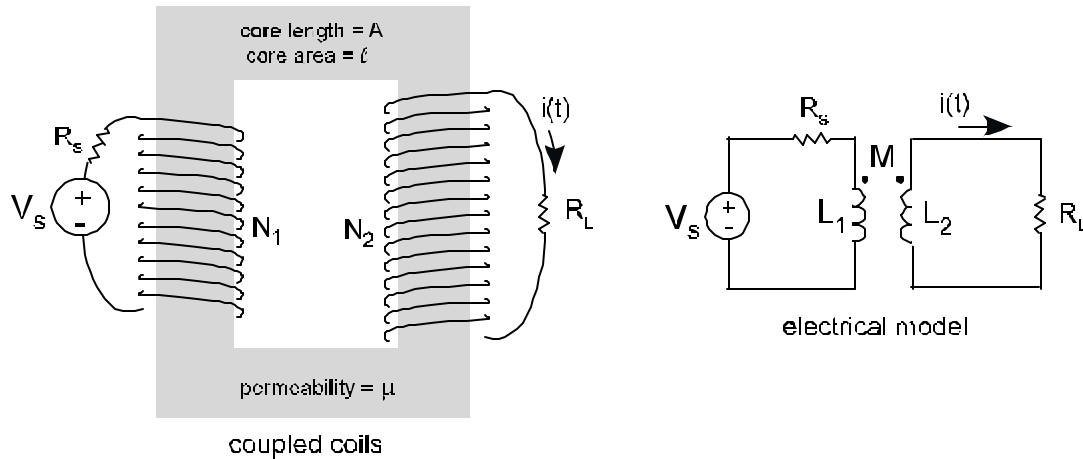
Figure 8.22: Induced Voltage Via Self and Mutual Inductance

The parameters which determine L_1, the self-inductance of coil 1, include the number of turns and the reluctance of the core.

$$L_1 = \frac{N_1^2}{R_1} = \frac{N_1^2}{l/mA}$$

For L_2 and M, similar formulas apply.

$$L_2 = \frac{N_2^2}{R_2} = \frac{N_2^2}{l/mA} \qquad\qquad M = \frac{N_1 N_2}{R_{12}} = \frac{N_1 N_2}{l/mA}$$

Increasing N_1 would increase L_1 and M. Increasing N_2 would increase L_2 and M. Increasing the core length, I, would decrease L_1, L_2, and M. Increasing the core permeability, μ, would increase L_1, L_2, and M. Increasing the core area, A, would increase L_1, L_2, and M.

Note: In this example, both self and mutual inductances share the same flux path. This need not be the case. Actually, the flux paths, even in this example, would not be the same. Not all the magnetic flux would be confined to the core. Some of the flux would "leak" out. The result would be the R_1 would actually be smaller than that of R_{12} since there would be additional paths in parallel with that defined by the core. In this example, the assumption is that this "leakage flux" is small compared to the flux in the core. This assumption is valid for sufficiently small core reluctance.

8.4.5   Using Inductive Sensors

The above discussion of magnetic circuits, Faraday's law, and inductance provides a physical understanding for use in the analysis and application of inductive sensors.

Many magnetic sensors use the reluctance as the parameter with which to couple the quantity being measured to inductance.

Example 8.9

Consider a variable reluctance sensor used to measure displacement.   In particular, since the inductance varies inversely with the reluctance of the magnetic circuit and since the reluctance will increase as the air gap increases, using the air gap to measure displacement can readily be done.
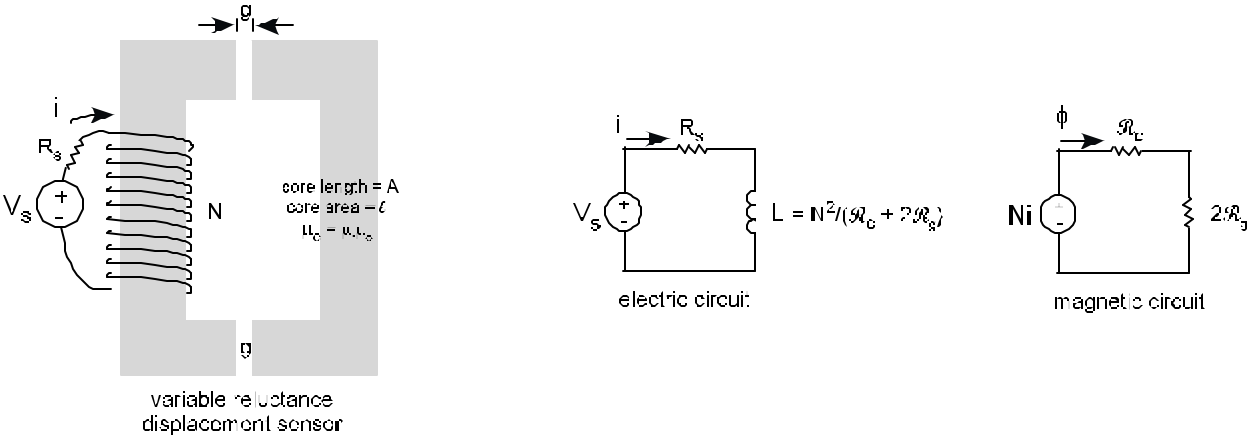
A variable reluctance sensor can be used to measure displacement.

Figure 8.23: Variable Reluctance Displacement Sensor

The inductance of the inductor shown at the left of Fig. 8.23 can be calculated from the path reluctance.

$$L = \frac{N^2}{R_c + 2R_g} = \frac{N^2}{\dfrac{l}{\mu_r \mu_0 A} + 2\dfrac{g}{\mu_0 A}}$$

$$L \cong \frac{N^2 \mu_0 A}{2g} \quad \text{for } \frac{l}{\mu_r} << g$$

This approximation is often valid since μ_r, the relative permeability of the core, is typically in the thousands.   Under this approximation, the change in inductance per change in air gap can be calculated.

$$\frac{dL}{dg} \cong -\frac{N^2 \mu_0 A}{2g^2}$$

The linear variable differential transformer (LVDT) is a widely used inductive displacement sensor. The LVDT is constructed of two coils wound on a common core. To one winding, an AC voltage (V_s in Fig. 8.24) is applied. A voltage is induced across the second winding via mutual coupling. The second winding is divided in two parts and two voltages with opposing polarities are taken from it.
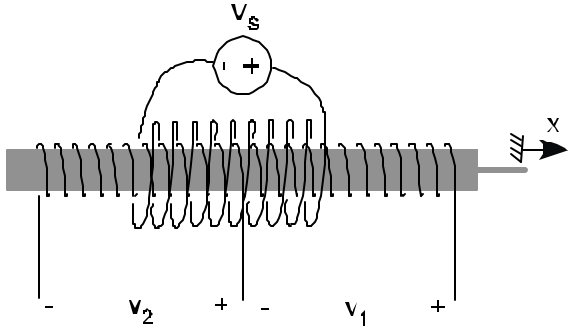

Figure 8.24: Linear Variable Displacement Transformer

The output voltage of the LVDT is $v_{out} = v_1 - v_2$, a measure of the position of x. The principle of the LVDT is that a highly permeable core moves with x which varies the coupling between the primary coil and that between the two sections of the second winding (the sections have v_1 and v_2 across them).

The LVDT is constructed so that the mutual coupling between the primary coil and section v_1 is equal to that of section v_2 for x=0; for x>0, the coupling to v_1 is greater than that to v_2; for x<0, the coupling to v_1 is less than that to v_2; In steady-state result is that, for x=0, $v_1 = v_2$; for x > 0, $v_1 > v_2$; and for x<0, $v_1 < v_2$.

Example 8.10

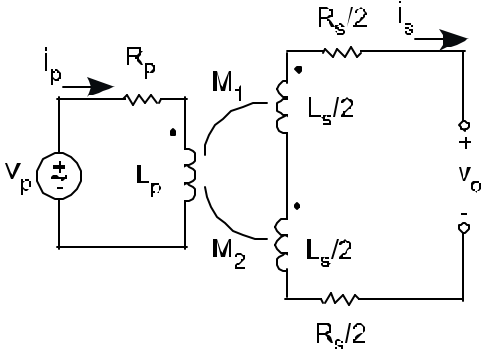Consider the LVDT model shown in Fig. 8.25.

Figure 8.25: LVDT Circuit Model

This model will be analyzed for the common case in which the current drawn from the LVDT is negligible so that the voltage drops across $L_s/2$ and $R_s/2$ can be neglected

and the back EMF into the primary circuit from the secondary current can also be neglected.

Using s-domain analysis and assuming i_s is negligible, the resulting system equations can be readily found.

$$I_p = \frac{V_p}{R_p + sL_p} \qquad\qquad V_o = (M_1 - M_2)sI_p$$

$$V_o = \frac{(M_1 - M_2)s}{R_p + sL_p} V_p$$

In sinusoidal steady-state, for $v_p = V \cos \omega t$.

$$V_o = \frac{|M_1 - M_2|}{\sqrt{R_p^2 + (wL_p)^2}} \cos\left[wt + 90° - \tan^{-1}\left(\frac{wL_p}{R_p}\right) \right] \qquad\text{for } M_1 > M_2$$

$$V_o = \frac{|M_1 - M_2|}{\sqrt{R_p^2 + (wL_p)^2}} \sin\left[wt - \tan^{-1}\left(\frac{wL_p}{R_p}\right) \right] \qquad\text{for } M_1 > M_2$$

In general,

$$V_o = \begin{cases} \dfrac{|M_1 - M_2|}{\sqrt{R_p^2 + (wL_p)^2}} \sin\left[wt - \tan^{-1}\left(\dfrac{wL_p}{R_p}\right) \right] & \text{for } M_1 > M_2 \\[4ex] -\dfrac{|M_1 - M_2|}{\sqrt{R_p^2 + (wL_p)^2}} \sin\left[wt - \tan^{-1}\left(\dfrac{wL_p}{R_p}\right) \right] & \text{for } M_1 < M_2 \end{cases}$$

Notice that the magnitude of the displacement can be found from the amplitude of the output. Whether x is greater or less than zero can be obtained from the sign. Signal conditioning for this type of LVDT often includes the use of a low-pass filter to extract the magnitude of the output (to determine the magnitude of x) and phase-sensitive circuitry (to determine the sign of x).

Another type of LVDT gives a DC voltage which increases with displacement of the LVDT core.
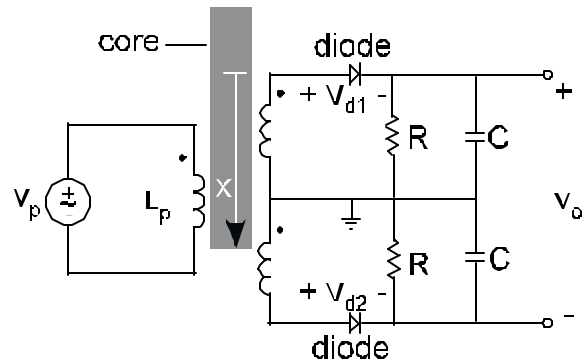
Figure 8.26: DC output LVDT Circuit Model

The secondary coil is center tapped and tied to common. The diodes allow current to flow only when the voltages shown are positive (about 0.7 V is sufficient for silicon diodes.). The RC circuits serve to reduce the ripple in the DC output voltage, v_o.

8.5 Other Sensor Types

There are many other physical principles that can be used to measure physical quantities. If a physical principle can be identified which can be used to establish a relation between two quantities, the principle can be used as the basis for a sensor. Some properties that determine whether the sensor is suitable for its intended purpose is its range, robustness, cost, stability, accuracy, sensitivity, and linearity.

8.5.1   Piezoelectric and Piezoresistive Sensors

In piezoelectric materials—quartz, zinc oxide, and lead zirconate titanate are examples—electrical polarization and strain are coupled. That is, if a piezoelectric crystal is strained, the crystal becomes polarized. This polarization charge may be used as an analog to the strain. Piezoelectric sensors can source very little current so that the required signal conditioning often includes the use of a charge amplifier. Piezoelectric crystals and films have very desirable characteristics including a large frequency range and a wide dynamic range.

The effect is reversible. This allows piezoelectric actuators where the input to the sensor is a voltage, and the output is mechanical displacement.

In piezoresistive material, a material's conductivity (the inverse of resistivity, $\sigma = 1/\rho$) is coupled to strain. Silicon is a widely used piezoresistive material, particularly in microsensors. The mechanism for piezoresistivity in silicon is that the mobility of the charge carriers is affected by strain.

A common silicon microsensor is the piezoresistive pressure sensor. An enclosed volume is formed in which part of the enclosure is a thin silicon membrane. As the pressure difference across this membrane varies the membrane will experience

changes in strain. By doping regions along the rim of the membrane, piezoresistors are placed in regions of greatest strain. The sensor's input is Δp and its output is ΔR. Typical signal conditioning involves using a Wheatstone bridge, amplification, and filtering.
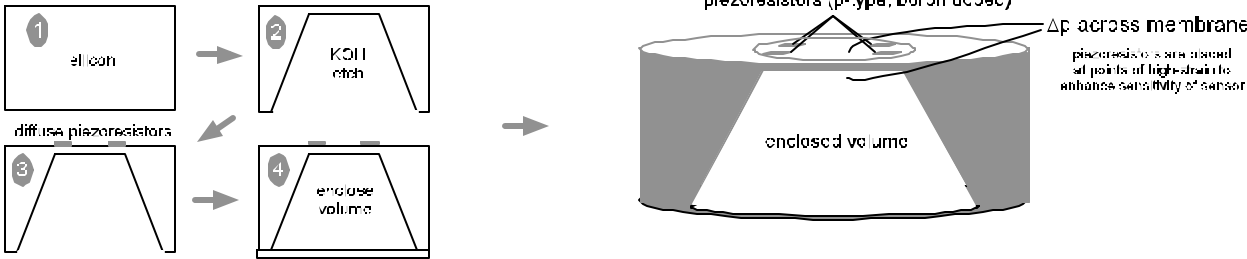

Figure 8.27: Piezoresistance-Based Pressure Sensor

8.5.2   Hall Effect Sensors

The Hall effect results from the Lorentz force which gives relates the electromagnetic field to the force on a charged particle.

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$$

F = force on particle (N)

q = charge on particle (C)

$\mathbf{v}$ = velocity of particle (m/s)

$\mathbf{E}$ = electric field (V/m)

$\mathbf{B}$ = magnetic flux density (Wb/m^2)

The Hall effect is used the measure magnetic fields. Since magnetic fields surround currents, the Hall Effect is used in many current sensors.
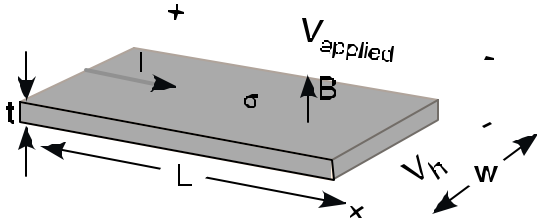
Figure 8.28: Hall Effect

From Ohm's law, $V_{applied} = \dfrac{L}{s\,wt}I = \dfrac{L}{s\,wt}Jwt = \dfrac{L}{s}\,r_v v$

J = current density (A/m^2)

r_v = charge density (C/m^3) .

v = drift velocity (m/s)

Since there is no current in the transverse direction, one can see from the Lorentz force relation that there must be a transverse electric field to balance the magnetic force. This field is referred to as the "Hall field."   The product of the Hall field and the width, w, gives the Hall voltage, V_h.

$$V_{applied} = \frac{L}{s}\, r_v\, \frac{E_h}{B} = \frac{Lwr_v}{s}\, \frac{V_h}{B}$$

As mentioned above, the Hall Effect is widely utilized in current sensors. The only adjustment needed is to calibrate the sensor with values of current rather than magnetic field.

8.5.3 Thermoelectric Sensors

The thermocouple is a widely used temperature sensor that uses an effect first observed by Thomas Seebeck, namely that a voltage—a Seebeck potential—appears across a junction of two dissimilar metals.
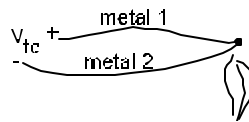

Figure 8.29: Thermocouple

There are several different types of thermocouples (different pairs of metals) available. They are designated with letters—for example K, E, J, N, T, B, R, and S— each with different ranges, stabilities, and sensitivities. See Design Example 2 for more detailed information.
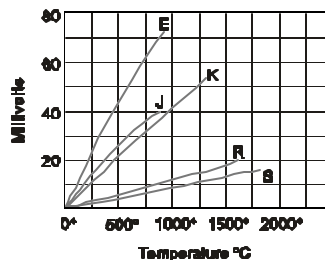
Figure 8.30: Thermocouple Curves (adapted from Omega Engineering)

8.5.4 Optical Sensors

Optical sensors are placed between the optical source and detector. Optical sensors function by the parameter under measurement affecting in some way the characteristics of the transmission path between source and detector. A partial list of parameters which optical sensors can measure include displacement, position, magnetic field, electric current, voltage, pressure, temperature, chemical identification, and fluid flow. Like other types of sensors, the parameter under measurement either changes the geometry or the physical properties of the transmission path between the optical source and detector.

Optical sensors are largely immune to electromagnetic interference—a significant advantage over electric and magnetic-based sensors. Optical sensors include simple optical encoders used to measure shaft position and/or speed to powerful interferometer-based instruments.

Example 8.11

One simple type of optical sensor can be used to sense the level of fluids and is based on total internal reflection at a dielectric interface. Consider a ray incident on a plane dielectric interface.
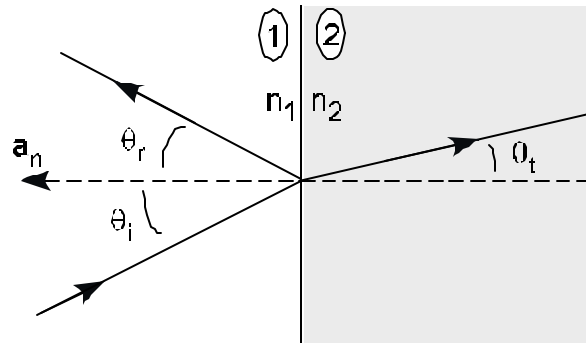

Figure 8.31: Reflection and Refraction at a Dielectric Interface

The law of reflection gives $\theta_i = \theta_r$. The law of refraction can be written as Snell's law, $n_1 \sin\theta_i = n_2 \sin \theta_t$, where n_1 and n_2 are the refractive indices of the two materials.

Notice that, if the speed of propagation is less in region 2 than region 1, the ray will refract toward the normal ($\theta_i < \theta_t$). On the other hand, if the speed of propagation is greater in region 2 than region 1, the ray will refract away from the normal ($\theta_i > \theta_t$).

Consider the second case, where the speed of propagation is less in region 2 than region 1, and $\theta_i < \theta_t$. This relation holds until, at some critical angle, $\theta_i = \theta_c$, the angle of the transmitted ray is 90°. This is the onset of *total internal reflection*. Total internal reflection is a very important phenomenon. It is by total internal reflection, for example, that optical fibers are able to guide beams of light.

Total internal reflection can be readily used to determine whether a fiber is in contact with a fluid or not as illustrated in Fig. 8.32.
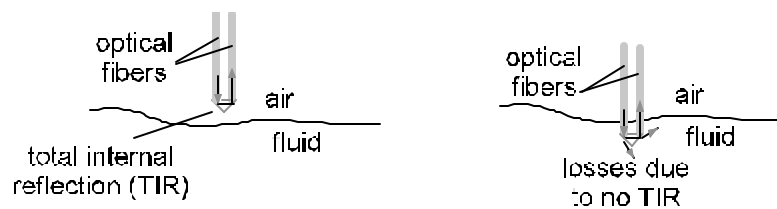

Figure 8.32: Fluid Level Sensor using Total Internal Reflection

Example 8.12

One versatile type of optical sensor is based on the *Mach-Zehnder Interferometer*. In spirit, the interferometer is analogous to bridge circuits in electrical circuits. One arm serves as a reference optical path. Another arm serves as the sensing path.
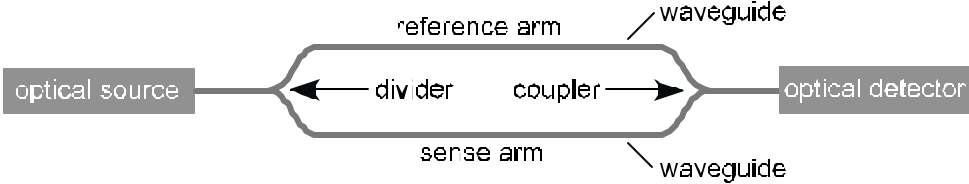

Figure 8.33: Mach-Zehnder Optical Sensor

The two optical signals, the reference and sensing signal, combine to form the input for the optical detector.

Consider the amplitude of the waves entering the reference and sense arms, after being split by the divider are A cos ωt.   Consider the case where the sense arm's optical path length is longer than that of the reference arm. The signal at the detector is the sum of the reference and sense signals.

$$\text{signal} = A \cos(wt - q) + A \cos(wt - q - \Delta q) \quad \begin{cases} A \cos(wt - q) & \text{from reference arm} \\ A \cos(wt - q - \Delta q) & \text{from sense arm} \end{cases}$$

$$\text{signal} = 2 \cos \frac{\Delta q}{2} \cos (wt - q - \frac{\Delta q}{2})$$

The difference in the optical path length directly affects the amplitude of the detected signal.   This scheme could be used, for example, to sense the strength of an electric field. By using an electro-optic material such as $LiNbO_3$, the Pockel's effect could be used. The Pockel's effect refers to a first-order effect whereby the refractive index varies with electric field.

Example 8.13

The Fabry-Perot Interferometer can be used to precisely measure displacement. Combined with appropriate structures, they can be used to measure pressure, force, or acceleration.   Several manufactures sell strain sensors which utilize Fabry-Perot Interferometers.
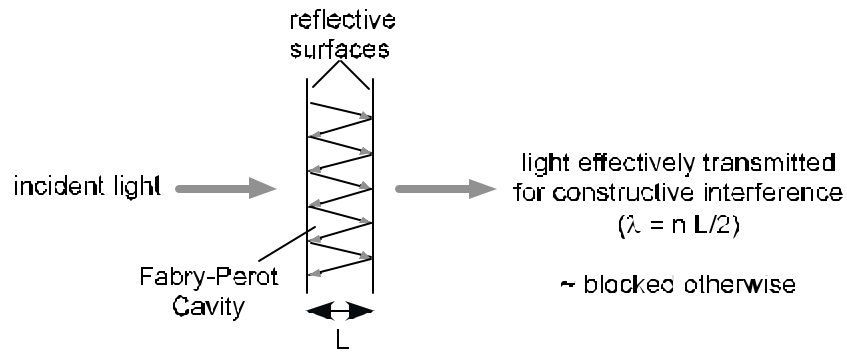
reflective
surfaces

incident light →

light effectively transmitted
for constructive interference
$(\lambda = n\, L/2)$

~ blocked otherwise

Fabry-Perot
Cavity

L

Figure 8.34: Fabry-Perot Interferometer

In these sensors, the Fabry-Perot cavity is placed into the fiber. Longitudinal strain across the sensor changes the cavity length which dramatically affects the intensity of the transmitted light.
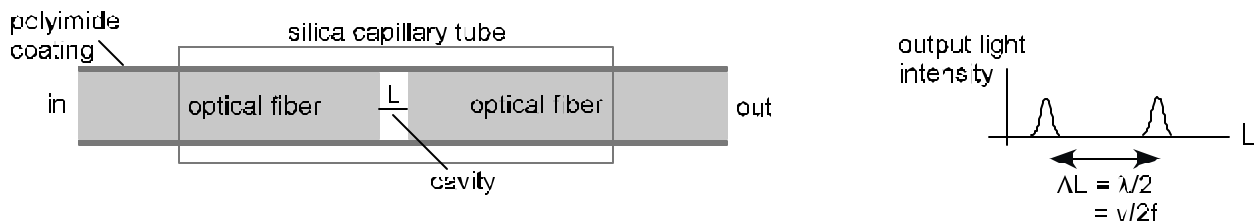
polyimide
coating

silica capillary tube

output light
intensity

in

optical fiber

L

optical fiber

out

cavity

L

$\Lambda L = \lambda/2$
$= v/2f$

Figure 8.35: Fiber Optic Strain Gage (adapted from NASA TP2000-210639)

8.5.5   Chemical Sensors

Chemical sensors selectively respond in the presence of particular chemical species. Although some chemical sensors can be used repeatedly, many cannot. Those that cannot often have as their mechanism an active element which reacts irreversibly with the substance under test.

Oxygen sensors are widely used in internal combustion engines to control the air-fuel ratio. Relative oxygen content can be measured with sensors having a layer of zirconium dioxide with a thin layer of microporous platinum deposited on both sides. This structure is exposed on one side to the engine exhaust and on the other side to the outside atmosphere.
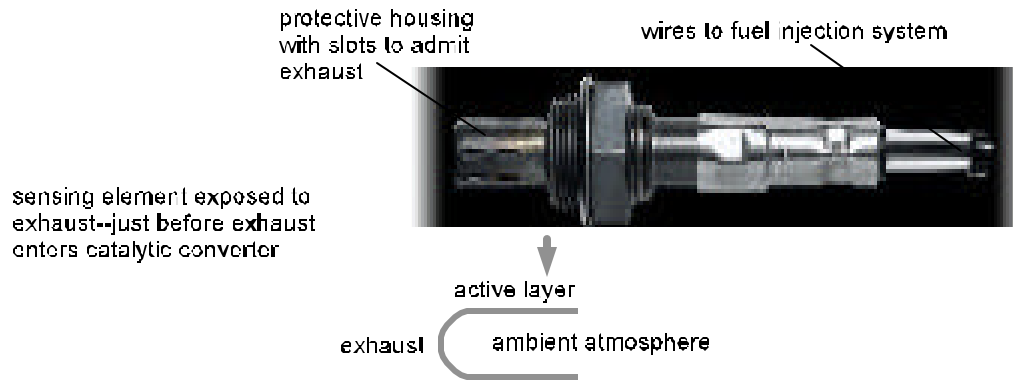
Figure 8.36: Automotive Oxygen Sensor

Platinum has an affinity for oxygen ions, and, at the working voltage of above 600°F, the ZrO_2 layer becomes conductive. A voltage is produced when there is a difference in oxygen content the two surfaces—one exposed to exhaust of the engine and the other to the ambient atmosphere—of the sensor. A voltage of 0.45 V indicates proper air-fuel mixture, a lower voltage indicates too lean an air-fuel mixture, and a higher voltage indicates too rich an air fuel mixture. This information is fed back to the fuel injection system and allows greater efficiencies and lower emissions.

In medicine, the partial pressure of oxygen (PO_2) in blood is measured using the Clark electrode, an electrochemical cell with a platinum cathode and a silver anode.
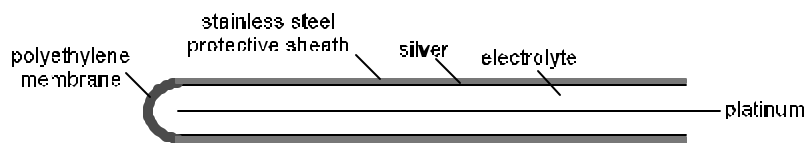
Figure 8.37: Clark Electrode

Optical fibers permit flexible use of spectroscopic measurement techniques. Consider a bundle of optical fibers, the end of each covered with a specific fluorescent dye. This arrangement would allow several measurements to be performed with a single probe.
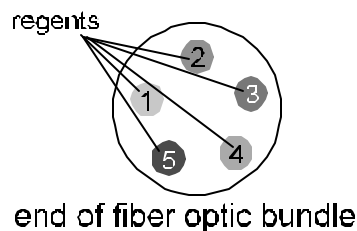

Figure 8.38: Fiber Optic Bundle for Multiple Clinical Measurements

Each optical fiber could both carry the optical excitation to the regent as well has the detected emissions back for analysis. This scheme can provide the platform for a wide variety of biomedical measurements.

8.6 Summary

Engineers and scientists need an understanding of sensors, of their capabilities, characteristics, and limitations. Sensors allow the physical world to be observed or measured. Being able to observe the physical world is a central requirement in most branches of engineering and science. Sensors provide a means of translating the value of the measurand (the parameter being measured) into another form. Resistive sensors translate values of temperature or strain, for example, into values of resistance. As another example, capacitive sensors translate values of the measurand into values of capacitance.

Most sensors depend upon their geometry and material properties. A sensor can often be designed when a well defined correlation between some measurand and some aspect of sensor geometry or material property.

8.7 Computer Tools and Other Resources

MATLAB SIMULINK is a good tool for sensor simulation. Consider the design of the MEMS accelerometer discussed above. Quasi-static operation cannot be assume during the design. Here, the designer works with the complete second-order lumped element model.
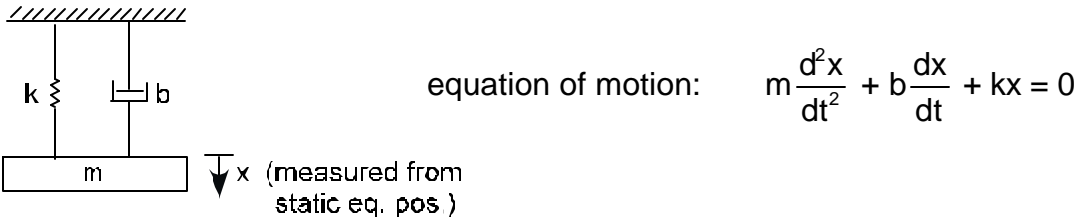

equation of motion: $m\dfrac{d^2x}{dt^2} + b\dfrac{dx}{dt} + kx = 0$

Figure 8.39: Mechanics of MEMS Accelerometer

The Simulink simulation diagram is shown in Fig. 8.38.
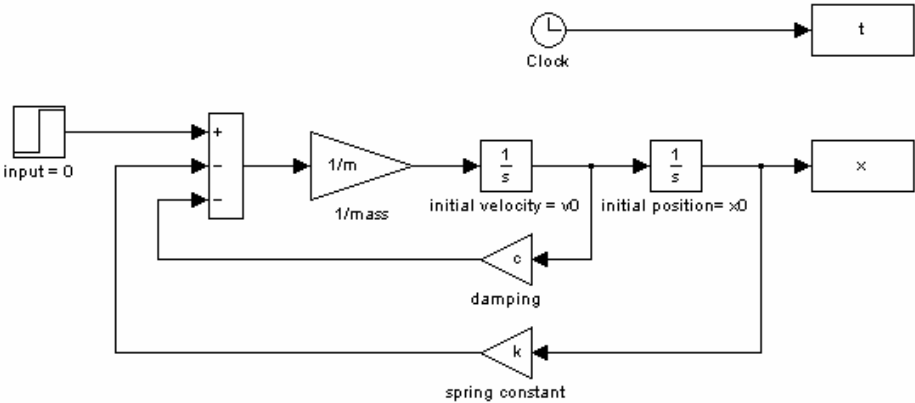

Figure 8.40: MEMS Accelerometer Simulink Simulation

The designer may find it convenient to develop the model so that changes are easily implemented. For this purpose, an m-file is used to drive the model. The parameters—mass, damping, spring constant, initial velocity—are easily changed in this manner. In this example, the damping, c, is varied.

```
% M-file to run Simulink model      t1 = t; % save for plot          c=8e-6;
                                     x1 = x; % save for plot          sim('sensor',[0,3.5e-5]);
clear;                                                                t4 = t;
v0=30;                               c=2e-6;                          x4 = x;
x0=0;                                sim('sensor',[0,3.5e-5]);
t0=1;                                t2 = t;                          % plot results
A=0;                                 x2 = x;                          plot(t1,x1,'k-',t2,x2,'k:',t3,x3,'k--',t4,x4,'k-.')
m=1e-12;                                                              xlabel('time (s)')
k=0.8;                               c=4e-6;                          ylabel('x(t) (m)')
                                     sim('sensor',[0,3.5e-5]);
% vary damping and save results      t3 = t;                          %last line
c=8e-7;                              x3 = x;
sim('sensor',[0,3.5e-5]);
```

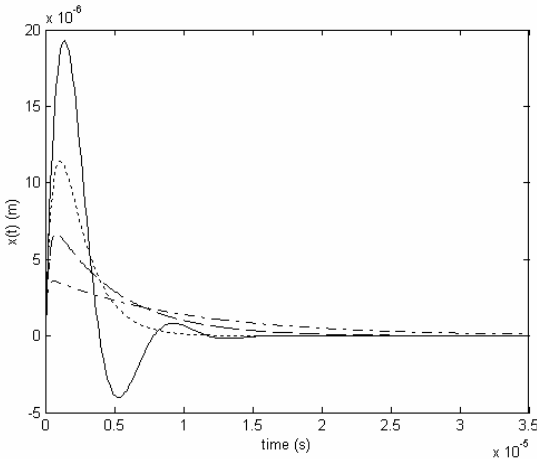**Figure 8.41: m-file Driver for Simulink Model**

Figure 8.42: Simulation Results

8.8 Design Examples

The strategy chosen in this chapter has been to give a broad overview of types of sensors, together with a more than superficial discussion and analysis of a few specific sensors.   These design examples build on this strategy.

8.8.1 Design Example 1: Experiment Using Metallic-Film Strain Gages in a Force Transducer

Metallic-film strain gages are resistors, the resistance of which varies with strain. Constantan is a popular metal since the resistivity of constantan varies only slightly with temperature.
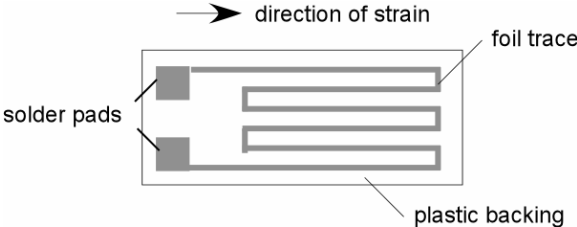
Figure 8.42: Metallic Film Strain Gage

For a resistor of uniform cross section A, of length L, and of resistivity ρ,

$$R = \frac{rL}{A}.$$

Taking differentials of the relation for resistance,

$$\frac{dR}{R} = \frac{dr}{r} + \frac{dL}{L} - \frac{dA}{A}.$$

Assuming a circular cross section (the same result is obtained for a general cross section).

$$A = p\,\frac{D^2}{4}, \qquad \frac{dA}{A} = 2\,\frac{dD}{D}$$

The axial strain is related to the transverse strain via ν, Poisson's ratio, a material property.

$$e_t = -n\ e_a$$

This permits us to write $\frac{dD}{D} = -n\,\frac{dL}{L} = -n\ e_a$. Substituting these relationships,

$$\frac{dR}{R} = \frac{dr}{r} + e_a + 2\,n\ e_a.$$

Strain gages are characterized by the strain gage factor, S.

$$S = \frac{dR/R}{e_a} = 1 + 2n + \frac{dr/r}{e_a}$$

For metallic strain gages, the gage factor is typically around 2-6, usually closer to 2. Semiconductor strain gages can have gage factors higher than 100. For metallic gages, strains as high as 0.04 (4% elongation) are measured routinely. For semiconductor gages, the strain is limited to about 0.003 (0.3% elongation).   Common terminology is in terms of "microstrain." For example, a strain of 0.04 is 40000 microstrain.

In this design example, the design and calibration of a force transducer is outlined in, what is hoped, sufficient detail to allow it to be followed. In Fig. 34 below, four 120 Ω metallic film strain gages are installed on an aluminum cantilever beam.
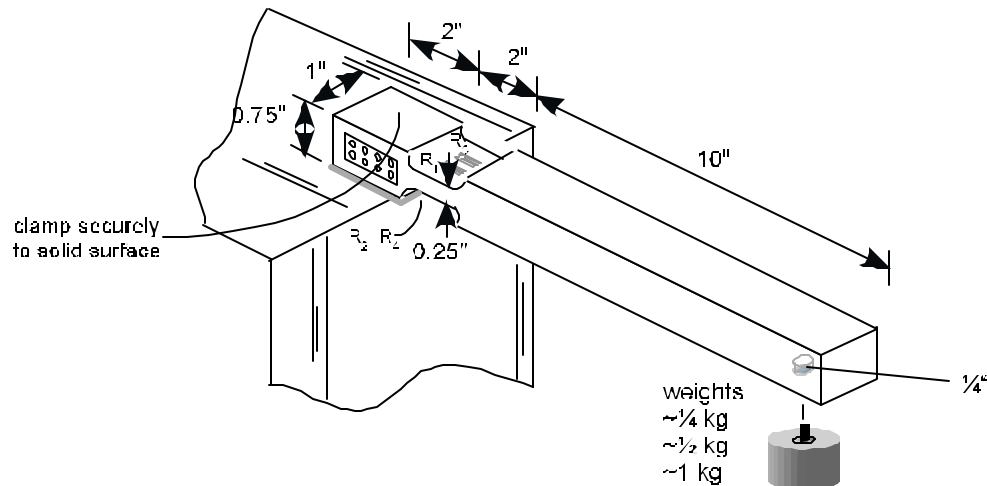
Figure 8.44: Cantilever Load Cell

The strain gage resistors are arranged in a four arm active Wheatstone bridge. The voltage output of the bridge, V_b, is then amplified to produce the output of the measurement system, V_o. For more information on signal conditioning using the Wheatstone bridge, see chapter 9.
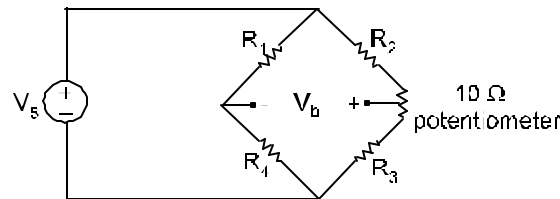

Figure 8.45: Wheatstone Bridge (DR ® DV)

Note: The potentiometer, a variable resistance, allows the instrument to be calibrated. Assuming a linear instrument, at least two points (preferable several more) are needed for calibration.

1. The first point will establish the zero or null point. Let V_s be some value between 5 Vdc and 25 Vdc. Adjust potentiometer so that $V_b = 0$ when no weights are connected to load cell.
2. Connect a relatively large weight to cell (perhaps 5 lbs). This establishes a second point on the instrument's input-output curve (weight-voltage curve).

This is the first stage of calibration. *Adding an amplifier (see below) will require the instrument to be recalibrated after connecting the amplifier to establish the input-output curve (the input being the weight and the output now V_0).*

The voltage output by the bridge circuit is typically a few millivolts. An amplifier is often used to obtain a larger voltage from the measurement system.
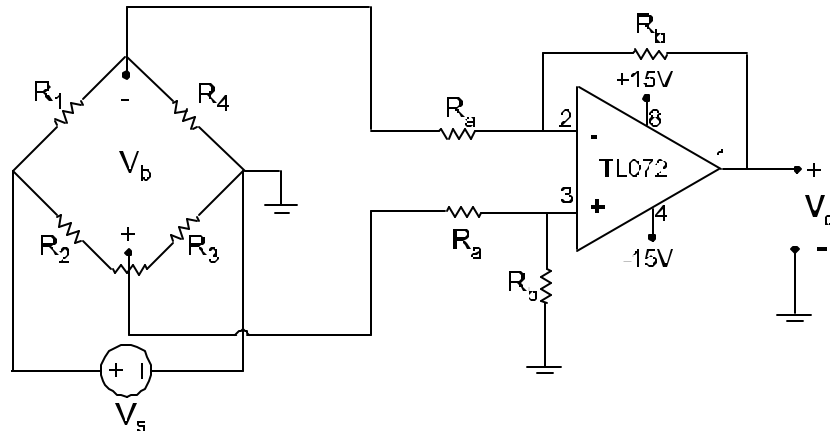
Figure 8.46: Signal Conditioning

$$V_o = \frac{R_b}{R_a + \dfrac{R}{2}} \frac{\Delta R}{R} V_s = \frac{R_b}{R_a + \dfrac{R}{2}} S e V_s, \text{ where R is the nominal strain gage resistance,}$$

ΔR is the magnitude of the strain gage resistance change, S is strain gage factor, and ε is the strain.

8.8.2 Design Example 2: Thermocouples (adapted from Omega® Engineering)

In 1821, Thomas Seebeck discovered that a current flows when two dissimilar metals, joined at both ends, is heated at one end.
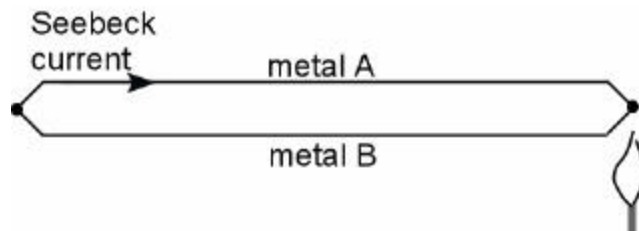
Figure 8.47: Seebeck Current

If the circuit is opened, a net voltage, the Seebeck voltage, is present. This voltage is a function of the two metals and the junction temperature.
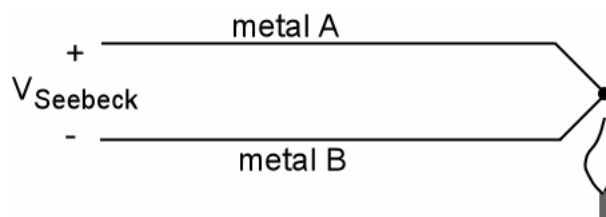
Figure 8.48: Seebeck Voltage

The voltage-temperature relationship is not linear for thermocouples, but the functional relationships are known and tabulated for common thermocouples.

	TYPE E	TYPE J	TYPE K	TYPE R	TYPE S	TYPE T
	Ni-10% Cr (+) -vs- Constantan -100°C to 1000°C ±0.5°C, 9th order	Fe (+) -vs- Constantan 0°C to 760°C ±0.1°C, 5th order	Ni-10% Cr (+) -vs- Ni-5% AlSi 0°C to 1370°C ±0.7°C, 8th order	Pt-13% Rh (+) -vs- Pt 0°C to 1000°C ±0.5°C, 8th order	Pt-10% Rh (+) -vs- Pt 0°C to 1750°C ±1°C, 9th order	Cu (+) -vs- Constantan -160°C to 400°C ±0.5°C, 7th order
a_0	0.104967248	-0.048868252	0.226584602	0.263632917	0.927763167	0.100860910
a_1	17189.45282	19873.14503	24152.10900	179075.491	169526.5150	25727.94369
a_2	-282639.0850	-218614.5353	67233.4248	-48840341.37	-31568363.94	-767345.8295
a_3	12695339.5	11569199.78	2210340.682	1.90002 E+10	8990730663	78025595.81
a_4	-448703084.6	-264917531.4	-860963914.9	-4.82704 E+12	-1.63565 E+12	-9247486589
a_5	1.10866 E+10	2018441314	4.83506 E+10	7.62091 E+14	1.88027 E+14	6.97688 E+11
a_6	-1.76807 E+11		-1.18452 E+12	-7.20026 E+16	-1.37241 E+16	-2.66192 E+13
a_7	1.71842 E+12		1.38690 E+13	3.71496 E+18	6.17501 E+17	3.94078 E+14
a_8	-9.19278 E+12		-6.33708 E+13	-8.03104 E+19	-1.56105 E+19	
a_9	2.06132 E+13				1.69535 E+20	

Table 4: Polynomial coefficients from the NATIONAL BUREAU OF STANDARDS

Thermocouple temperature-voltage relation (assumes reference junction at 0°C):

$$T = a_0 + a_1 V_{tc} + a_2 (V_{tc})^2 + \ldots$$

When measuring the thermocouple voltage, one must be aware that the act of measuring can create additional thermoelectric junctions.

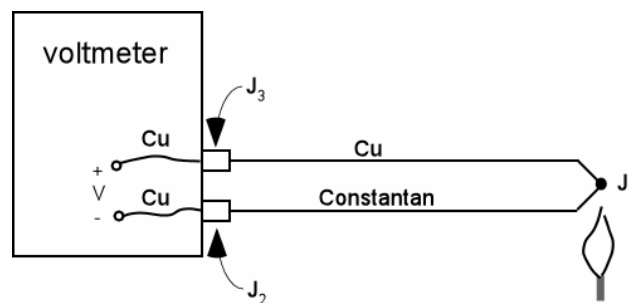Consider measuring the voltage across a Cu-Constantan (type T) thermocouple

Figure 8.49: Type-T Thermocouple

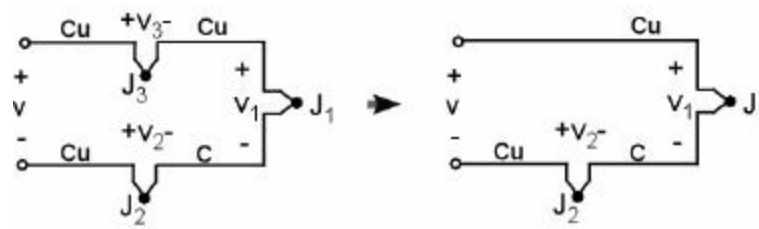Consider the equivalent thermoelectric circuit that describes the system above.

Figure 8.50: Thermoelectric Circuit

To know the temperature at J_1, two things must be known: 1) the type of junction (in this case Cu-Constantan) and 2) the junction voltage, v_1.

In this case, $v_3 = 0$ since the junction is Cu-Cu. But v_2 is Cu-C, and v_1 cannot be determined from v (the voltmeter reading) unless v_2 is known. v_2 can be fixed by placing J_2 in a known temperature.

We cannot determine the temperature at J_1 without knowing the temperature at J_2. This can be accomplished using an ice bath. ($T_{j2} = 0°C$)

$$v = v_1 - v_2 \approx \alpha(t_{j1} - t_{j2}) = \alpha$$
$$[(T_{j1(°C)}+273.15) - (T_{j2(°C)}+273.15)]$$
$$v = \alpha\,(T_{j1(°C)} - T_{j2(°C)}) = \alpha\,(T_{j1(°C)} - 0) = \alpha\,T_{j1}$$
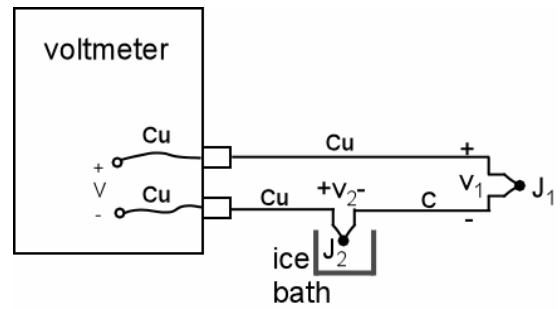
Figure 8.51: Reference Junction I

Now suppose a different type of thermocouple is used, suppose Type J, which is Fe-Constantan.

If J_3 and J_4 are at the same temperature, their junction voltages will cancel and $v = \alpha\,T_{j1}$.
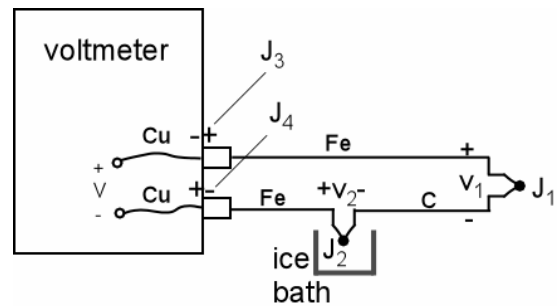

Figure 8.52: Reference Junction II

An isothermal block (high thermal conductivity and low electrical conductivity) can help to insure the junction temperatures remain equal.
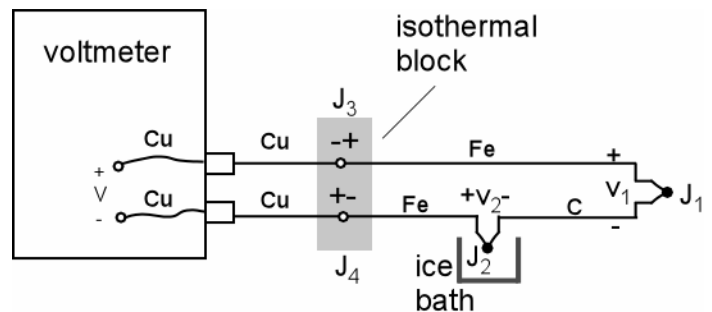

Figure 8.53: Reference Junction III

This circuit still requires two thermocouples, J_1 and J_2. It is possible to eliminate the reference thermocouple. Step one is to eliminate the ice bath.

Replace the ice bath with an isothermal block at temperature T_{ref}.

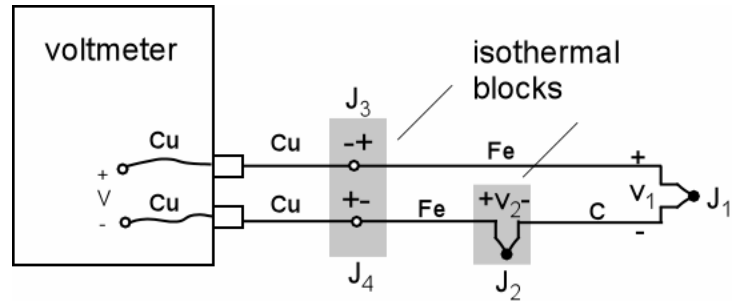$$v = \alpha \, (T_{j1} - T_{ref})$$

Figure 8.54: Eliminating Reference Junction I

Step 2 is to join the isothermal blocks which are at a temperature T_{ref}.

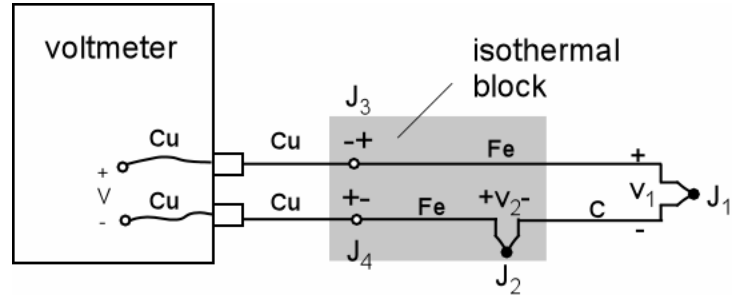$$v = \alpha \, (T_{j1} - T_{ref})$$

Figure 8.55: Eliminating Reference Junction II

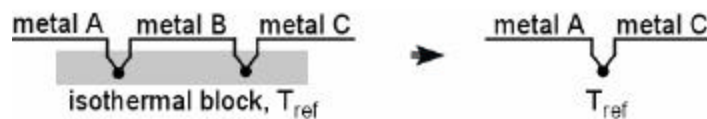Step 3 is to eliminate J_2 using the law of intermediate metals.

Figure 8.56: Law of Intermediate Metals

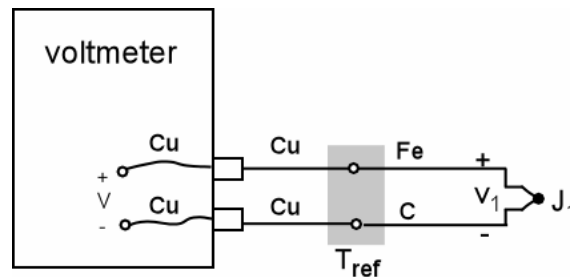$$v = \alpha \, (T_{j1} - T_{ref})$$

Figure 8.57: Reference Junction

To measure T_{ref}, we can use an RTD or some other convenient temperature sensor. This raises the question of why, when another temperature transducer

must be used to measure T_{ref}, does one even bother with the thermocouple in the first place?   Why not just use the other temperature transducer to measure the temperature of interest and be done with it?

There are two primary reasons: temperature range and ruggedness. RTDs can be used to measure temperatures that RTDs, thermistors, and IC sensors simply cannot. They are also very rugged; thermocouples can be welded to a metal part; or they can be clamped under a screw.

Their wide temperature range and ruggedness makes them the workhorses of temperature measurement in a wide variety of applications.

Several thermocouples can be switched so that all use the same isothermal block.

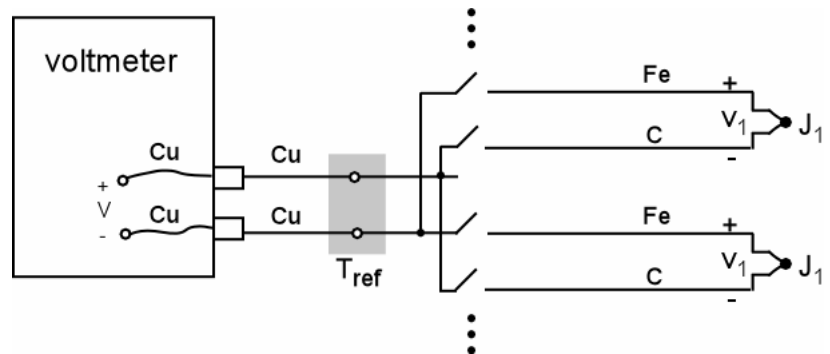This allows one temperature sensor to service many thermocouples.

Figure 8.58: Reference Junction

Type	Characteristics
K	Type K is the present general purpose thermocouple The type K TC is low cost and is available in a wide variety of probes. Thermocouples are available in the -200°C to +1200°C range. Sensitivity is approx 41uV/°C. Although Type K is presently most popular with users, the improved performance of Type N will serve to supplant Type K as the most popular thermocouple.
E	Type E has a high output which makes it well suited to low temperature (cryogenic) use.
J	The old standard. Limited range (-40 to +750°C) makes type J less popular than type K. The main application is with old equipment that can not accept 'modern' thermocouples. J types should not be used above 760°C as an abrupt magnetic transformation will cause permanent decalibration.
N	High stability and resistance to high temperature oxidation makes type N suitable for high temperature measurements without the cost of platinum (B,R,S) types. Designed to be an 'improved' type K, it is becoming more popular.
	Thermocouple types B, R and S are all 'noble' metal thermocouples and exhibit similar characteristics. They are the most stable of all thermocouples, but due to their low sensitivity (approx 10uV/$^{\rho}$C) they are usually only used for high temperature measurement (>300°C).
B	Suited for high temperature measurements up to 1800°C. Type B thermocouples give the same output at 0°C and 42°C. This makes them useless below 50°C.
R	Suited for high temperature measurements up to 1600°C. Low sensitivity (10uV/°C) and high cost makes them unsuitable for general purpose use.
S	Suited for high temperature measurements up to 1600°C. Low sensitivity (10uV/vC) and high cost makes them unsuitable for general purpose use. Due to its high stability type S is used as the standard of calibration for the melting point of gold (1064.43°C).

Table 5: Thermocouple Characteristics (*from Picotech*)