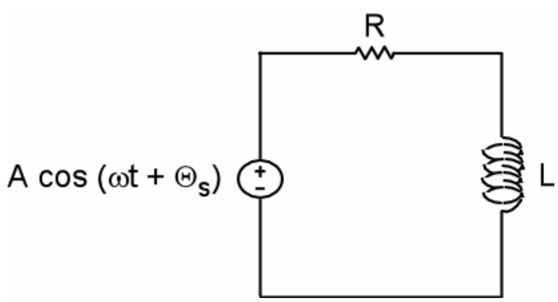
Sinusoidal steady-state analysis

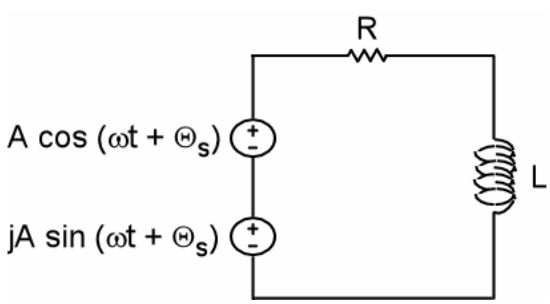
Phasor analysis is a technique to find the steady-state response when the system input is a sinusoid. That is, phasor analysis is sinusoidal analysis.

Phasor analysis is a powerful technique with which to find the steady-state portion of the complete response. Phasor analysis does **not** find the transient response. Phasor analysis does not find the complete response.

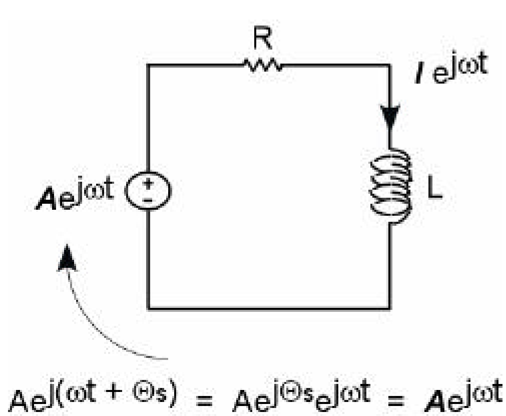
Original circuit



Add an imaginary sine source to obtain



Use Euler's relation to obtain



The differential equation becomes

$$R\,\boldsymbol{I}_L\, e^{j\omega t} + L\, d(\boldsymbol{I}_L\, e^{j\omega t})/dt = \boldsymbol{V}_s\, e^{j\omega t}$$

$$R\,\boldsymbol{I}_L\, e^{j\omega t} + j\omega L\,\boldsymbol{I}_L\, e^{j\omega t} = \boldsymbol{V}_s\, e^{j\omega t}$$

$$R\,\boldsymbol{I}_L + j\omega L\,\boldsymbol{I}_L = \boldsymbol{V}_s$$

The complex currents and voltages in the equations above are called phasors—phasor currents and phasor voltages.

Many use the convention of using RMS values when using phasor analysis in electrical circuits. That's what we'll do in this course from now on.

Notice that, in the equation above, the inductance appears as a "resistance" of $j\omega L$. This quantity is referred to as the inductance's impedance.

Impedance

The algebraic relationship between a phasor voltage and a phasor current is a generalization of resistance and is termed an element's impedance.

The unit of impedance is the ohm.

Resistance

Let's assume that all the voltages are of the form $\boldsymbol{V}\, e^{j\omega t}$ and all the currents are of the form $\boldsymbol{I}\, e^{j\omega t}$.

Let's look at the resistance's element relation, ohm's law.

$$\boldsymbol{V}\, e^{j\omega t} = R\,\boldsymbol{I}\, e^{j\omega t}$$

$$\boldsymbol{V} = R\,\boldsymbol{I}$$

The impedance of the resistance $\boldsymbol{Z}_r$ is just its resistance. That is,

$$\boldsymbol{Z}_r = \boldsymbol{V} / \boldsymbol{I} = R$$

Inductance

From the inductance's element relation:

$$\boldsymbol{V} e^{j\omega t} = L\, d(\boldsymbol{I} e^{j\omega t})/dt = j\omega L\, \boldsymbol{I} e^{j\omega t}$$
$$\boldsymbol{V} = j\omega L\, \boldsymbol{I}$$

$$\boldsymbol{Z}_L = j\omega L$$

Capacitance

From the capacitance's element relation:

$$\boldsymbol{I} e^{j\omega t} = C\, d(\boldsymbol{V} e^{j\omega t})/dt = j\omega C\, \boldsymbol{V} e^{j\omega t}$$
$$\boldsymbol{I} = j\omega C\, \boldsymbol{V}$$

$$\boldsymbol{Z}_C = 1/ j\omega C = -j/\omega C$$

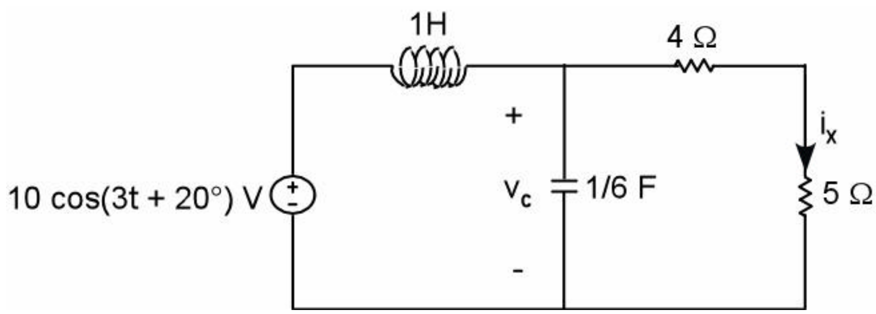
	Resistance	*Inductance*	*Capacitance*
impedance	$\boldsymbol{Z}_r = R$	$\boldsymbol{Z}_L = j\omega L$	$\boldsymbol{Z}_c = -j/\omega C$

Example

Find $v_c(t)$ and $i_x(t)$.

1. Find phasor circuit (give sources in RMS)
2. Write nodal equations
3. Solve for nodal analysissystem
4. Find the phasor voltages and/or currents of interest
5. Use the phasor information to provide the sinusoidal responses.

Time-domain circuit

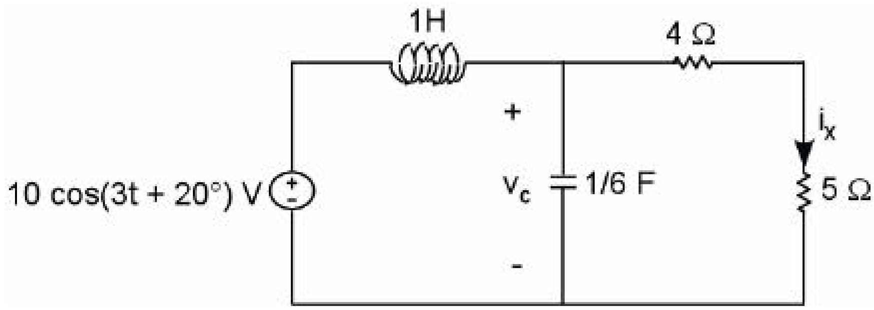


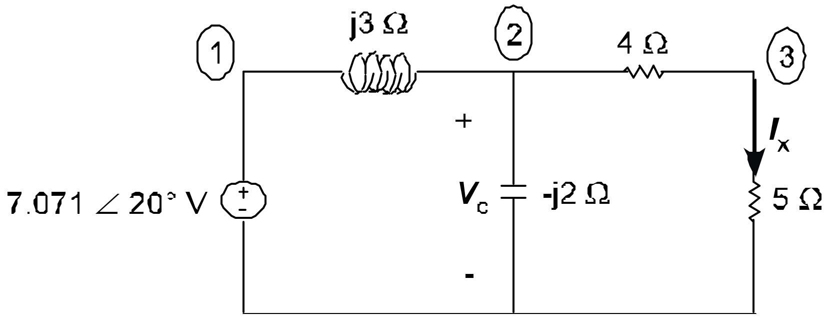
a) Find phasor circuit

b) Write the nodal equations to find the phasor voltage across the capacitance, $\boldsymbol{V}_c$ (express in polar form with the phase in degrees).

c) Find $v_c(t)$, the voltage across the capacitance as a function of time.

Perform nodal analysis on this circuit:





Using Maple

Phasor analysis example

```
> restart:
> alias(I=`I`,j=sqrt(-1)):
> eqns:={v1=7.071*exp(j*20*Pi/180),
> (v2-v1)/(j*3)+v2/(-j*2)+(v2-v3)/4=0,
> (v3-v2)/4+v3/5=0};
> soln:=solve(eqns):
> assign(soln):
> vc:=evalf(polar(v2),4);

  eqns := {9/20 v3 - 1/4 v2 = 0, v1 = (10/Ö2) exp(1/9 j Pi),

        - 1/3 j (v2 - v1) + 1/2 j v2 + 1/4 v2 - 1/4 v3 = 0}

                  vc := polar(11.77, -2.205)

> evalf(-2.205*180/Pi,4);

                          -126.3
```

check your work:

$v_c(t) = 11.77 \cos(3t - 126.3°)$ V, $i_x(t) = 1.31 \cos(3t - 126.3°)$ A

Discuss

Again, compare with the big gun technique. How many variables would have been required? How many equations?

Really good advice on phasor analysis

Far and away the best tool for phasor analysis is a good engineering calculator. ***Experience shows that you will avoid pain, work less, and learn more by taking this simple step.***