## Frequency Response Problems

## Conceptual Questions

$$
\tau \dot{x}+x=K\left\{(t) \quad \begin{array}{l}
\text { Used for the next } \\
\text { twe probems }
\end{array}\right.
$$

1) $\boldsymbol{T} / \boldsymbol{F}$ Given $f(t)=A \cos (\omega t+\theta)$ : The amplitude of the output in sinusoidal steady-state increases as K increases and decreases as $\omega$ increases.
2) $\boldsymbol{T} / \boldsymbol{F}$ Given $f(t)=A \cos (\omega t+\theta)$ : The magnitude of the phase shift (sinusoidal steady-state) between $x(t)$ and $f(t)$ increases as either $\tau$ or $\omega$ increases. The phase shift does not depend upon K or A .

used for the next three questions
3) What order is the above system? Explain your answer.
4) Give the transfer function for the above system.
5) What is the the magnitude of the static gain? Explain your answer.

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\text { Given the transfer function, } T F(s)=\frac{V_{0}(s)}{V_{\mathrm{in}}(\mathrm{~s})}=\frac{1000}{\mathrm{~s}+20} \text {, answer the next six questions. }
$$

6) $\quad \mathbf{T} / \boldsymbol{F}$ The low frequency magnitude in dB will be $20 \log _{10} 1000$.
7) $\quad T / F$ The high frequency slope for $|T F|_{\mathrm{dB}}$ will be $-20 \mathrm{~dB} /$ decade.
8) Give the differential equation describing the relationship between $v_{\text {in }}(t)$ and $v_{0}(t)$. Identify the order, the time constant, and the static gain coefficient.
9) $\boldsymbol{T} / \boldsymbol{F}$ If $v_{\text {in }-1}=5 \cos (10 \mathrm{t}) \mathrm{V}$ and $\mathrm{v}_{\text {in- }-2}=50 \cos (100 \mathrm{t}) \mathrm{V}$, the amplitude of $\mathrm{v}_{\text {out-1 }}$ will be greater than $v_{\text {out-2. }}$ Why or why not?
10) $T / F$ If $v_{\text {in }-1}=1000 \cos \left(10^{4} t\right) V$ and $v_{\text {in-2 }}=10 \cos (500 \mathrm{t}) \mathrm{V}$, the amplitude of $\mathrm{v}_{\text {out-2 }}$ is greater than $v_{\text {out-1 }}$. Why or why not?
11) $T / F$ The high frequency phase of $T F$ will be $-90^{\circ}$.

12) What order system is described by the above Bode magnitude plot.
13) Give a transfer function which would have the above Bode magnitude plot.

Note: The answer is not unique. There's more than one answers possible.
14) Give the differential equation describing the relationship between the output [assume $x(t)$ as the output] and the input [assume $f(t)$ as the input].
15) Identify the static gain coefficient, the damping ratio, and the natural frequency. Is the system overdamped or underdamped?

16) $\boldsymbol{T} / \boldsymbol{F}$ Increasing C in the high-pass filter will lower its break frequency.
17) $T / F$ Increasing $R_{\text {in }}$ in the bandpass filter has no effect on its lower break frequency .
18) $\boldsymbol{T} / \boldsymbol{F}$ Increasing C in the high-pass filter has no effect on its high-frequency gain.
19) $\boldsymbol{T} / \boldsymbol{F}$ Increasing $\mathrm{C}_{\text {in }}$ in the bandpass filter has no effect on its passband gain.


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20) T/F Time-domain response. Increasing $C$ will lower the magnitude of the static gain coefficient.
21) $T / F$ Time-domain response. Increasing $R_{f}$ will increase the time constant.
22) $T / F$ Frequency-domain response. Lowering $R_{\text {in }}$ will lower the break frequency.
23) $T / F$ Frequency-domain response. Increasing $R_{f}$ will increase magnitude of the $D C$ gain.


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24) $\boldsymbol{T} / \boldsymbol{F}$ If $\mathrm{V}_{\text {in }}=20 \cos$ (2000t) mV , the amplitude of $\mathrm{V}_{0}$ will be approximately 2 V .
25) $\boldsymbol{T} / \boldsymbol{F}$ If $\mathrm{R}_{\mathrm{in}}=2 \mathrm{k} \Omega, \mathrm{C}_{\text {in }}$ will be approximately 50 nF . Why or why not?
26) $T / F$ If $\mathrm{V}_{0}=50 \cos (30 \mathrm{t}) \mathrm{mV}$, the amplitude of $\mathrm{V}_{\text {in }}$ will be approximately $1 / 2 \mathrm{~V}$.
27) $T / F$ If a signal $V_{\text {in-signal }}=100 \cos (3000 \mathrm{t}) \mathrm{mV}$ were corrupted with low frequency noise $\mathrm{V}_{\text {in-noise }}=50 \cos (30 \mathrm{t}) \mathrm{mV}$, the signal-to-noise ratio at the output will be approximately 200.

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28) $T / F$ Circuit on left: $R_{f}=100 \mathrm{k} \Omega, R_{i n}=10 \mathrm{~K} \Omega, C=0.01 \mu \mathrm{~F}$. The circuit is a low-pass filter with a DC gain of 10 and a break frequency of $10^{4} \mathrm{r} / \mathrm{s}$.
29) $T / F$ Circuit in middle: $\mathrm{R}_{\mathrm{f}}=20 \mathrm{k} \Omega, \mathrm{R}_{\mathrm{in}}=4 \mathrm{~K} \Omega, \mathrm{C}=0.025 \mu \mathrm{~F}$. For $\mathrm{V}_{\mathrm{in}}=20 \cos (10 \mathrm{t}) \mathrm{mV},\left|\mathrm{V}_{\mathrm{o}}\right|$ is approximately 0.02 mV .
30) T/F Circuit on right:. $R_{f}=10 \mathrm{k} \Omega, R_{\text {in }}=5 \mathrm{~K} \Omega, C_{\text {in }}=0.1 \mu \mathrm{~F}, \mathrm{C}_{\mathrm{f}}=100 \mathrm{pF}$. The circuit is a bandpass filter with a passband gain of 2 , a lower corner frequency of $2000 \mathrm{r} / \mathrm{s}$ and an upper corner frequency of $10^{6} \mathrm{r} / \mathrm{s}$.

## Workout Problems

1. Using nodal analysis, find $\mathrm{H}(\mathrm{s})=\mathrm{V}_{\mathrm{o}}(\mathrm{s}) / \mathrm{V}_{\text {in }}(\mathrm{s})$.

2. Using nodal analysis and superposition to express $\mathrm{V}_{0}(\mathrm{~s})$ in terms of $\mathrm{V}_{1}(\mathrm{~s})$ and $\mathrm{V}_{2}(\mathrm{~s})$.
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3. The band-pass filter shown below has an upper corner frequency of $250 \mathrm{krad} / \mathrm{sec}$ and a lower corner frequency of $10 \mathrm{krad} / \mathrm{sec}$. The band-pass gain of this filter should be 40 dB . The largest resistor is to be 1 MO . Design this filter by determining:

i) $\quad R_{\text {in }}, C_{\text {in }}, R_{f}$, and $C_{f}$.
ii) Filter transfer function in the s-domain.
iii) Using semilog paper, sketch the Bode magnitude plot by hand.
iv) Using Matlab, find the Bode magnitude and phase plot.
4. Given the step response,

i) find $V_{0}(s)$.
ii) find $\mathrm{H}(\mathrm{s})$.
iii) Using semilog paper, sketch the Bode magnitude plot by hand.
5. Given the Bode magnitude plot below for $|\mathrm{H}|_{\mathrm{dB}}$,
$|\mathrm{H}| \mathrm{dB}$

i) Find the transfer function, $\mathrm{H}(\mathrm{s})$, which will have this magnitude plot.
ii) Design a circuit which will have this $\mathrm{H}(\mathrm{s})$.
iii) Find the output for this circuit, $v_{0}(t)$, when the input, $v_{\text {in }}(t)=u(t)$.
6. Given the transfer function $H(s)=240 \frac{s}{s^{2}+120 s+90000}$,
i) Determine the filter type
ii) Using semilog paper, sketch the Bode magnitude plot by hand
iii) Using Matlab, give the Bode magnitude and phase plot
iv) The sinusoidal steady-state response for $v_{0}(t)$ if $\mathrm{v}_{\text {in }}(\mathrm{t})=\left[2+1.414 \cos \left(360 \mathrm{t}+30^{\circ}\right)+2 \cos \left(10000 \mathrm{t}+50^{\circ}\right)\right] \mathrm{V}$.
7. A measurement system has been found, through experiment, to have the following equation of motion.
i) Determine the Static Gain and Time Constant.
ii) Write the transfer function, in the $s$ domain, for $v_{\text {measured }}$ as output and $v(t)_{\text {actual }}$ as input.
iii) Find $v(t)_{\text {actual, }}$ in steady-state, given $v(t)_{\text {measured }}=6+14 \cos \left(1500 t-16.9^{\circ}\right) \mathrm{V}$.
8. For the circuit shown below,
i) Find the general expression for the transfer function $\mathrm{V}_{0} / \mathrm{V}_{s}$.
ii) For $R_{L}=R_{s}=100 \Omega$ and $C=1 \mu F$, give the transfer function in Bode form
iii) Using semilog paper, sketch the Bode magnitude and phase plots by hand
iv) What are the circuit's filtering properties?
v) Find $v_{o}(t)$ for $v_{s}(t)=10+5 \cos \left(200000 t+30^{\circ}\right) V$

9. For the circuit shown below,
i) Find the general expression for the transfer function $V_{0} / V_{s}$.
ii) For $\mathrm{R}_{\text {in }}=2 \mathrm{~K} \Omega, \mathrm{R}_{\mathrm{f}}=10 \mathrm{~K} \Omega$ and $\mathrm{C}=0.01 \mu \mathrm{~F}$, give the transfer function in Bode form
iii) Using semilog paper, sketch the Bode magnitude and phase plots by hand
iv) What are the circuit's filtering properties?
v) Find $v_{o}(t)$ for $v_{s}(t)=10+5 \cos \left(200000 t+30^{\circ}\right) V$

10. Given the circuit below with $R_{s}=R_{L}=100 \Omega$ and $C=1 \mu F$,

i) Find the transfer function $V_{0}(\mathrm{~s}) / \mathrm{V}_{\text {in }}(\mathrm{s})$.

ii) Given a pulse train input for $v_{s}(t)$, find the first four non-zero terms in the Fourier series for $v_{s}(t)$. Perform the necessary integrals by hand.
iii) Using Maple or some other mathematical tool, plot the output waveform for $\mathrm{v}_{\mathrm{o}}(\mathrm{t})$.
11. Given the circuit below with $R_{s}=R_{L}=100 \Omega$ and $C=1 \mu F$,

i) Find the transfer function $\mathrm{V}_{0}(\mathrm{~s}) / \mathrm{V}_{\mathrm{in}}(\mathrm{s})$.

ii) Given a pulse train input for $\mathrm{v}_{\mathrm{s}}(\mathrm{t})$, find the first four non-zero terms in the Fourier series for $v_{s}(t)$. Perform the necessary integrals by hand.
iii) Using Maple or some other mathematical tool, plot the output waveform for $\mathrm{v}_{\mathrm{o}}(\mathrm{t})$.
12. Given the circuit below,
i) Find the transfer function $V_{0}(s) / V_{i}(s)$.
ii) Using semilog paper, sketch the Bode magnitude plot.

13. For the transfer function below,
i) Using semilog paper, sketch the Bode magnitude plot.
ii) Using computer tools, give the Bode magnitude and phase plots.
iii) What type of filtering does this system display?

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H(s)=\frac{5000 s}{s^{2}+105 s+500}
$$

14. For the transfer function below,
i) Using semilog paper, sketch the Bode magnitude plot.
ii) Using computer tools, give the Bode magnitude and phase plots.
iii) What type of filtering does this system display?

$$
H(s)=-\frac{5000}{s+1000}
$$

15. Given the straight-line approximation for a Bode magnitude plot shown below, write a corresponding transfer function. (Note: There is more than one correct solution.)

