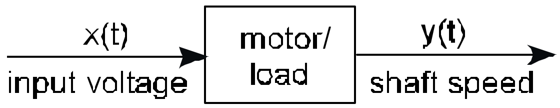
Control Systems

Control systems are used to control parameters such as the position of a cutting tool, the temperature of a chemical bath, or the speed of a motor. Measurement is a critical element in advanced control systems utilizing "feedback."

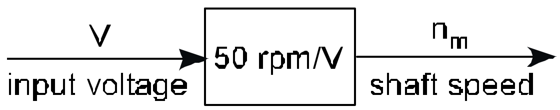
Open-loop control (no feedback)

Consider a system of a motor, together with its load. The input, x(t), is the motor voltage, and the output, y(t), is the shaft speed of the motor.



A control system is a system by which the behavior of the output is controlled by controlling the input—here the shaft speed of the DC motor is controlled by the input motor voltage.

Consider the motor in steady state, with a load such that the input voltage/shaft speed relationship is 50 rpm/volt.



How could the shaft speed be controlled in this case? Simple. Just divide the desired shaft speed in rpm by 50 to obtain the required input voltage in volts.

This is an open-loop control system. It works fine for household fans, etc.

More sophisticated control systems are often needed. The crucial weakness in open-loop systems is that they cannot compensate for changes in the motor/load system. Motor loads vary constantly. Motors age. The 50 rpm/V transfer characteristic is only accurate for a particular load at a particular time.

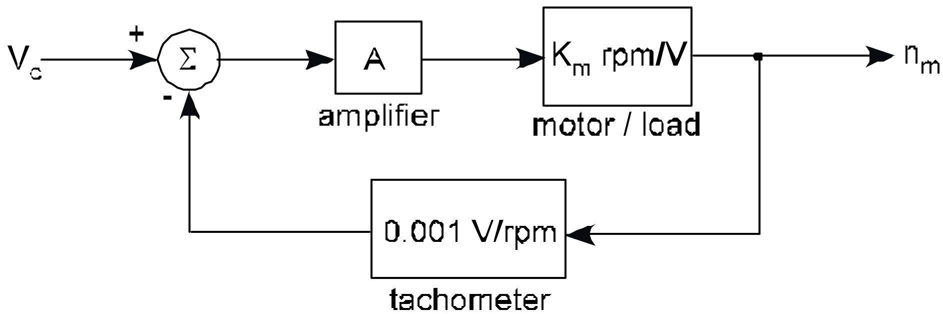
The design of more sophisticated control systems usually uses feedback—that is the output is measured and fed back into the system input. *Accurate measurement of the output is crucial.*

Closed-loop control systems (with feedback)

A measurement of the output shaft speed can be used to design a motor speed control system which is relatively insensitive to changes in motor load—that is, where variations in motor speed are small when compared to changes in motor load.

Suppose the desired motor speed is 2000 rpm regardless of the load to which the motor is attached. Take 2V as the input voltage which is to produce 2000 rpm. (1V produces 1000 rpm, etc.)

For this case a shaft speed transducer (a tachometer) is need that produces 2V when measuring 2000 rpm.



The actual speed which corresponds to 2V will be very close to 2000 rpm **if** AK_m is "large."

$$n_m = \frac{V_c}{\frac{1}{AK_m} + 0.001} = \frac{V_c}{\frac{1}{AK_m} + \frac{1}{1000}} \cong 1000V_c \qquad \text{for } AK_m >> 1000$$

Two key items in this control scheme are

1)accurate measurement of the output shaft speed and

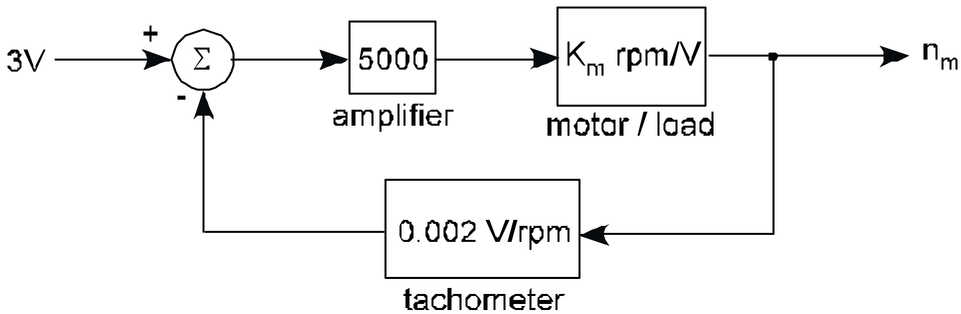
2)a large "forward gain" (the product of A and K_m).

Example

Design a feedback control system for motor/load speed control.
The desired speed is 1500 rpm for a 3V input.

required tachometer: $\frac{1}{500}\ \text{V/rpm}$

amplifier gain: A = 5000

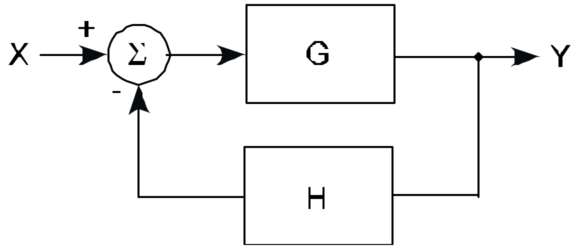


Find the actual motor shaft speed when

i) $K_m = 50$ rpm/V

ii) $K_m = 25$ rpm/V

General Analysis



$G(X - HY) = Y$? $\frac{Y}{X} = \frac{G}{1+GH}$

approximations

$$\frac{Y}{X} = \frac{G}{1 + GH} = \frac{1/H}{1 + 1/GH}$$

$$\frac{Y}{X} \cong \frac{1}{H}\left(1 - \frac{1}{GH}\right) \quad \text{for } \frac{1}{GH} << 1 \qquad \left(\frac{1}{GH} \text{ is the error from the "ideal" of } \frac{1}{H}\right)$$

Example

The system shown below is a system for controlling the temperature of a chemical reaction. An electrically-controlled valve mixes two sources of water to produce water with a temperature of T_1, which then flows through the reactor jacket which brings the chemical solution to the same temperature (T_1=T_2 in steady-state).

An RTD together with a Wheatstone bridge monitors the temperature T_2 and develops a voltage v_2 which is compared with a reference voltage v_x to produce an error voltage v_e which drives the controller whose gain is 1000, which in turn drives the valve.

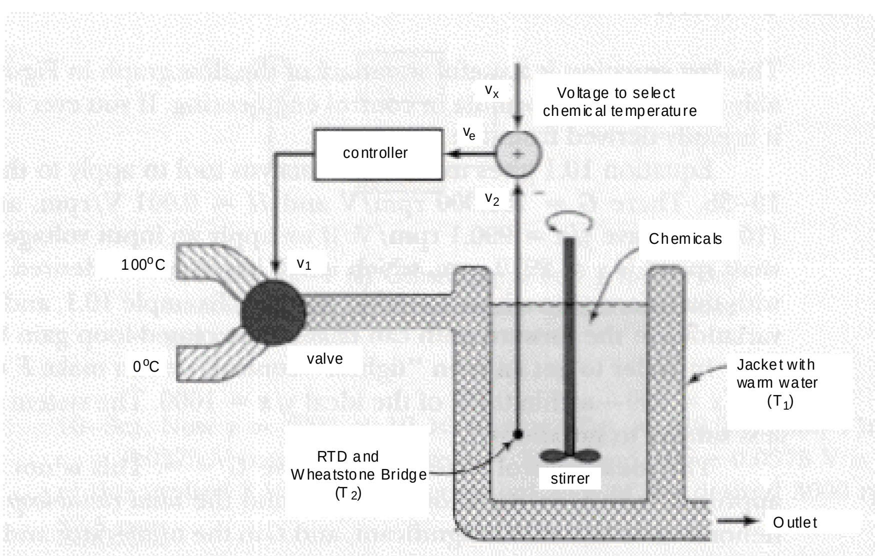
The gain of the valve is such that 0V corresponds to 0°C and 5V corresponds to 100°C. The RTD/Wheatstone combination has a transfer function of 5mV/°C.

i) Draw the closed-loop block diagram that represents the control system.

ii) Determine the ideal closed-loop gain.

iii) Determine the exact closed-loop gain and the percent error in part ii).

iv) Determine a new value of controller gain that would reduce the error in the closed-loop gain to 0.5%.

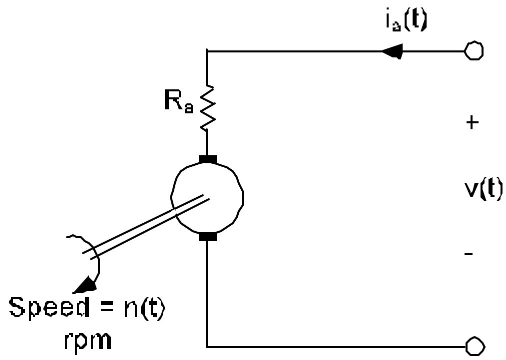


Example (cont.)

Control of dynamic systems

In the previous example, T_1=T_2 in steady-state. Transient behavior was not considered.

When transient behavior is of interest, system dynamics must be included in the analysis. Consider a DC motor/load with a constant field.



Energy storage is typically dominated by that stored in mechanical inertia, and, consequently, the system behaves as a first-order system.

With a step function input, the resulting speed response is exponential
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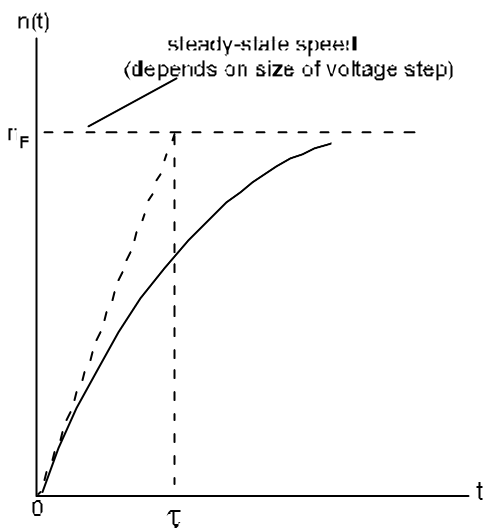
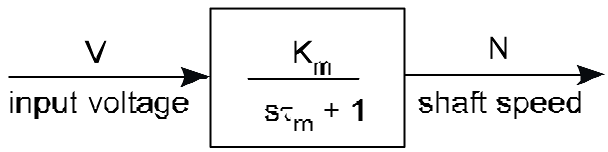


$n(t) = n_F(1 - e^{-t/\tau})$

n_F is the steady-state speed

τ is the motor time constant

In the s-domain this is modeled as:

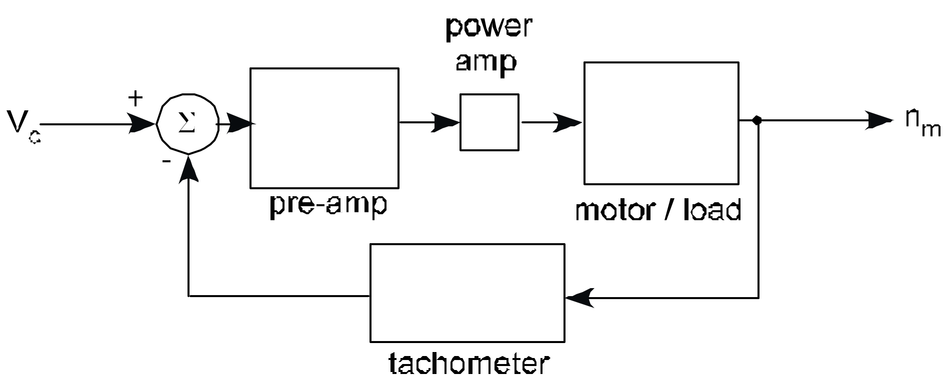


K_m is the static gain coefficient, and τ_m is the motor-load time constant.

Example

The following diagram depicts a motor speed control system in which the pre-amp has a gain of 2000 with a time constant of 200μs, the power amp has a gain of 10, the motor has a gain of 50 rpm/V with a time constant of 200ms, and the tachometer has a gain of 1 mV/rpm.

i) Draw the block diagram that represents the control system in the s-domain.



ii) Determine an expression for the closed-loop gain in polynomial form.

iii) Compare the steady-state value of part ii) with the ideal closed-loop gain.

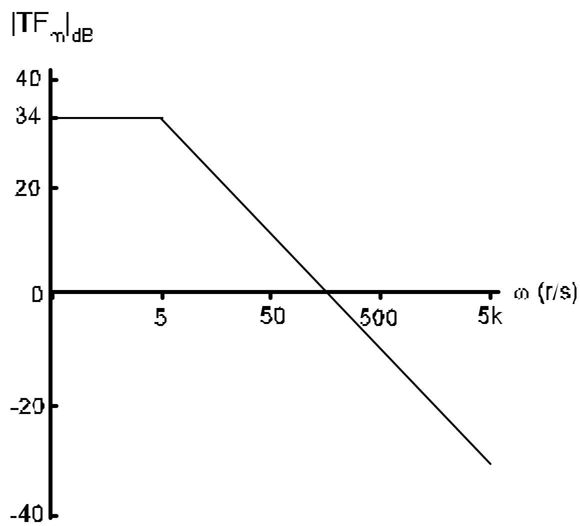
Example (cont.)

Frequency response of dynamic control systems

Consider the motor used in a previous example

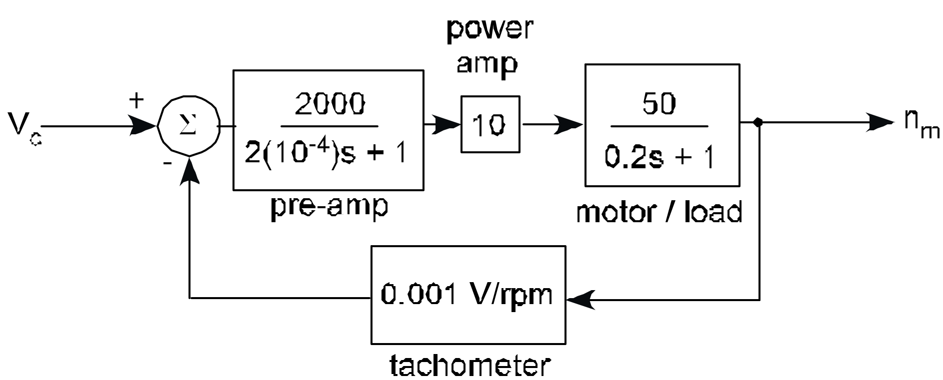
$$TF_{motor} = \frac{50}{1+0.2s}$$

Taking the Bode magnitude plot



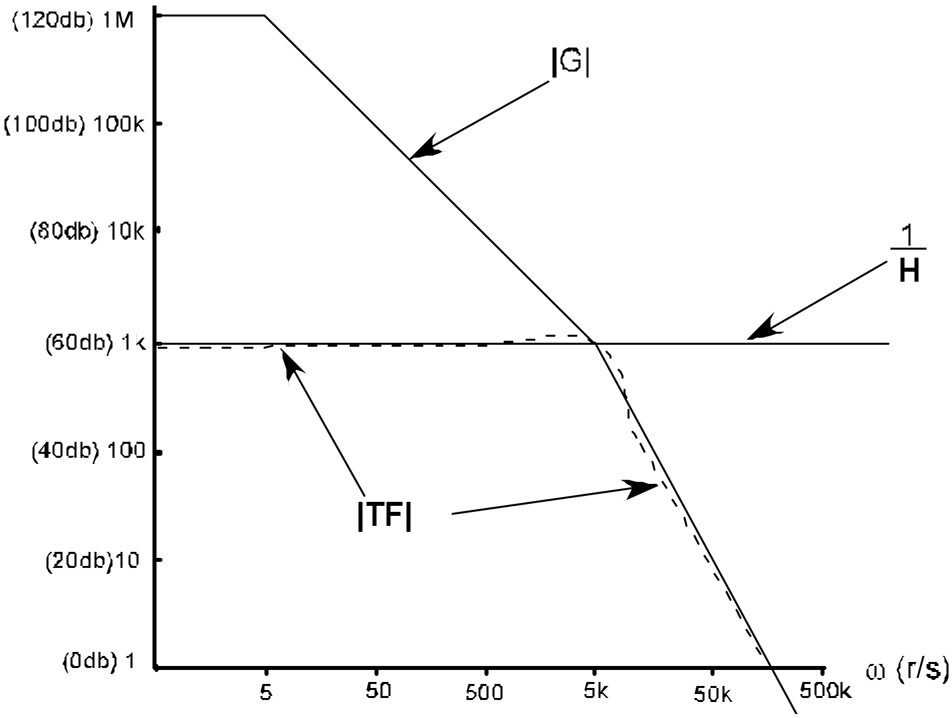
The plot shows that the rpm/V response of the motor goes down for frequencies higher than about 5 r/s.

Now consider this motor as part of the closed-loop control system.



1) draw the Bode magnitude plot for the forward path (G)
2) draw the Bode magnitude plot for the feedback path (H)

$$\frac{G}{1+GH} \cong \begin{cases} \frac{1}{H} & \text{for } G \gg \frac{1}{H} \\ G & \text{for } G \ll \frac{1}{H} \end{cases}$$



1) Notice that for frequencies less than about 5000 r/s, the system gain is much as it is at DC. Compare this to the response of the motor alone which has a break frequency at 5 r/s.

   Using feedback can extend the frequency range of useful operation. Put another way, using feedback can actually reduce a system's time constant! Feedback can make a system quicker!

2) Notice that 1/H crosses |G(s)| at the 2nd breakpoint which results in a damping ratio $\zeta \cong 0.5$. If 1/H crosses |G(s)| before the 2nd breakpoint, then $\zeta > 0.5$. If 1/H crosses |G(s)| after the 2nd breakpoint, then $\zeta < 0.5$, which is unacceptable for most control systems.

Example

For a system with the Bode mag. plot shown below determine:

i) an expression for the gain without feedback, G.
ii) an expression for the gain with feedback G/(1+GH).
iii) the damping ratio ζ.
iv) an estimate for the gain with feedback when ω=3000 r/s.
v) the percent error in the estimate in part (iii).
vi) repeat parts iv) and v) for ω=30 r/s.