SIGNAL CONDITIONING: FILTERING

Frequency Response

The frequency response of a circuit is its steady-state response to a sinusoidal input as the frequency of the sinusoidal input varies.

$$\mathbf{V_i} = V_i\angle\theta_i \quad \text{and} \quad \mathbf{V_o} = V_o\angle\theta_o$$

Recall in Laplace analysis, the transfer function is obtained by taking the ratio between the output and input. The system's frequency response can be found by substituting $j\omega$ for s.

$$\boldsymbol{TF}(s = j\omega) = TF\angle\theta_{TF}$$

The relationship between input and output is

$$V_o\angle\theta_o = TF\angle\theta_{TF} \; V_i\angle\theta_i$$

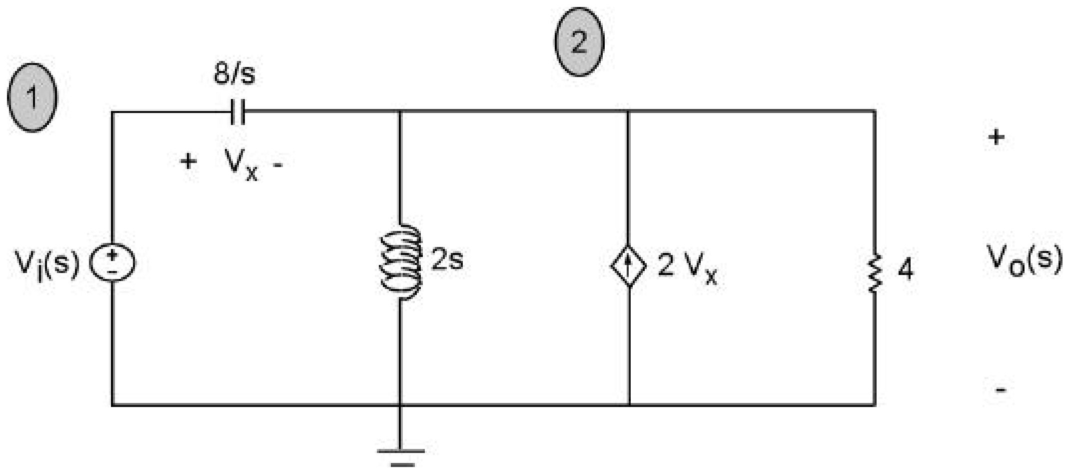
A circuit's frequency response is a phasor relationship involving magnitude and phase..

1. The magnitude response is the ratio of output and input magnitudes. TF is a function of ω.

2. The phase response is the phase shift between input and output. That is $\theta_{TF} = \theta_o - \theta_i$. θ_{TF} is a function of ω.

*OUR INTEREST IN FILTERING IS USUALLY CONFINED TO THE **MAGNITUDE RESPONSE**.*

Find the frequency response of this system.



The nodal equations read:

Define control variable: $V_x = V_1 - V_2$

nodal equations

voltage source equation: $V_1 = V_i$

KCL at node 2: $\dfrac{V_2 - V_1}{8/s} + \dfrac{V_2}{2s} - 2\,(V_1 - V_2) + \dfrac{V_2}{4} = 0$

Use Maple to find $V_o(s) = TF(s)\, V_i(s)$

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> restart;
> eqns:={v1=vi,
> (v2-v1)*s/8+v2/(2*s)-2*(v1-v2)+v2/4=0}:
> soln:=solve(eqns,{v1,v2}):
> assign(soln);
> vo:=v2:
> TF:=vo/vi;
```

$$TF := \frac{s\,(s + 16)}{s^2 + 4 + 18\,s}$$

This is the system transfer function. To obtain the system frequency response, substitute $j\omega$ for s.

Filtering

Filters are frequency selective systems. In filtering, the magnitude response is often of most interest.

Looking at the filter as an input/output relation, filters are classified by how the input and output magnitudes are related at different frequencies.

We'll use four types of filters: **lowpass**, **highpass**, **bandpass**, and **band-reject**.

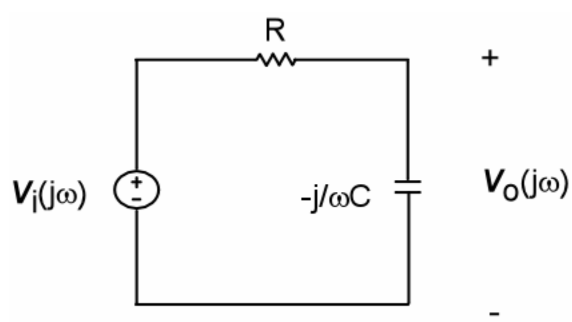
Another way to categorize filters is to classify them passive or active.

Passive filters consist solely of passive components (R's, C's, L's, transformers, etc.) Passive filters receive energy only from the input.

Active filters also use active components such as transistors or op-amps. Active filters can provide a gain >1. They receive energy from the input and from external power supplies.

Passive Low Pass

Low pass filters pass low frequencies from input to output and attenuate high frequencies.

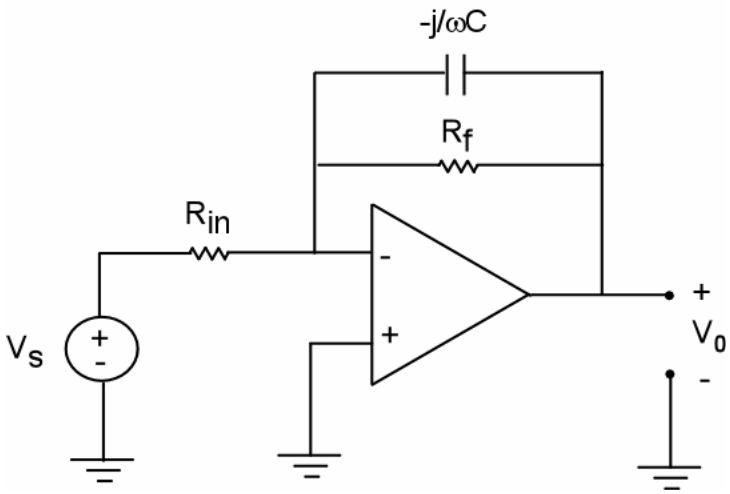


The capacitance impedance decreases at high frequencies. By voltage division, V_o/V_{in} will go down as well.

Active Low Pass

This lowpass filter has a break frequency (in rad/s) of $1/R_fC$ and a dc gain of R_f/R_{in}.

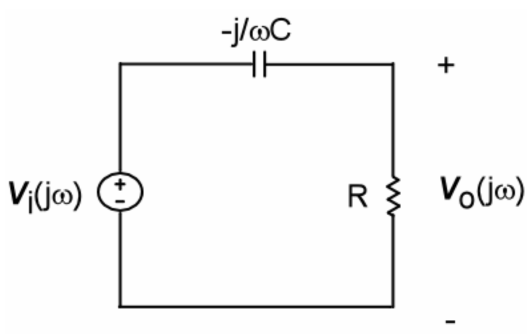
There is also inversion since this is the inverting configuration.

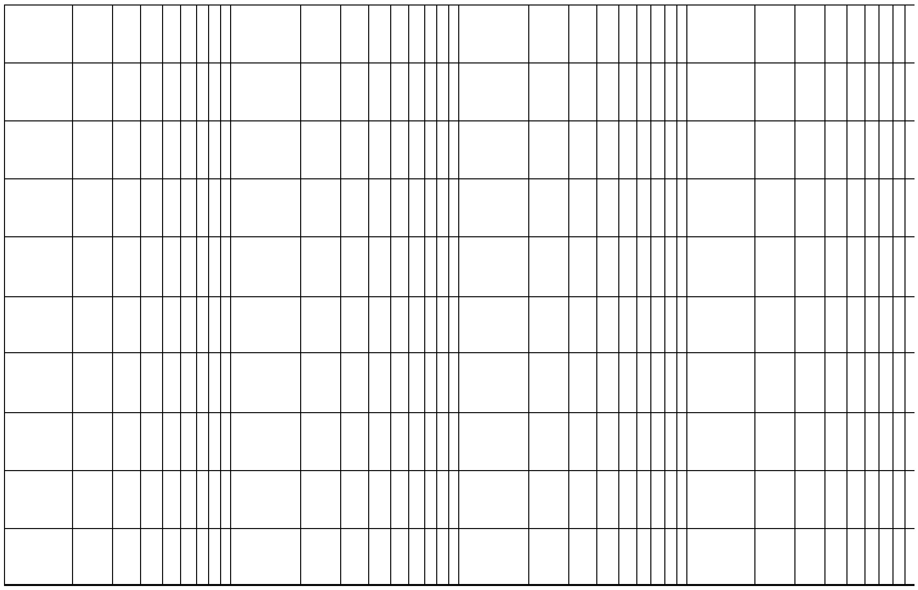


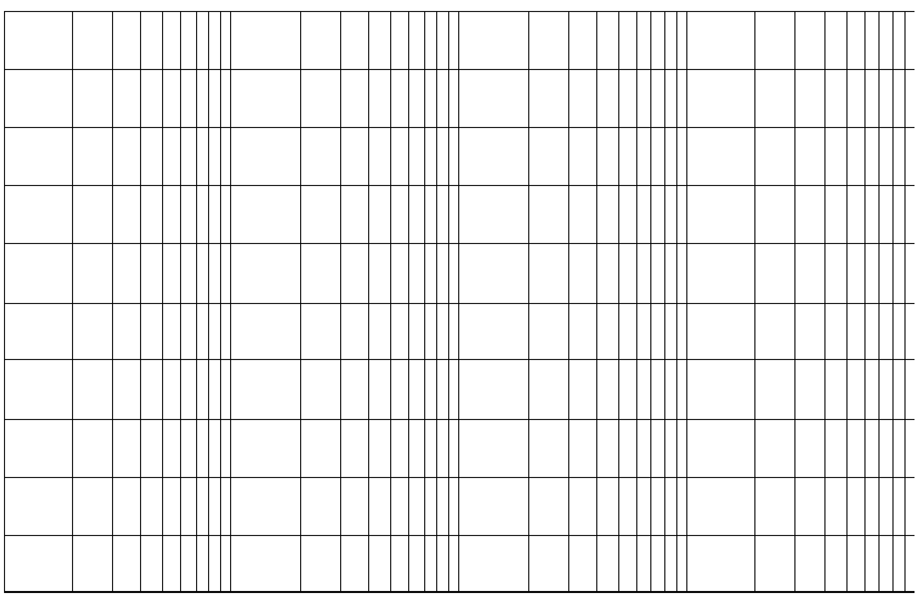
Passive High Pass

High pass filters pass high frequencies from input to output and attenuate low frequencies.

The capacitance impedance increases at low frequencies. By voltage division, V_o/V_{in} will decrease at low frequencies.



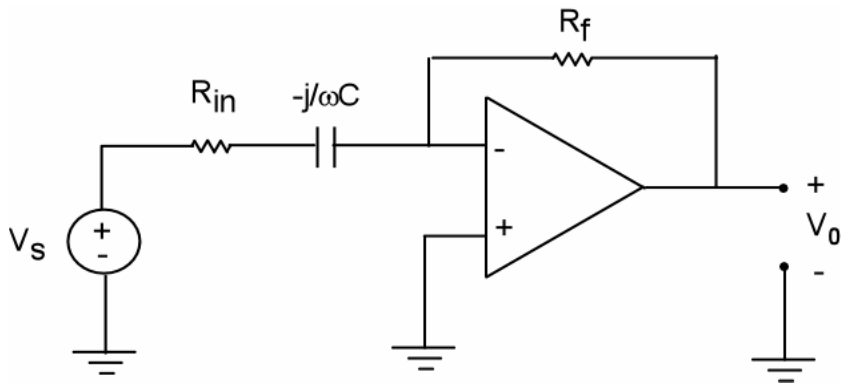




Active High Pass

This highpass filter has a break frequency (in rad/s) of $1/R_{in}C$ and a high frequency gain of R_f/R_{in}.

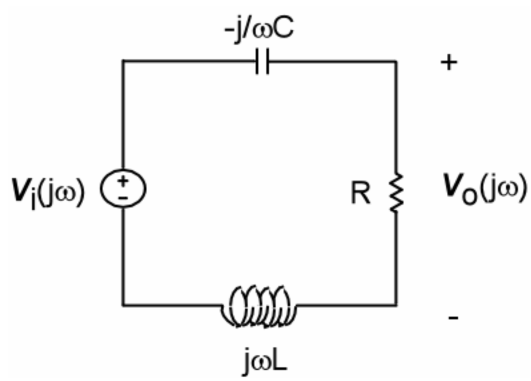
There is also inversion since this is the inverting configuration.



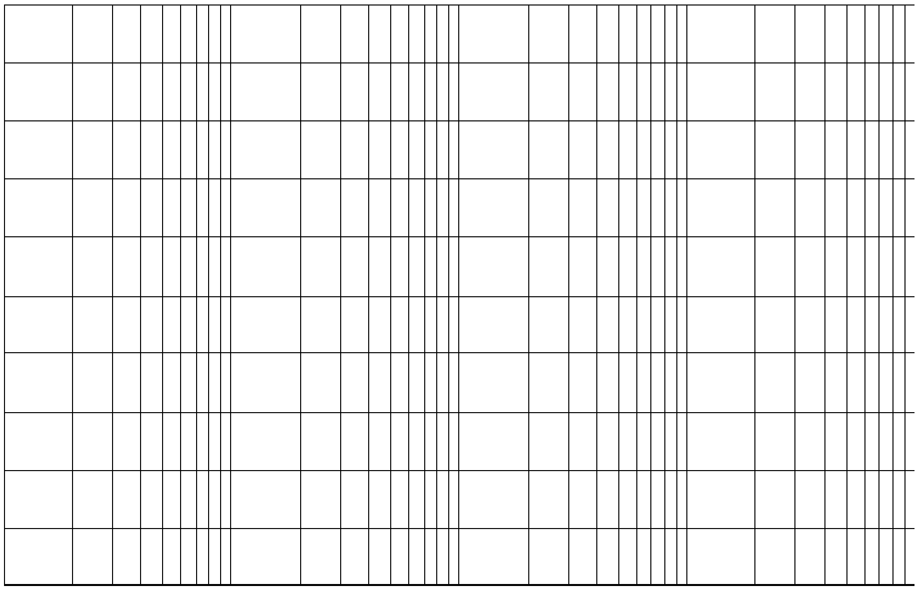
Passive Band Pass

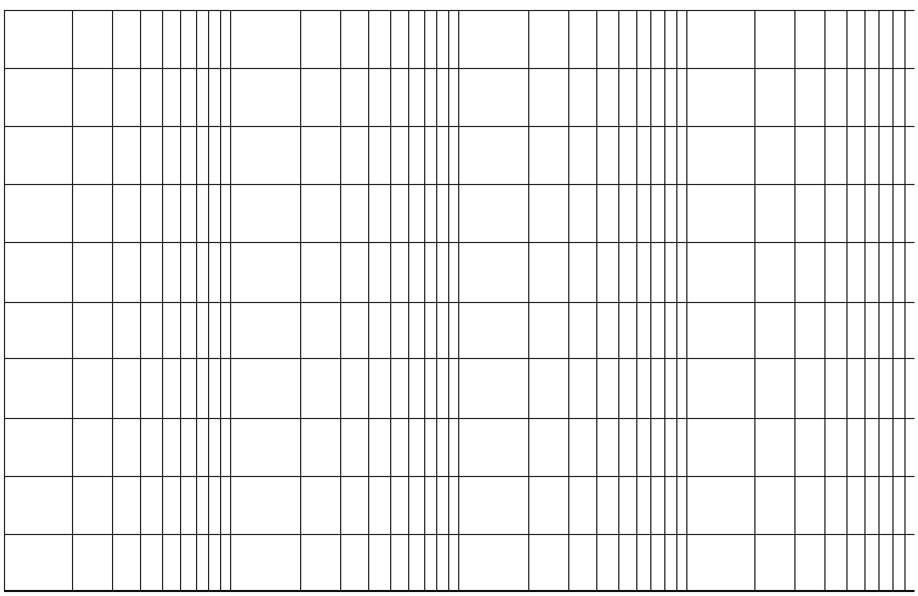
Band pass filters only pass a band of frequencies from input to output.

The capacitance's impedance grows large at low frequency, and the inductance's impedance grows large at high frequency. By voltage division V_o/V_{in} will grow small at high or low frequencies



The peak amplitude of the transfer function is 1 at $w = 1/\sqrt{LC}$.





Active Band Pass

Find the transfer function of this circuit.