Ideal Transformers

Transformers are enormously important. Without them, electrical power would not be as available and widespread as it is today. They allow changes in voltage levels. In particular, voltage can be increased to allow power to be transported at lower current levels.

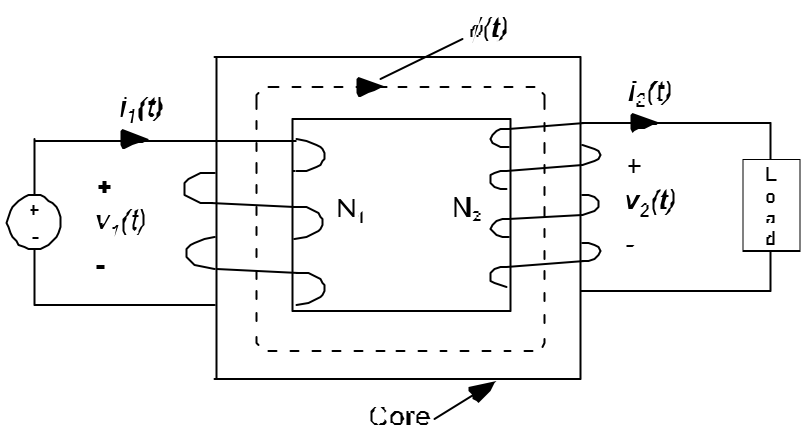
Transformers have two principal components.

- A core of high permeability, read ferromagnetic, material which is able to confine a magnetic field

- Windings which are wound about the core. An externally supplied AC current on one winding (call the primary winding) produces a magnetic field in the core by Ampere's law. AC voltages will then be induced by Faraday's law on other coils (secondary winding) that are wound on the core.

The ideal transformer model is based on two assumptions.

- The core of an ideal transformer has infinite permeability. The result is that all flux is confined to the core.
- There is no power loss in the ideal transformer. The result is that power in must equal the power out.



Transformer action

The sinusoidal currents, $i_1(t)$ and $i_2(t)$, flow in the primary coil and secondary coils, the coils having N_1 and N_2 turns, respectively.

The two currents, working together, produce a sinusoidal magnetic flux, $\phi(t)$, in the core. Since the core is assumed to have infinite permeability, all the flux is confined to the core, no flux leaks out.

The result is that the same flux links both coils. Sinusoidal voltages are induced in the coils via Faraday's law. Since they are caused by the same flux, they must be in phase. Their magnitudes are related by the turn's ratio.

$$v_1(t)=N_1\frac{df}{dt} \qquad v_2(t)=N_2\frac{df}{dt}$$

$$v_1(t) = V_1 \cos(\omega t + \theta_v) \qquad v_2(t) = V_2 \cos(\omega t + \theta_v)$$

$$\frac{v_1(t)}{v_2(t)} = \frac{N_1}{N_2} = \frac{V_1}{V_2} = a$$ (a is called the "turns ratio)

For an ideal transformer the permeability of the core is infinite and therefore has no reluctance. The magnetic KVL gives

$$N_1 I_1 = N_2 I_2$$

For an ideal transformer, $P_{in} = P_{out}$ (no losses)

$$V_1 I_1 \cos(\theta_v - \theta_{i1}) = V_2 I_2 \cos(\theta_v - \theta_{i2})$$

Therefore, $\theta_{i1} = \theta_{i2}$

$$i_1(t) = I_1 \cos(\omega t + \theta_i) \qquad i_2(t) = I_2 \cos(\omega t + \theta_i)$$

Phasor relations for the ideal transformer

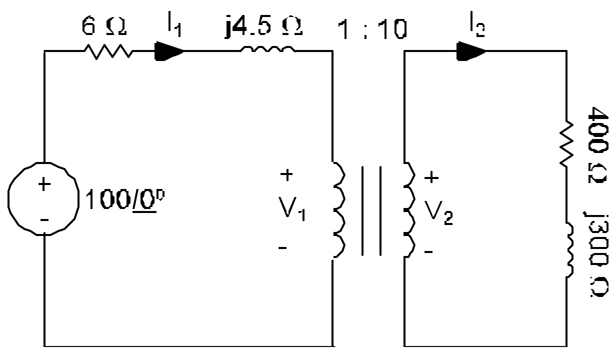
$$\frac{\boldsymbol{V}_1}{\boldsymbol{V}_2} = \frac{N_1}{N_2} \qquad \frac{\boldsymbol{I}_1}{\boldsymbol{I}_2} = \frac{N_2}{N_1}$$

The ideal transformer

application: impedance reflection

Example

Determine the impedance seen by the source and then determine the primary and secondary currents and voltages.



Notice that when the load was referred to the primary side, its value reduced. This is because the load was taken from the high voltage side across to the low voltage side. Impedance will go up when it is moved to a higher voltage and will go down when it is moved to a lower voltage.

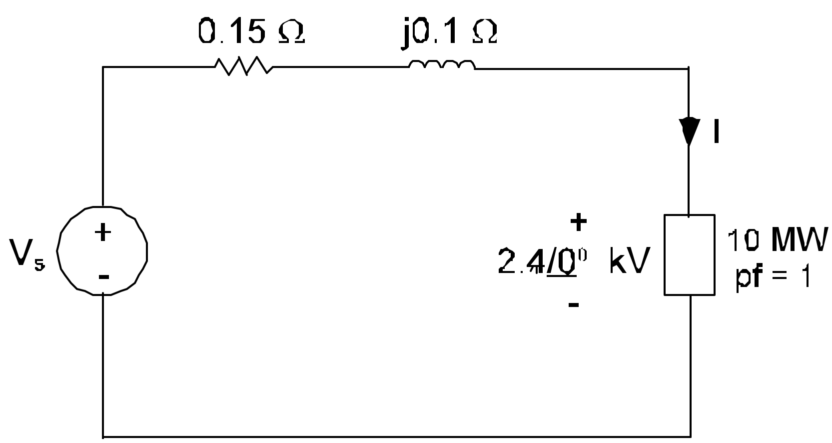
EXAMPLE

This example demonstrates how transformers reduce the cost of supplying electricity.

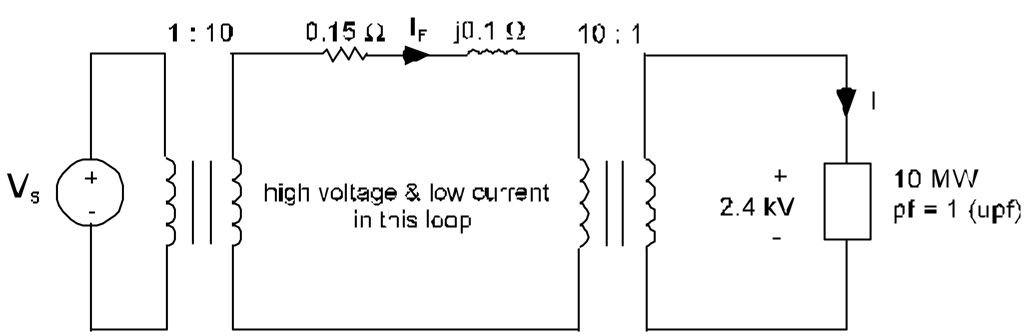
A single-phase load of 10MW with pf=1 is to be supplied at 2.4kV via a distribution feeder with (0.15 + j0.1)Ω impedance. Electric energy costs 3¢/kWh. Calculate the yearly cost of supplying the line losses if the load is constant and continuous if:

a) no transformers are used, and
b) two transformers each with a = 10 are used.

original system



system with step-up and step-down transformers



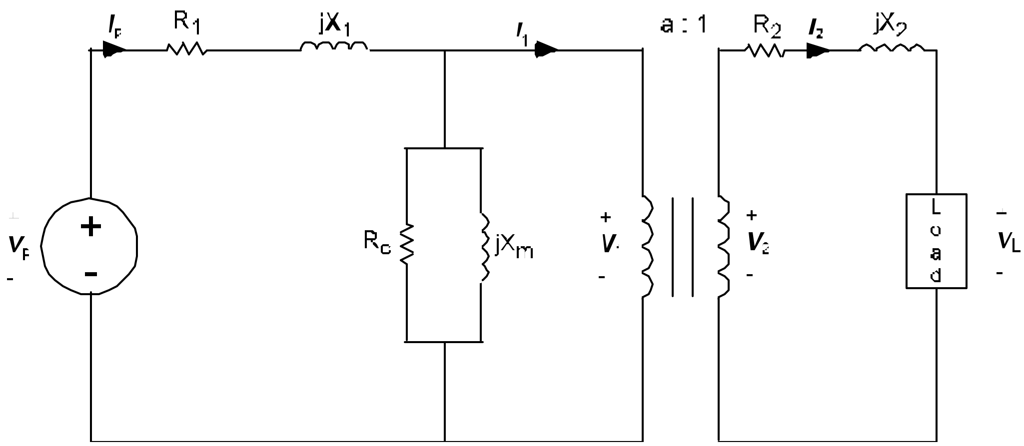
Energy Costs:

%η

%VR

Practical Transformer Model

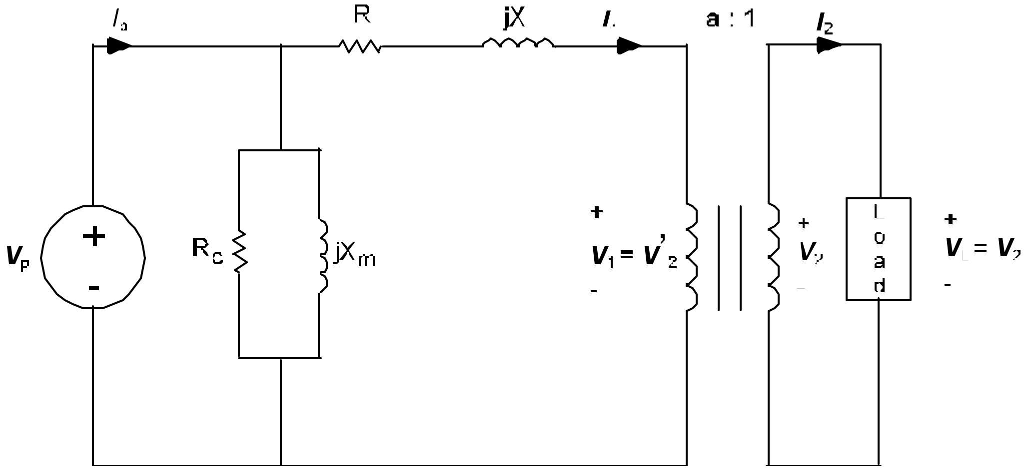
A model that accounts for energy storage and energy dissipation is shown below.



The model includes an ideal transformer with components added to account for non-ideal behavior of real transformers.

- Real transformers are not lossless: R_1 and R_2 account for energy dissipation in the primary and secondary windings.
- In real transformers, the flux is not entirely confined to the core; some flux "leaks out." X_1 and X_2 account for the energy stored in this leakage flux.
- The core is not lossless. Some energy is lost magnetizing and demagnetizing the core—R_c represents the real power that is dissipated in the core of the transformer due to hysteresis and eddy currents.
- X_m accounts for the magnetic energy stored in the core.
- V_p and V_2 are the primary and secondary voltages that appear at the terminals of the transformer.

The circuit above is known as the "exact equivalent circuit." A simpler model that still gives accurate results is shown below.



The parallel elements, R_c and X_m, are moved in front of R_1 and X_1 and placed across the supply. This does not introduce much error since R_c and X_m are much larger impedances than R_1 and X_1 and so appear to be almost open circuits.

R_2 and X_2 are referred to the primary side by impedance reflection. They are added to R_1 and X_1 since they are in series with them,
$R = R_1 + a^2R_2$ and $X = X_1 + a^2X_2$.

This circuit is called the "approximate equivalent circuit." It's the one we'll work with.

Efficiency and Regulation

Efficiency and regulation are figures of merit for a transformer; they measure well a transformer performs, how close to ideal it is.

For %VR, use V_p and V_1.

For %η, use $P_{out} = \text{Re}\{\boldsymbol{V}_1\boldsymbol{I}_1^*\}$ and $P_{in} = \text{Re}\{\boldsymbol{V}_p\boldsymbol{I}_p^*\}$

General Analysis

P_{loss} has two components, P_{coil} and P_{core}

- P_{coil} varies quadratically with current (goes up with load).
- P_{core} is almost constant since the voltage V_p doesn't vary much.

Example

A 300 kVA, single-phase transformer is rated to transform 2.4 kV to 600 V and has the following equivalent circuit parameters referred to the high voltage side.

$$R = 0.75\,\Omega, \quad X = 1.5\,\Omega, \quad R_c = 500\,\Omega, \quad X_m = 60\,\Omega$$

Calculate %VR and %η when the transformer supplies rated load at 0.7 lag pf.

Example (cont)

Example

Re-work with the load pf corrected to unity (P remains the same). The results are::

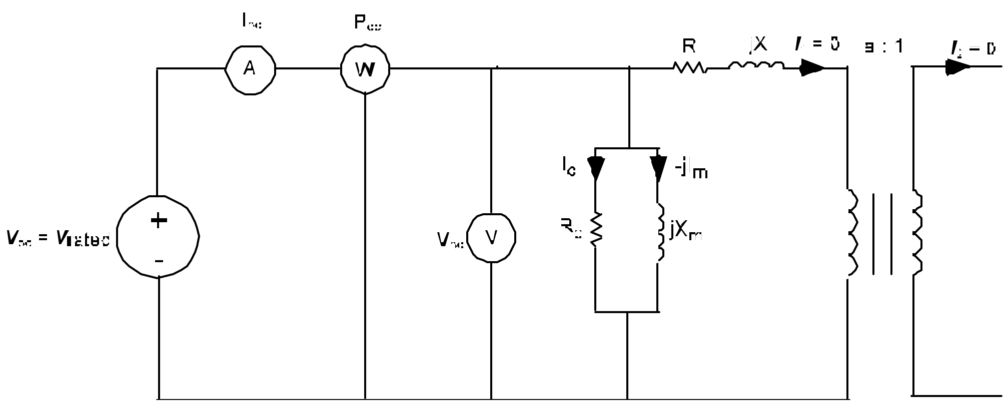
- %VR = 2.88%
- %η = 92.1%
- P_{coil} = 7.96 kW
- P_{core} = 12.2 kW

Experimental determination of equivalent circuit

The equivalent circuit parameters can determined from the open-circuit (OC) and short-circuit (SC) tests. Required measurements are V_p, I_p and P_{in}.

OC test

The test is performed at rated voltage and no load—the load is an open-circuit ($I_1 = 0$)—R and X can be neglected. Only R_c and X_m load the supply.



V_{oc}, I_{oc}, and P_{oc} are measured once the supply has been adjusted to V_{rated}. The procedure is:

1. determine θ_{oc} from: $pf_{oc} = \frac{P_{oc}}{V_{oc} I_{oc}}$ then: $?_{oc} = \cos^{-1} pf_{oc}$

2. determine $\boldsymbol{I_{oc}}$ from: $\boldsymbol{I_{oc}} = I_{oc} \angle -\theta_{oc} = I_c - jI_m$

   The negative signs associated with θ_{oc} and I_m are due to the lagging current. (How does one determine it's lagging?)

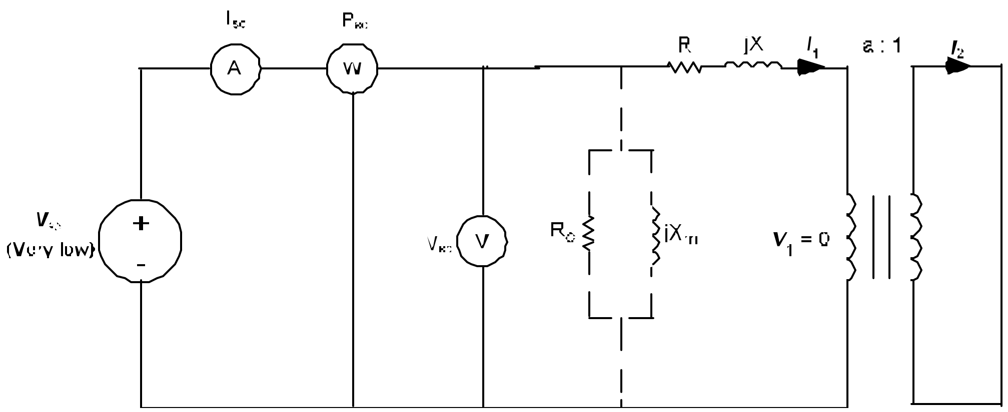
   Calculate $R_c = \frac{V_{oc}}{I_c}$ and $X_m = \frac{V_{oc}}{I_m}$

SC Test

This test is performed at rated current with the secondary terminals short circuited. (V_1 =0).

> **The supply voltage is turned down to zero and <u>gently</u> increased until rated current flows.**

R_c and X_m are usually ignored in this test since they are very large compared to R and X. (That is, $\boldsymbol{I}_{sc} \sim \boldsymbol{I}_2$)



V_{sc}, I_{sc}, and P_{sc} are measured once the I_{sc} has been adjusted to I_{rated}. The procedure is:

1. Determine θ_{sc} from: $pf_{sc}=\dfrac{P_{sc}}{V_{sc}I_{sc}}$ then: $?_{sc}=\cos^{-1}pf_{sc}$

2. Determine $\boldsymbol{I}_1$ from: $\boldsymbol{I}_1 = I_{sc}\angle- \theta_{sc}$

3. Determine $\boldsymbol{Z}$ from: $\boldsymbol{Z}=\dfrac{V_{sc}\angle 0}{I_{sc}\angle -?_{sc}}$ $= R + jX$

Example

Find the approximate equivalent circuit for the following transformer:

ratings	100 kVA	2500 V : 125 V	
oc test	V_{oc} = 2500 V	I_{oc} = 2.5 A	P_{oc} = 2750 W
sc test	V_{sc} = 178 V	I_{sc} = 40 A	P_{sc} = 2250 W