## **Frequency Response Problems**

## **Conceptual Questions**

$$\tau \mathbf{\dot{x}} + \mathbf{x} = \mathbf{K} \mathbf{f}(\mathbf{t}) \qquad \boxed{ \begin{array}{c} \text{Used for the next} \\ \text{two problems} \end{array} }$$

- 1) **T/F** Given  $f(t) = A \cos (\omega t + \theta)$ : The amplitude of the output in sinusoidal steady-state increases as K increases and decreases as  $\omega$  increases.
- 2) **T/F** Given  $f(t) = A \cos(\omega t + \theta)$ : The magnitude of the phase shift (sinusoidal steady-state) between x(t) and f(t) increases as either  $\tau$  or  $\omega$  increases. The phase shift does not depend upon K or A.



- 3) What order is the above system? Explain your answer.
- 4) Give the transfer function for the above system.
- 5) What is the the magnitude of the static gain? *Explain your answer.*

Given the transfer function,  $TF(s) = \frac{V_o(s)}{V_o(s)} = \frac{1000}{s+20}$ , answer the next **six** questions.

- 6) **T/F** The low frequency magnitude in dB will be  $20\log_{10}1000$ .
- 7) **T/F** The high frequency slope for  $|TF|_{dB}$  will be -20dB/decade.
- B) Give the differential equation describing the relationship between v<sub>in</sub>(t) and v<sub>o</sub>(t). Identify the order, the time constant, and the static gain coefficient.
- 9) **T/F** If  $v_{in-1} = 5 \cos(10t)$  V and  $v_{in-2} = 50 \cos(100t)$  V, the amplitude of  $v_{out-1}$  will be greater than  $v_{out-2}$ . *Why or why not*?
- 10) **T/F** If  $v_{in-1} = 1000 \cos(10^4 \text{ t}) \text{ V}$  and  $v_{in-2} = 10 \cos(500 \text{ t}) \text{ V}$ , the amplitude of  $v_{out-2}$  is greater than  $v_{out-1}$ . Why or why not?
- 11) *T/F* The high frequency phase of *TF* will be -90°.



- 12) What order system is described by the above Bode magnitude plot.
- 13) Give a transfer function which would have the above Bode magnitude plot. *Note:* The answer is not unique. There's more than one answers possible.
- 14) Give the differential equation describing the relationship between the output [assume x(t) as the output] and the input [assume f(t) as the input].
- 15) Identify the static gain coefficient, the damping ratio, and the natural frequency. Is the system overdamped or underdamped?



used for next **fou**r questions

- 16) *T/F* Increasing C in the *high-pass* filter will lower its break frequency.
- 17) T/F Increasing R<sub>in</sub> in the *bandpass* filter has no effect on its lower break frequency.
- 18) *T/F* Increasing C in the *high-pass* filter has no effect on its high-frequency gain.
- 19) **T/F** Increasing C<sub>in</sub> in the *bandpass* filter has no effect on its passband gain.



- 20) **T/F** Time-domain response. Increasing C will lower the magnitude of the static gain coefficient.
- 21) T/F Time-domain response. Increasing  $R_f$  will increase the time constant.
- 22) **T/F** Frequency-domain response. Lowering R<sub>in</sub> will lower the break frequency.
- 23) T/F Frequency-domain response. Increasing R<sub>f</sub> will increase magnitude of the DC gain.



- 24) T/F If  $V_{in} = 20 \cos (2000t)$  mV, the amplitude of  $V_0$  will be approximately 2 V.
- 25) **T/F** If  $R_{in} = 2 k\Omega$ ,  $C_{in}$  will be approximately 50 nF. Why or why not?
- 26) T/F If  $V_o = 50 \cos (30t) \text{ mV}$ , the amplitude of  $V_{in}$  will be approximately  $\frac{1}{2} \text{ V}$ .
- 27) **T/F** If a signal  $V_{in-signal} = 100 \cos (3000t) \text{ mV}$  were corrupted with low frequency noise  $V_{in-noise} = 50 \cos (30t) \text{ mV}$ , the signal-to-noise ratio at the output will be approximately 200.



- 28) **T/F** Circuit on **left**:  $R_f = 100k\Omega$ ,  $R_{in} = 10 K\Omega$ ,  $C = 0.01\mu$ F. The circuit is a low-pass filter with a DC gain of 10 and a break frequency of 10<sup>4</sup> r/s.
- 29) **T/F** Circuit in **middle**:  $R_f = 20k\Omega$ ,  $R_{in} = 4K\Omega$ ,  $C = 0.025\mu$ F. For  $V_{in} = 20 \cos(10t)$  mV,  $|V_o|$  is approximately 0.02 mV.
- 30) *T/F* Circuit on **right**:.  $R_f = 10k\Omega$ ,  $R_{in} = 5K\Omega$ ,  $C_{in} = 0.1\mu$ F,  $C_f = 100$  pF. The circuit is a bandpass filter with a passband gain of 2, a lower corner frequency of 2000 r/s and an upper corner frequency of  $10^6$  r/s.

## **Workout Problems**

1. Using nodal analysis, find  $H(s) = V_o(s)/V_{in}(s)$ .



2. Using nodal analysis and superposition to express  $V_0(s)$  in terms of  $V_1(s)$  and  $V_2(s)$ .



 The band-pass filter shown below has an upper corner frequency of 250 krad/sec and a lower corner frequency of 10 krad/sec. The band-pass gain of this filter should be 40 dB. The largest resistor is to be 1 MO. Design this filter by determining:



- i)  $R_{\text{in}}$  ,  $C_{\text{in}}$  ,  $R_{\text{f}}$  , and  $C_{\text{f}}.$
- ii) Filter transfer function in the s-domain.
- iii) Using semilog paper, sketch the Bode magnitude plot by hand.
- iv) Using Matlab, find the Bode magnitude and phase plot.
- 4. Given the step response,



- i) find  $V_o(s)$ .
- ii) find H(s).
- iii) Using semilog paper, sketch the Bode magnitude plot by hand.

5. Given the Bode magnitude plot below for  $|H|_{dB}$ ,



- i) Find the transfer function, H(s), which will have this magnitude plot.
- ii) Design a circuit which will have this H(s).
- iii) Find the output for this circuit,  $v_o(t)$ , when the input,  $v_{in}(t) = u(t)$ .
- 6. Given the transfer function H(s) =  $240 \frac{s}{s^2 + 120s + 90000}$ ,
  - i) Determine the filter type
  - ii) Using semilog paper, sketch the Bode magnitude plot by hand
  - iii) Using Matlab, give the Bode magnitude and phase plot
  - iv) The sinusoidal steady-state response for  $v_o(t)$  if

 $v_{in}(t) = [2 + 1.414 \cos (360t + 30^{\circ}) + 2 \cos (10000t + 50^{\circ})] V.$ 

7. A measurement system has been found, through experiment, to have the following equation of motion.



- i) Determine the Static Gain and Time Constant.
- ii) Write the transfer function, in the s domain, for  $v_{measured}$  as output and  $v(t)_{actual}$  as input.
- iii) Find  $v(t)_{actual}$ , in steady-state, given  $v(t)_{measured} = 6 + 14 \cos(1500t 16.9^{\circ})$  V.

- 8. For the circuit shown below,
  - i) Find the general expression for the transfer function  $V_o / V_s$ .
  - ii) For  $R_L = R_s = 100 \Omega$  and  $C = 1 \mu F$ , give the transfer function in Bode form
  - iii) Using semilog paper, sketch the Bode magnitude and phase plots by hand
  - iv) What are the circuit's filtering properties?
  - v) Find  $v_o(t)$  for  $v_s(t) = 10 + 5 \cos (200000t + 30^\circ) V$



- 9. For the circuit shown below,
  - i) Find the general expression for the transfer function  $V_o / V_s$ .
  - ii) For  $R_{in} = 2 \text{ K}\Omega$ ,  $R_f = 10 \text{ K}\Omega$  and  $C = 0.01 \mu\text{F}$ , give the transfer function in Bode form
  - iii) Using semilog paper, sketch the Bode magnitude and phase plots by hand
  - iv) What are the circuit's filtering properties?
  - v) Find  $v_o(t)$  for  $v_s(t) = 10 + 5 \cos (200000t + 30^\circ) V$



10. Given the circuit below with  $R_s$  =  $R_L$  = 100  $\Omega$  and C = 1  $\mu F,$ 

i)



- ii) Given a pulse train input for  $v_s(t)$ , find the first four non-zero terms in the Fourier series for  $v_s(t)$ . Perform the necessary integrals by hand.
- iii) Using Maple or some other mathematical tool, plot the output waveform for  $v_o(t)$ .
- 11. Given the circuit below with  $R_s$  =  $R_L$  = 100  $\Omega$  and C = 1  $\mu F,$



i) Find the transfer function  $V_o(s)/V_{in}(s)$ .



- ii) Given a pulse train input for  $v_s(t)$ , find the first four non-zero terms in the Fourier series for  $v_s(t)$ . Perform the necessary integrals by hand.
- iii) Using Maple or some other mathematical tool, plot the output waveform for  $v_o(t)$ .

- 12. Given the circuit below,
  - i) Find the transfer function  $V_o(s)/V_i(s)$ .
  - ii) Using semilog paper, sketch the Bode magnitude plot.



- 13. For the transfer function below,
  - i) Using semilog paper, sketch the Bode magnitude plot.
  - ii) Using computer tools, give the Bode magnitude and phase plots.
  - iii) What type of filtering does this system display?

$$H(s) = \frac{5000s}{s^2 + 105s + 500}$$

- 14. For the transfer function below,
  - i) Using semilog paper, sketch the Bode magnitude plot.
  - ii) Using computer tools, give the Bode magnitude and phase plots.
  - iii) What type of filtering does this system display?

$$H(s) = -\frac{5000}{s + 1000}$$

15. Given the straight-line approximation for a Bode magnitude plot shown below, write a corresponding transfer function. (*Note: There is more than one correct solution.*)

