Stress and Strain Measurement

Introduction

Stress and strain can be measured in many ways, typically based in some fashion upon the fact that a structure is deformed, or strained, when it experiences stress.

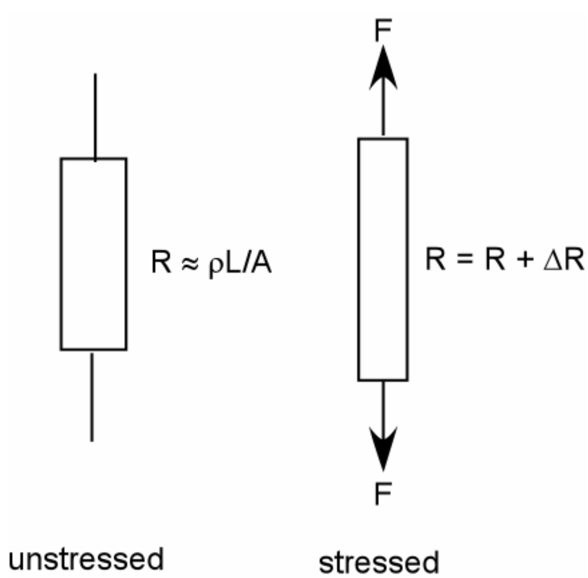
Piezoelectric transducers use the fact that, in piezoelectric materials, electric charge is separated when the materials is strained (and vice versa). Other transducers are often based on the resultant changes that the strain produces in capacitance, inductance or resistance.

We are going to focus on perhaps the most widely used transducer, the metallic strain gage. Its fundamental principles of operation, use and application, necessary signal conditioning, and its limitations will be explored.

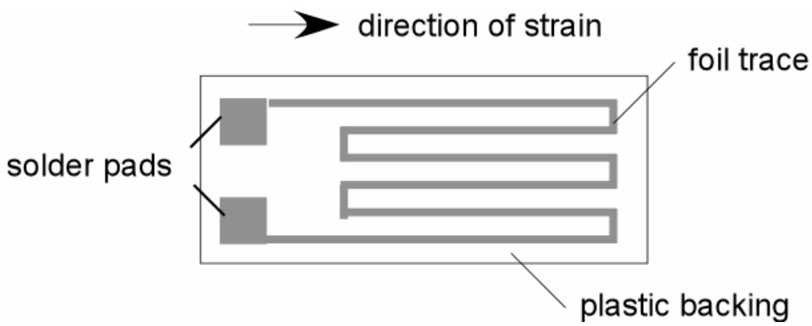
Metallic electric resistance strain gages

Metallic ER strain gages use the fact that a metal is deformed, or is strained, when it experiences stress.

The strain results in a change in its resistance.



The shape is usually not a simple bar.



The most common form for metallic strain gages is for the wire grid to be formed from constantan (a Cu-Ni alloy) and the backing to be polyimide (a high-temperature polymer).

This metal foil is bonded to the element to be strained so that it will experience the same strain as that of the element.

While this one type is most common, metallic-foil strain gages are manufactured from a variety of different metals and alloys, with a range of resistance values, a range of sensitivity of resistance to strain, and varying resistance-to-temperature characteristics.

Common nominal resistance values (the resistance of the unstrained strain gage) are 120 Ω, 350 Ω, and 1000 Ω.

Applications

Strain gages can be related directly to stress through the elastic modulus (Young's modulus, E). Other typical measurements are force, torque, and pressure.

1. If the relationship between applied force and strain can be determined for a given structure, then strain gages can be used to measure the force.
2. If the relationship between applied torque and strain can be determined for a given structure, then strain gages can be used to measure the torque.
3. If the relationship between applied pressure and strain (examples include a container or a membrane) can be determined for a give structure, then strain gages can be used to measure the pressure.

Theoretical background

For a resistor of uniform cross section A, of length L, and of resistivity ρ:

$$R = \frac{\rho L}{A}$$

If we take the differentials of the relation for resistance, we obtain.

(1) $$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A}$$

Assuming a circular cross section (same end result is obtained, the boxed equation below, if other cross sections are used).

$$A = \pi \frac{D^2}{4}, \qquad \frac{dA}{A} = 2\frac{dD}{D}$$

The axial strain is related to the transverse strain via ν, Poisson's ratio, a material property.

$$\varepsilon_t = -\nu \, \varepsilon_a$$

This permits us to write $$\frac{dD}{D} = -\nu \frac{dL}{L} = -\nu \, \varepsilon_a$$

Substituting these relationships into (1).

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \varepsilon_a + 2\,\nu\,\varepsilon_a$$

Strain gages are characterized by the strain gage factor, S.

$$\boxed{S = \frac{dR/R}{\varepsilon_a} = 1 + 2\nu + \frac{d\rho/\rho}{\varepsilon_a}}$$

Overview of their characteristics

For metallic strain gages, the gage factor is typically around 2-6, usually closer to 2. Semiconductor strain gages can have gage factors higher than 100.

For metallic gages, strains as high as 0.04 (4% elongation) are measured routinely. For semiconductor gages, the strain is limited to about 0.003 (0.3% elongation).

For a constant temperature, S is fairly constant with strain. Depending on the material, S can be sensitive to temperature changes.

Specific examples

Measuring force: Strain gages can be used to measure force. Given a structure's geometry and composition, the relationship between applied force and resulting stress is calculated. Knowing material properties, the resulting stress is related to the strain, ε.

For a given gage factor, S, the resulting change in resistance is easily calculated.

$$\boxed{\Delta R \approx S e R}$$

Measuring torque: Strain gages can be used to measure torque. Given a structure's geometry and composition, the relationship between applied torque and resulting stress can be calculated. Knowing material properties, the resulting stress is related to the strain, ε.

For a given gage factor, S, the resulting change in resistance is easily calculated.

Using strain gages in a measurement system

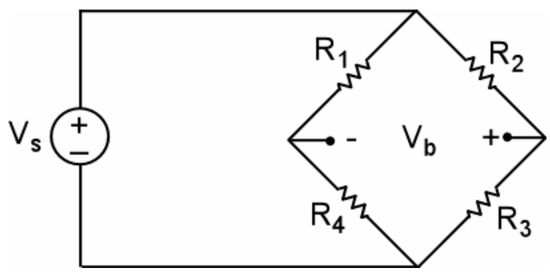
In a system designed to measure (for example, stress), the strain gage provides a change in resistance. This is the signal available from the strain gage.

This change in resistance is typically transformed to a change in voltage, often by using a Wheatstone bridge. Then, since the change in voltage is usually too small to be useful, an amplifying circuit is used to raise the voltage changes to a useful level.

Signal conditioning for metallic strain gages

Wheatstone Bridge

The Wheatstone Bridge is used to convert a change of resistance into a change of voltage.



With V_b measured across an open circuit,

- the current through R_1 is the same as that through R_4
- the current through R_2 is the same as that through R_3

By voltage division, we have

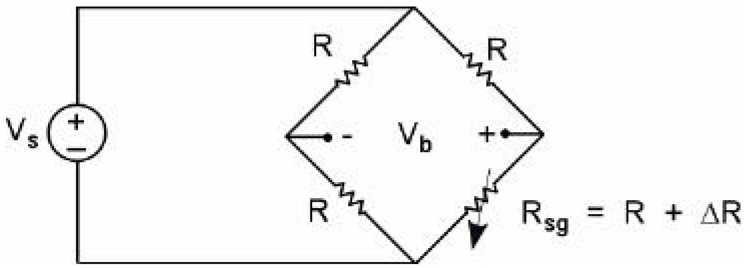
$$V_b = \left(\frac{R_3}{R_2 + R_3} - \frac{R_4}{R_1 + R_4} \right) V_s$$

The bridge is "balanced" (that is, $V_b = 0$), for

$$V_b = 0 \quad \text{for} \quad \frac{R_4}{R_1} = \frac{R_3}{R_2}$$

Now, let's replace one of the resistances on the bridge with a strain gage resistor and make the other three resistances the resistance of the strain gage's nominal resistance. Note that, when the strain gage experiences no strain, all four resistances in the bridge will have the same value.

Analysis to find V_b



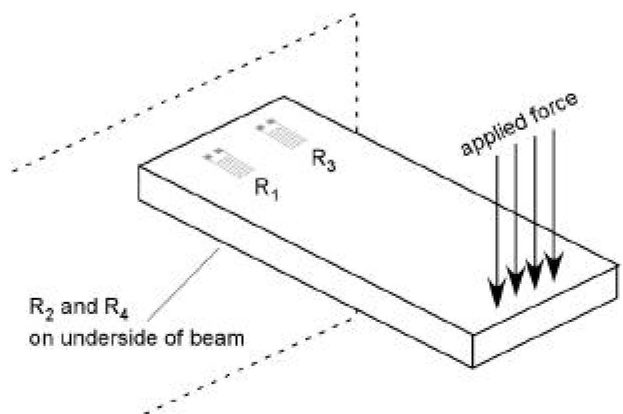
By voltage division,

$$V_b = \left(\frac{R + \Delta R}{R + R + \Delta R} - \frac{R}{R + R}\right)V_s = \left(\frac{2R^2 + 2R\Delta R - 2R^2 - R\Delta R}{4R^2 + 2R\Delta R}\right)V_s$$

$$V_b = \frac{\Delta R}{4R + \Delta R} V_s \cong \frac{\Delta R}{4R} V_s \quad \text{for} \quad \Delta R \ll R \text{ (almost always the case)}$$

This configuration is called the one-active-arm bridge since only one of the arms responds to the measurand.

$$V_b \cong \frac{\Delta R}{4R} V_s \text{ for one-active-arm bridge}$$



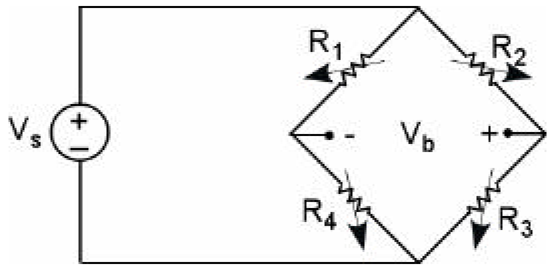
Often benefits are available by placing more than one strain gage resistance into the bridge. Consider a cantilever beam.

Place R_1 and R_3 in tension and R_2 and R_4 in compression.

Benefits are

1) elimination of torsion loading effects
2) more sensitivity
3) temperature compensation

This configuration is called the four-active-arm bridge since all of the arms respond to the measurand.



Analysis to find V_b

Again, by voltage division

$$V_b = \left(\frac{R + \Delta R}{R + R} - \frac{R - \Delta R}{R + R}\right)V_s = \left(\frac{R + \Delta R - R + \Delta R}{2R}\right)V_s$$

and we obtain for the four-active-arm bridge

$$\boxed{V_b = \frac{\Delta R}{R} V_s}$$

be sure to note the assumptions that this relation rests on:

- identical strain gages
- common temperature variations

Typical voltages

What voltages should we typically expect for V_b? That is, for typical strain gages with typical strains, is V_b going to be 10 V?, 1 V? 1mV? or 1μV?

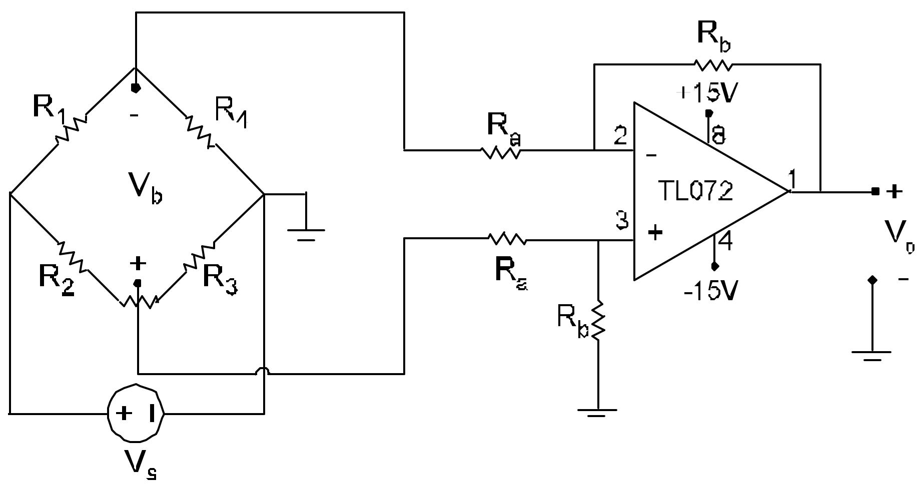
Four-active-arm Wheatstone bridge:

typical R (metallic strain gage) ~ 120 Ω
typical gage factor, S ~ 2
typical strain, ε ~ 0.001
typical V_s ~ 10 V

V_b = S e V_s = 20 mV

This is a fairly small voltage—amplification is often required.

Differential Amplifier



$$V_o = \frac{R_b}{\frac{R}{2} + R_a} V_b, \qquad \text{where } R \approx R_1 \approx R_2 \approx R_3 \approx R_4$$