Three-phase power

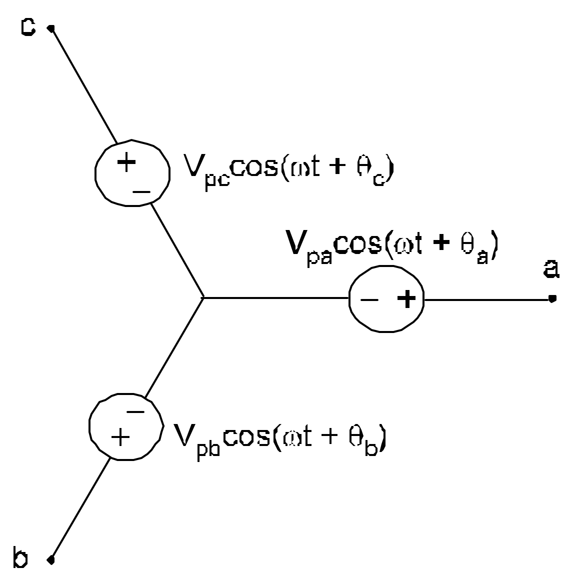
AC power is typically generated and delivered via 3ϕ systems. There are two principal advantages to using 3ϕ power.

1. The power delivered to a balanced 3ϕ load does not vary with time as do 1ϕ loads. This results in less vibration and lower stresses in 3ϕ motors and in systems driven by them.
2. A 3ϕ system can deliver the same amount of power at lower current levels than can 1ϕ systems. This allows the use of smaller conductors.

A three-phase source is made from three independent sources. Each of these sources is a sinusoidal source of a certain frequency—all three sources have the same ω, and differing phases—thus the term "three-phase."

Three-phase source

This 3ϕ source is connected in the Y-configuration. Here the "Y" can be viewed sideways.



Another 3ϕ connection, the Δ-configuration, is also possible. For sources, the Y-configuration is most often used.

The source is "balanced" **if**
$V_{pa} = V_{pb} = V_{pc} = V_p$ and if θ_a, θ_b, and θ_c differ by 120°

If any of these conditions are not met, then the 3ϕ source is unbalanced.

To be a balanced 3ϕ source, the source amplitudes must be equal and their phases must differ from each other by 120°.

Three-Phase Loads

Y-configuration.

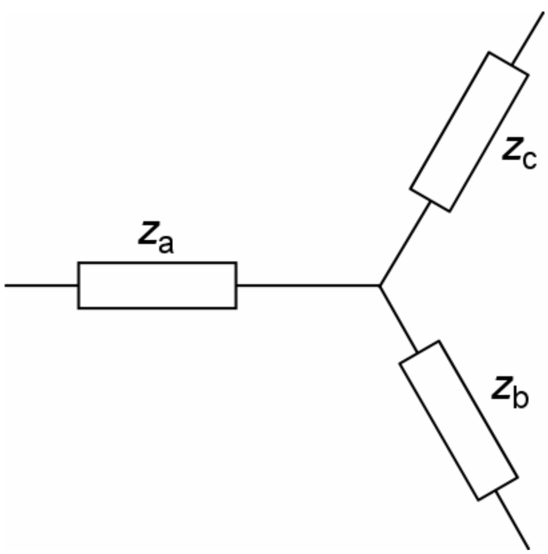
The Y-connected 3ϕ load is said to be "balanced" if all impedances are equal. That is, if $\boldsymbol{Z}_a = \boldsymbol{Z}_b = \boldsymbol{Z}_c = \boldsymbol{Z}_Y$.

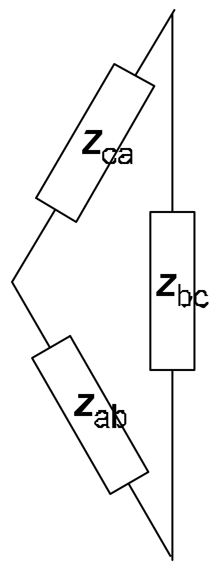
If these impedances are not equal, the load is unbalanced.

Δ-configuration.

Likewise, the Δ -connected 3ϕ load is said to be "balanced" only if all impedances are equal. That is, if $\boldsymbol{Z}_{ab} = \boldsymbol{Z}_{bc} = \boldsymbol{Z}_{ca} = \boldsymbol{Z}_{\Delta}$.

If these impedances are not equal, the load is unbalanced.



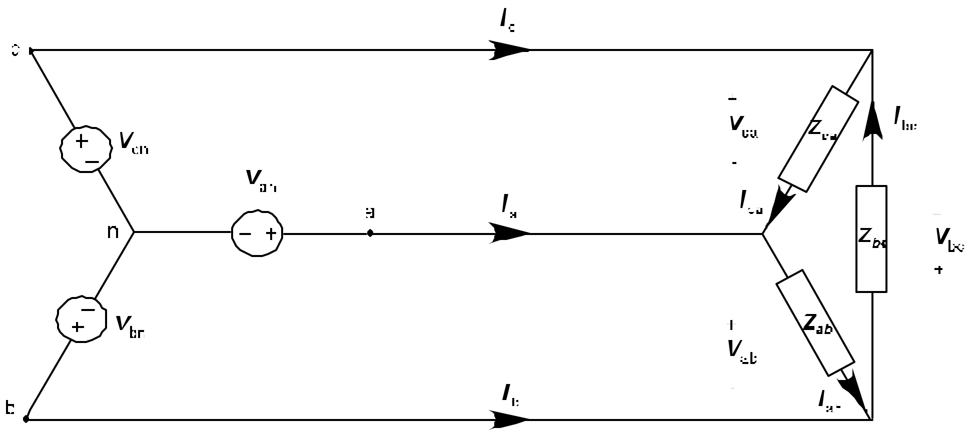


A balanced 3ϕ, Δ-connected load can be converted to an equivalent Y-connected load by dividing $\boldsymbol{Z}_{\Delta}$ by 3.

A three-phase system—the source and load—is said to be balanced only if both the source and load are balanced. If either is unbalanced, the system is also unbalanced.

- For balanced 3ϕ systems, we can simplify our task quite a bit if we are familiar with the characteristics of balanced 3ϕ system.
- On the other hand, the way to treat an unbalanced 3ϕ system is like any other phasor circuit.

Y-D system



Phase Quantities

Phase currents (I_ϕ) and phase voltages (V_ϕ) are associated with the individual loads and sources

.

For the Y-connection, the phase currents are $\boldsymbol{I}_a$, $\boldsymbol{I}_b$, $\boldsymbol{I}_c$. and the phase voltages are $\boldsymbol{V}_{an}$, $\boldsymbol{V}_{bn}$, $\boldsymbol{V}_{cn}$.

For the Δ-connection, the phase currents are $\boldsymbol{I}_{ab}$, $\boldsymbol{I}_{bc}$, $\boldsymbol{I}_{ca}$. and the phase voltages are $\boldsymbol{V}_{ab}$, $\boldsymbol{V}_{bc}$, $\boldsymbol{V}_{ca}$.

Line Quantities

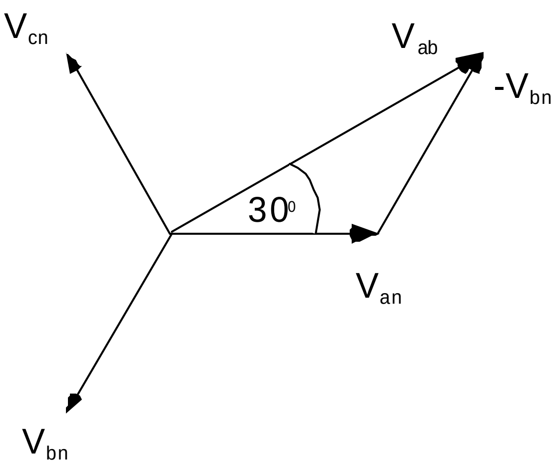
Line currents (I_L) and line voltages (V_L) are the currents in, and the voltages between, the lines between load and source.

They are the same for Y or Δ systems: the line voltages are $\boldsymbol{V}_{ab}$, $\boldsymbol{V}_{bc}$, $\boldsymbol{V}_{ca}$, while the line currents are $\boldsymbol{I}_a$, $\boldsymbol{I}_b$, $\boldsymbol{I}_c$.

In 3ϕ systems, voltages and currents are understood to be line quantities unless otherwise specified. Powers are understood to be 3ϕ powers.

KVL and KCL give the relations between phase quantities and line quantities.

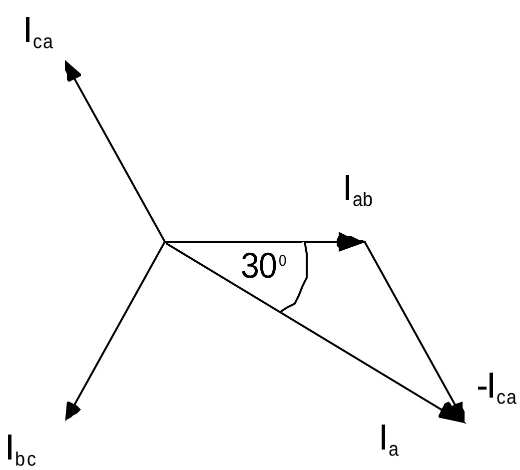
Y: $\boldsymbol{I}_L = \boldsymbol{I}_\phi$
$V_L = \sqrt{3}V_\phi$
the line voltage's phase leads the phase of phase voltages by 30°



KVL gives
$\boldsymbol{V}_{ab} + \boldsymbol{V}_{bn} - \boldsymbol{V}_{an} = 0$

Re-arranging gives:
$\boldsymbol{V}_{ab} = \boldsymbol{V}_{an} - \boldsymbol{V}_{bn}$

D: $\boldsymbol{V}_L = \boldsymbol{V}_\phi$
$I_L = \sqrt{3}\, I_\phi$
the line current/s phase lags the phase of phase voltages by 30°



KCL
$\boldsymbol{I}_a - \boldsymbol{I}_{ab} + \boldsymbol{I}_{ca} = 0$

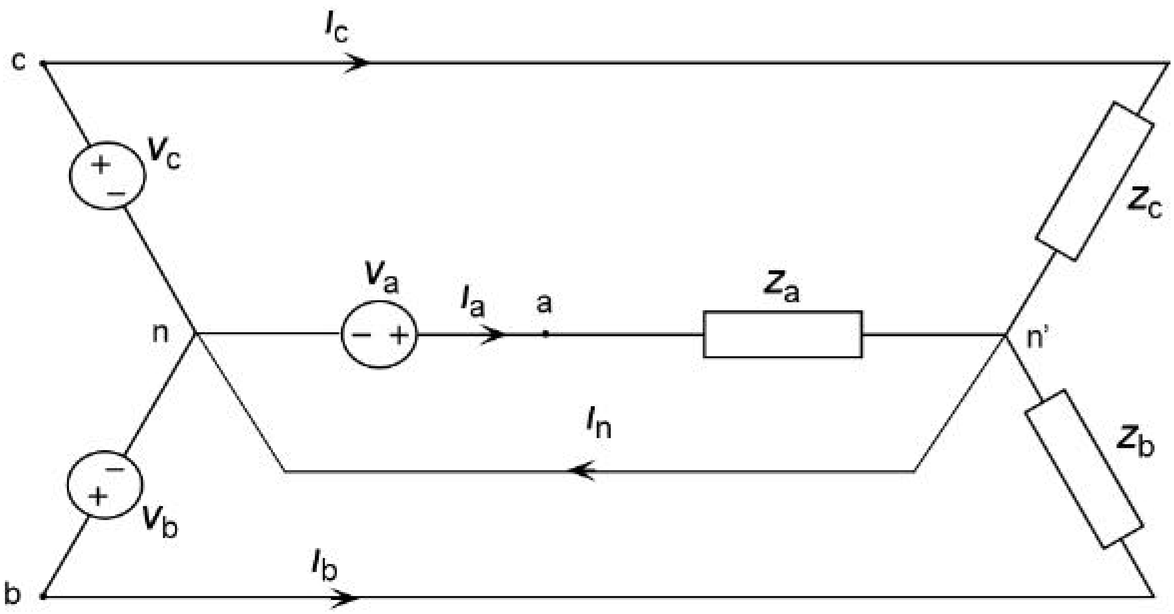
Re-arranging gives:
$\boldsymbol{I}_a = \boldsymbol{I}_{ab} - \boldsymbol{I}_{ca}$

Example

A 208V 3ϕ supply feeds a balanced Δ-connected load with $\boldsymbol{Z} = 120\angle 35^\circ\ \Omega$. Determine

i) the line and phase voltages and currents
ii) the real, reactive and apparent power supplied to the 3ϕ load.

Y-Y system



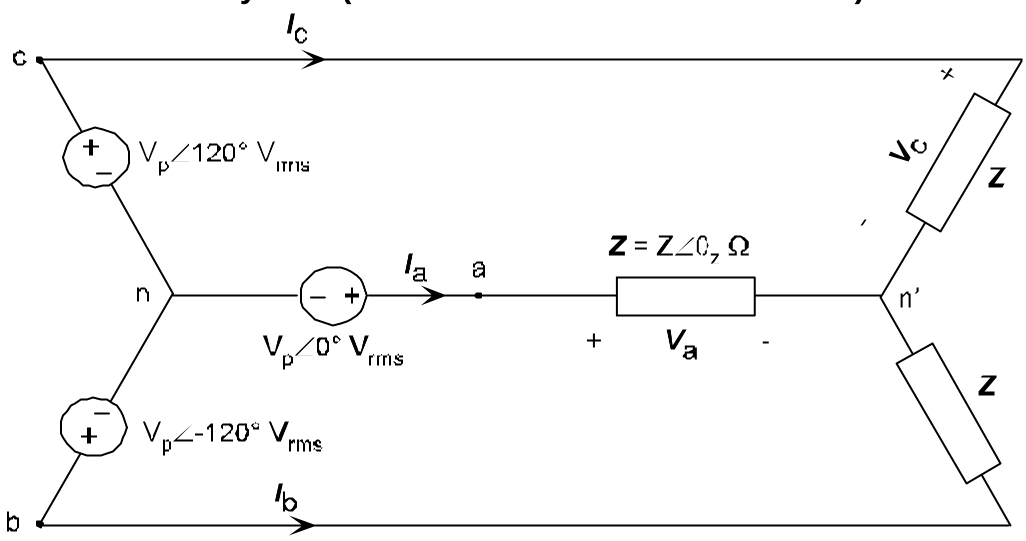
For this to be a balanced system Y-Y 3ϕ system

1. $\boldsymbol{Z}_a = \boldsymbol{Z}_b = \boldsymbol{Z}_c = \boldsymbol{Z}$
2. The source voltages have the same magnitude and differ 120° in phase.

For a balanced Y-Y system, KCL shows that $\boldsymbol{I}_n = 0$. This is important since the voltage between n and n' must be zero.

> The connection between n and n' has no current through it and no voltage across it. **Nothing would be changed if the connection were removed.** In particular, the voltage between n and n' would remain zero even without the connection present.

Balanced Y-Y System (with neutral connection removed)



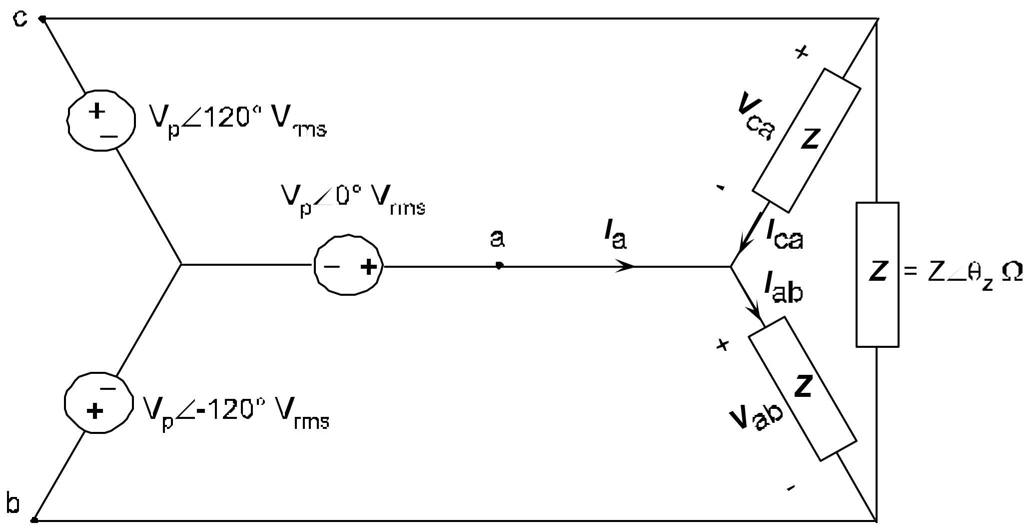
Analyzing balanced 3f systems

1ϕ equivalent from 3ϕ

- If loads are given as impedances, convert all Ds to **Y**s by dividing $\boldsymbol{Z}_{\Delta}$ by 3.
- If loads are given as powers, the powers will be 3ϕ powers. Divide 3ϕ power by 3 to obtain 1ϕ power.
- Divide all line voltages by v3 to produce Y phase voltages.
- Analyze the 1ϕ equivalent system.
- Multiply resulting phase voltages by v3 to produce line voltage results.
- Multiply resulting powers by 3 to give 3ϕ power.

1ϕ equivalent circuit

Balanced Y-D System



1φ equivalent circuit

Example

Two 3ϕ loads are connected in parallel and supplied at 480 V.

Load 1 is 15 kW, 0.7 lag pf, Y-connected.
Load 2 is 15 kVA, 0.8 lag pf, Δ-connected.

The feeder from the source has an impedance of 0.25 + j0.5 O/phase.

Determine

i) %VR
ii) P, Q, and S drawn from the source
iii) pf seen by the source

There are usually two main factors in the cost of electricity:

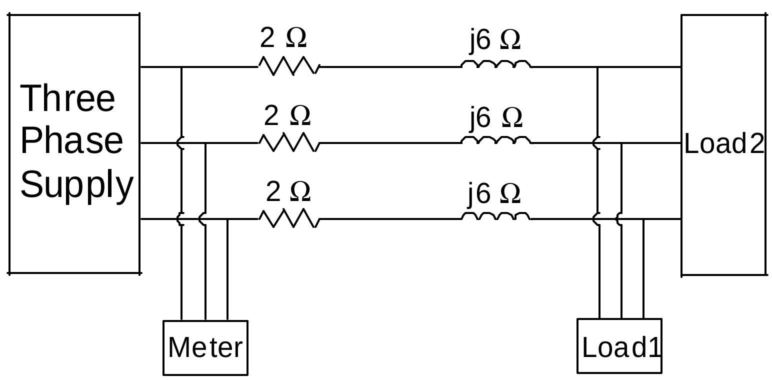
i) **Demand charge** which is based on the maximum kVA demanded in the month.

ii) **Energy charge** which is based on the total kWh consumed in the month.

Electric bills are greatly reduced if the pf is corrected to near unity. It is worth noting that power factors are not corrected to exactly unity because this would lead to the pf being over-correction when the load is small and produce a leading pf.

Example

The following diagram shows two loads supplied from a common 3ϕ feeder. Load 1 is 10 MW @ 0.65 lag, while load 2 is 15 MVA @ 0.75 lag. Both loads are supplied at 34.5 kV.



Notice that the meter is placed before the feeder. This means that the feeder losses will be part of the electricity bill.

Determine:

i) Current in the feeder

ii) Voltage regulation

iii) Cost of electricity ($/yr) for constant loads, energy costs 2.5 ¢/kWh, and demand charge is $7.00/KVA/mth.

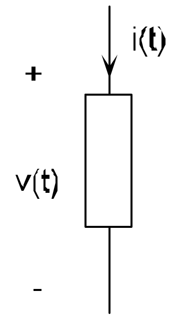
iv) The capacitance (µF/ph) of a wye-connected capacitor bank needed to improve the combined load pf to 0.98 lag.

v) Repeat parts (i) - (iii) with the new pf.

General formulas, nomenclature, units, and conventions in power

Time domain

$v(t) = V_m \cos(\omega t + \theta_v) = V_p \cos(\omega t + \theta_v)$
$i(t) = I_m \cos(\omega t + \theta_i) = I_p \cos(\omega t + \theta_i)$



Phasors with magnitude in peak value

$\boldsymbol{V}_p = V_p\angle\,\theta_v$
$\boldsymbol{I}_p = I_p\angle\,\theta_i$

Phasors in rms

$\boldsymbol{V} = V\angle\theta_v = (V_p/\sqrt{2})\angle\theta_v$
$\boldsymbol{I} = I\angle\theta_i = (I_p/\sqrt{2})\angle\theta_i$

If a phasor current or voltage is given, without other qualification, it's assumed to be in rms.

Power

$\boldsymbol{S} = S\angle(\theta_v-\theta_i) ? S\angle\theta_s ? P_{av} + jQ$

$\boldsymbol{S}$ ~ complex power, in VA
S ~ apparent power, in VA
P_{av} ~ average power, in W
Q ~ reactive power, in VAR
$\theta_s ?\ \theta_v - \theta_I = \theta_z$

If power is referred to, without any other qualification, it's assumed to be average power

$\boldsymbol{S} = \boldsymbol{VI}^* = V\angle\theta_v\ I\angle{-\theta_i} = VI\angle(\theta_v-\theta_i) = VI\cos(\theta_v-\theta_i) + j\,VI\sin(\theta_v-\theta_i)$

$S = VI$
$P_{av} = VI\cos(\theta_v-\theta_i) = VI\cos\theta_s$
$Q = VI\sin(\theta_v-\theta_i) = VI\sin\theta_s$

power factor = $\cos(\theta_v-\theta_i)$
lagging for $\theta_v>\theta_I$ ($\theta>0$, $Q>0$, inductive loads)
leading for $\theta_v<\theta_I$ ($\theta<0$, $Q<0$, capacitive loads)

3f power

- *Unless otherwise specified, voltages and currents are assumed to be line quantities.*
- *Power, unless otherwise specified, is assumed to be average 3f power.*
- *Any power, unless otherwise specified, is assumed to be 3f.*

A 3ϕ system is balanced if

1. Phase voltages are equal in magnitude and displaced 120° in phase from each other.
2. Phase load impedances are equal.

for balanced Y-connection, $V_L = \sqrt{3}V_p$, $I_L = I_p$
for balanced Δ-connection, $V_L = V_p$, $I_L = \sqrt{3}I_p$

$\theta_S = \theta_Z = \theta_V - \theta_i$
$\boldsymbol{S} = S\angle\theta_S = P + jQ$
$= \sqrt{3}\,\boldsymbol{VI^*} = \sqrt{3}\,VI\cos\theta_S + j\sqrt{3}\,VI\sin\theta_S$