Measurement – begin reviewing text material on measurement

Measurement is one area that all engineers and scientists in research and industry rely upon.

Exercise

Suppose a design engineer is trying improving the efficiency of an internal combustion engine. What measurements might be required in experiments on how to improve efficiency?

Hints: temperature, strain, flow, speed, angular velocity, force, strain, stress, position, timing, chemical composition …

variable to be measured	purpose of measurement	importance of measurement	allowable error	measurement techniques available

Methods of Measurement

Measurement can be split into two types: 1) direct comparison with a primary or secondary standard and 2) measurement via a calibrated system.

Direct comparison is used when a length is measured by comparing the length to a ruler, the ruler being a secondary standard.

Using a calibrated system is the most common type of measurement—voltage with a calibrated voltmeter, power with a calibrated power meter, and weight with a calibrated scale.

Why Measure?

Observation is central to the scientific method, and observation requires measurement. Likewise, engineering design requires experimentation. Experimentation requires accurate measurement.

Measurement is critical for research, development, production, and process control.

Historical Background

In the past, measurement was usually based on the length of a ruler's arm, or foot, or hand. In fact, the word ruler, as in a county's ruler, is still used to refer to a standard length.

In 1790, the French National Assembly directed the French Academy of Sciences to develop a model for weights and measures. The Academy's recommendation, the metric system, included the stipulation that multiples of all basic units be in factors of 10.

In the United States, the U.S. Constitution gives congress the authority to maintaining a systems of weights and measures.

In 1821, John Quincy Adams, in a report to congress, states the necessity for a system of weights and measures in any society.

In 1832, the U.S. Treasury Department introduced a system of weights and measures. In 1936, these standards were approved by congress.

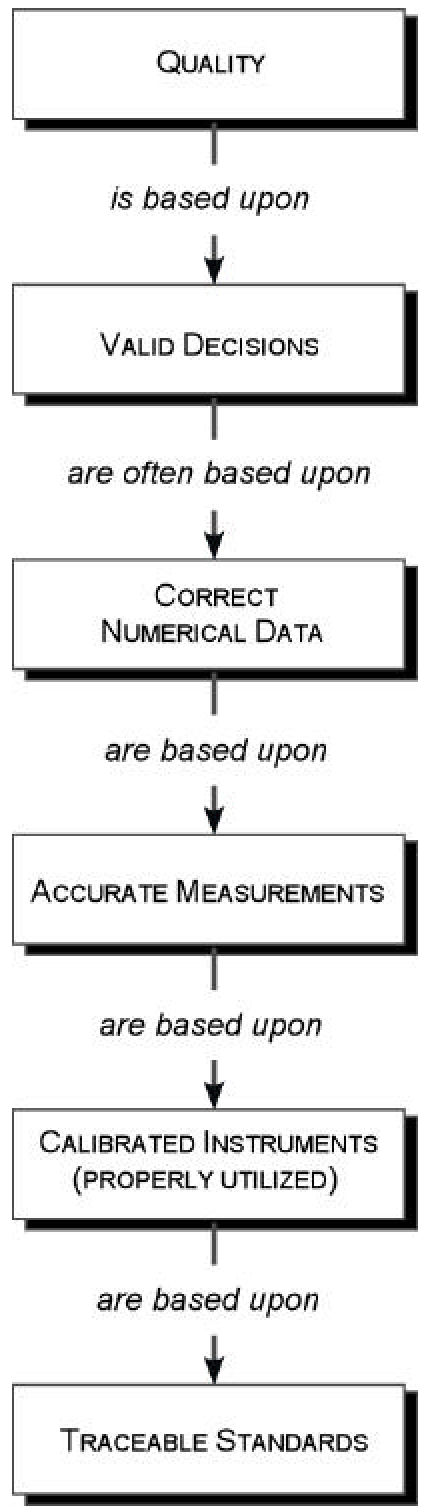
In 1875, the U.S. and 16 other countries sign the "Treaty of the Meter." This established a central bureau of standards in France.

In 1901, the U.S. Congress passed an act to establish the National Bureau of Standards (NBS) in 1910. In 1950, the name was changed to the National Institute of Standards and Technology (NIST).

International uniformity is maintained by scheduled meetings every six years by most industrialized counties.

Conceptual Basis for Why Measurement is Important

(adapted from *The Measurement, Instrumentation, and Sensors Handbook*, ed. John G. Webster, CRC Press, 1999)



Quality of product, research, or organization

Decisions—valid ones at least—are often based on numerical or quantifiable information.

If our decisions are based upon numerical data, it had better be correct.

This requires our measurements be accurate.

Accurate measurements requires calibrated instruments (sensors, signal conditioning, display and recording)

Accurate measurement is ultimately based on traceable standards.

Overview of Measurement via a Calibrated System

Measurement involves using some quantity that has a known relationship to the variable being measured

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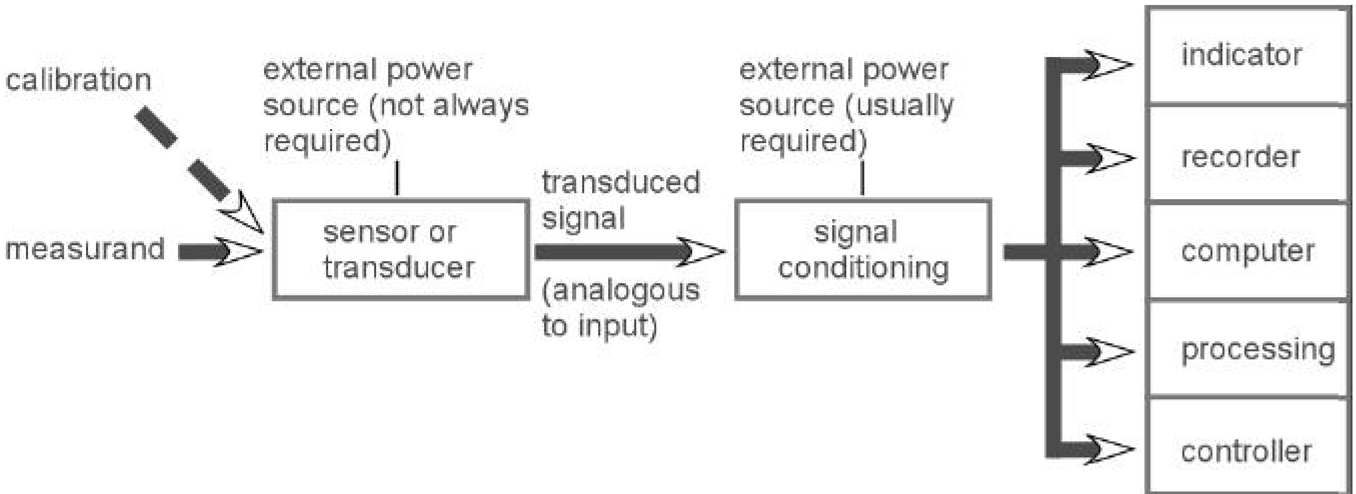

figure from *Mechanical Measurements*, 5th ed., Beckwith et al, Addison-Wesley (1993).

For example, when measuring strain, strain is the **measurand**. When measuring strain with a strain gage, **signal** is a change in the strain gage resistance (for resistive strain gages). **Signal conditioning** might include transforming the change in resistance to a change in voltage via a Wheatstone bridge and subsequent amplification.

Since measurement is central to all engineering and scientific work, we must take special care to really understand all aspects of measurement, from the physical principles of the sensors to proper choices for signal conditioning to knowledge of how to measurement errors interact in a system.

There are attributes shared by most measurement systems.

ERROR, BIAS ERROR, PRECISION ERROR, RANGE, ACCURACY, RESOLUTION, STABILITY, DRIFT, SENSITIVITY, HYSTERESIS, and LINEARITY

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Error

Error refers to the difference between the measured or observed value and the actual value of a measurand.

error = measured value - actual value

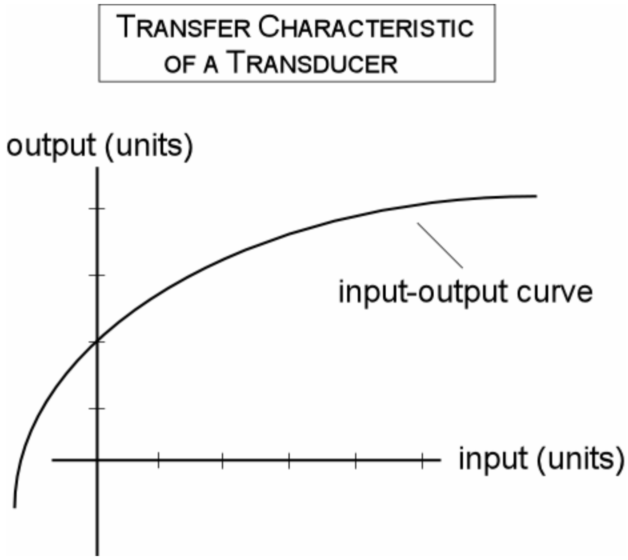
There are two general types of errors

bias error = average of measurements - actual value

precision error = measurement - average of measurements

BIAS, OR SYSTEMATIC, ERROR	PRECISION, OR RANDOM, ERROR
Bias errors give the same error in each measurement trial. Bias errors can be caused by faulty or outdated calibration. ZERO-OFFSET errors cause each measurement to deviate a given amount. SCALE ERRORS change the sensitivity of the measurement depending upon value the measurement Bias errors can also be introduced by the very act of measurement. LOADING ERROR occurs when the act of measurement changes that which is to be measured. Placing a thermometer having a different initial temperature on a small piece of metal can alter the temperature of the metal. Placing a voltmeter in parallel with a high resistance can change the voltage across the resistance.	Precision errors are errors that fluctuate, sometimes causing the measurement to be lower than the true reading and sometimes causing the measurement to be higher. The fluctuations can often be characterized by the Normal or Gaussian probability distribution. (the proverbial bell-shaped curve) Precision errors can be caused by 1)environmental variations 2)electrical or magnetic interference 3)random variations in the performance of the measurement system 4)uncontrolled variations in the measurand

Consider some parameters associated with this transducer input/output characteristic



The **range** of a sensor or measurement system refers to the range of inputs for which the sensor or measurement system functions within specifications.

Sensitivity refers to change of output per change of input. It's the magnitude of the slope in the above curve.

The input/output characteristic is **linear** to the extent in which its sensitivity does not change as the input changes.

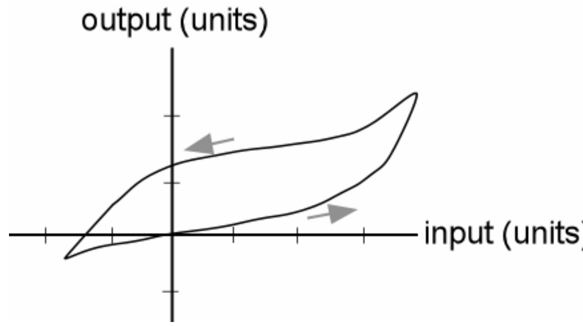
Stability refers to the degree of insensitivity that the input/ouput relation possesses with respect to environmental changes. Stability w.r.t. temperature, w.r.t. humidity, w.r.t. electromagnetic interference, etc.

Changes in the input/output characteristic over time (stability w.r.t. time) is referred to as **drift**. Drift is especially used to designate changes in the input/output characteristic with time.

Accuracy refers to how closely the measurement corresponds to the actual or true value. The lower the accuracy the greater the error.

Resolution is the smallest increment of change in the measured value that can determined from the system's readout.

Hysteresis refers to a dependence of the input/output relationship, depending on whether the input is increasing or decreasing. Hysteresis is indicates that the system has memory, that the past values of inputs influence present behavior. It's usually associated with some type of energy loss mechanism.



Example: Suppose the actual or true value of a force is 16.7 N.

In ten trials, we measure 16.9, 17.3, 17.0, 16.9, 17.1, 16.8, 16.7, 17.0, 17.2, and 17.0 N.

i) What is the bias error?

ii) What is the range of precision error?

Review of dynamic systems – read text chapter 11

0th-order systems

$$x = K\, f(t)$$

1st-order systems

Standard form $$t\, x' + x = K\, f(t)$$

The general solution consists of two pieces—the forced response and the natural or transient response.

Let's look at four cases

1. f(t) = 0 (natural response)
2. f(t) = Au(t) (step response)
3. f(t) = Atu(t) (ramp response)
4. f(t) = A cos ($\omega t + \theta$) (sinusoidal response)

<u>f(t) = 0</u>

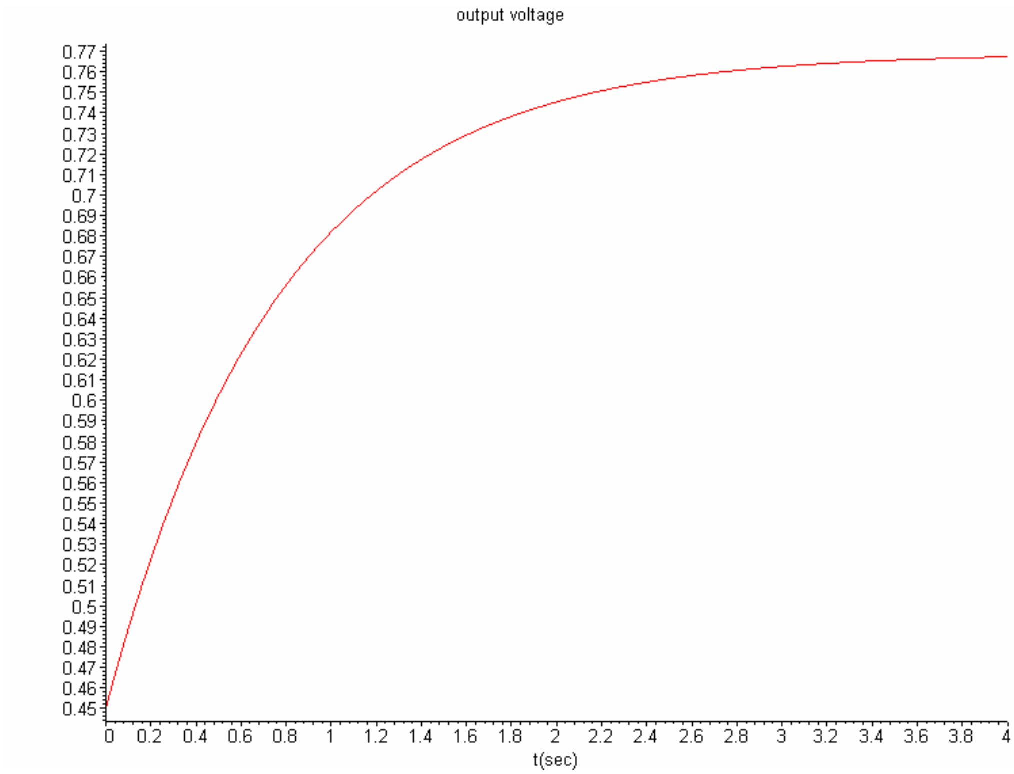
<u>f(t) = Au(t)</u>

$$x(t) = KA - \left(KA - x_o\right)e^{-\frac{t}{t}}$$

Parameter Estimation

Accurate values for τ can be obtained by using all the available data. by developing z(t), the incomplete response, and plotting z(t) vs t and finding the slope, m. $\tau = -m^{-1}$

Also, the slope at t = 0 is related to τ.



<u>$f(t) = At\,u(t)$</u>

<u>$f(t) = A\cos(\omega t+\theta)$</u>

2nd order systems

Standard form $$\frac{x''}{\omega_n^2} + \frac{2\zeta}{\omega_n} x' + x = K f(t)$$

Just as in the first-order case, the complete solution consists the forced response and the natural or transient response.

Consider three cases

1. f(t) = 0
2. f(t) = Au(t)
3. f(t) = A cos(ωt + θ)

f(t) = 0

Looking at the characteristic equation

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

solutions for of the characteristic equation are

$$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

Four cases:

$\zeta = 0$	(undamped)
$0 < \zeta < 1$	(underdamped)
$\zeta = 1$	(critically damped)
$\zeta > 1$	(overdamped)

$\zeta = 0$ (undamped)

$0 < \zeta < 1$ (underdamped)

Parameter Estimation

Logarithmic decrement: The natural log of the ratio of any two successive amplitudes. (w.r.t. the steady-state value)

$$\delta = \ln \frac{x_{t1}}{x_{t1 + t_d}}$$

$$\zeta = \frac{\delta}{\sqrt{\delta^2 + (2\pi)^2}}$$

For non-sequential max or min:

$$\delta = \frac{1}{N} \ln \frac{x_{t1}}{x_{t1 + N t_d}}$$

Where N is the number of elapsed cycles.

$\zeta = 1$ (critically damped)

$$x = C_1 e^{-\zeta \omega_n t} + C_2 t e^{-\zeta \omega_n t}$$

The critically damping case is really never seen. It is useful as a limiting case.

$\zeta > 1$ (overdamped)

$$X = C_1 e^{-r_1 t} + C_2 e^{-r_2 t}$$

f(t) = Au(t)

The form of the natural response is not affected by the form or presence of the source.

Sketch of step response

(picture assumes we have an underdamped system and that x(0) = 0 and x'(0) =0)

Time Reponse Specifications

-- **Steady State value**: $x_{ss} = KA$

-- **Peak Time**: time to 1st peak $= \pi/\omega_d$

-- **% overshoot** (% of KA)

$$\%OS = 100\, e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$$

-- <**2% settling time**

$$T_s = \frac{4}{\zeta\omega_n}$$

-- <**1% settling time**

$$T_s = \frac{5}{\zeta\omega_n}$$

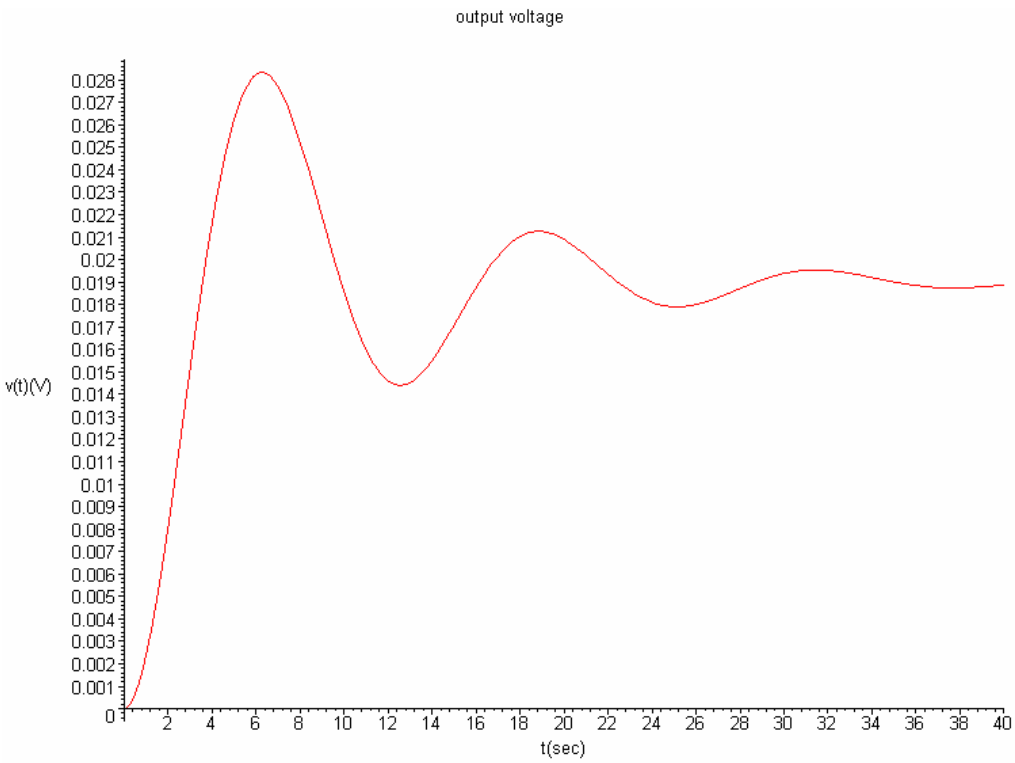
-- **rise time** (time to cross KA)

$$T_r = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}}{\omega_n\sqrt{1-\zeta^2}}$$

***Note**: A different rise time is utilized in electrical systems where it is defined as the time required for a signal to go from 10% to 90% of its final value.*

$\underline{f(t) = A \cos(\omega t+\theta)}$

Example: Given the unit step response below, find the differential equation relating the input f(t) to the output x(t). Give the differential equation in standard form.



i) Identify the undamped resonant frequency, the damped resonant frequency, static gain coefficient, the damping ratio.

ii) Calculate the percent overshoot, peak time, <2% settling time, and rise time. Compare with measurement.

Workspace for example