

Modules development under *NSF CCLI EMD project DUE-0088904*.

The topics for the modules listed below have been chosen to help achieve one or more of the following goals:

- 1) enhancing conceptual understanding of fundamental concepts (1, 2, 10, 11, 12, 14),
- 2) providing an interactive tool for a widely used technique (3, 4, 13), or
- 3) providing additional focus on often poorly understood topics (5, 6, 7, 8, 9).

While only one goal is matched to each module, it is clear that many of the modules listed below also help with one or more of the other goals.

***Please feel free to suggest any changes or modifications which improve the modules!***

#### List of Modules

Module 1	Parallel and series in simple DC circuit (voltage source)
Module 2	Parallel and series in simple DC circuit (current source)
Module 3	Nodal analysis (DC)
Module 4	Nodal analysis (AC)
Module 5	Voltage regulation (VR) and efficiency ( $\eta$ ) in simple AC circuit
Module 6	Power factor correction in simple AC circuit
Module 7	Using transformers to lower VR and $\eta$ in simple AC circuit
Module 8	Balanced 3 $\phi$ systems (Y-Y)
Module 9	Balanced 3 $\phi$ systems (Y- $\Delta$ )
Module 10	1 <sup>st</sup> -order systems: step, ramp, and sinusoidal response
Module 11*	2 <sup>nd</sup> -order systems: Step response, sinusoidal steady-state response
Module 12*	Dynamical systems and sinusoidal steady-state response
Module 13	Sketching Bode diagrams ( <b><i>may be the most challenging module</i></b> )
Module 14*	Simple feedback control system

The purpose of the conceptual questions (the ones beginning with ***True or false?***) accompanying some of the modules is to help students assess their level of understanding.

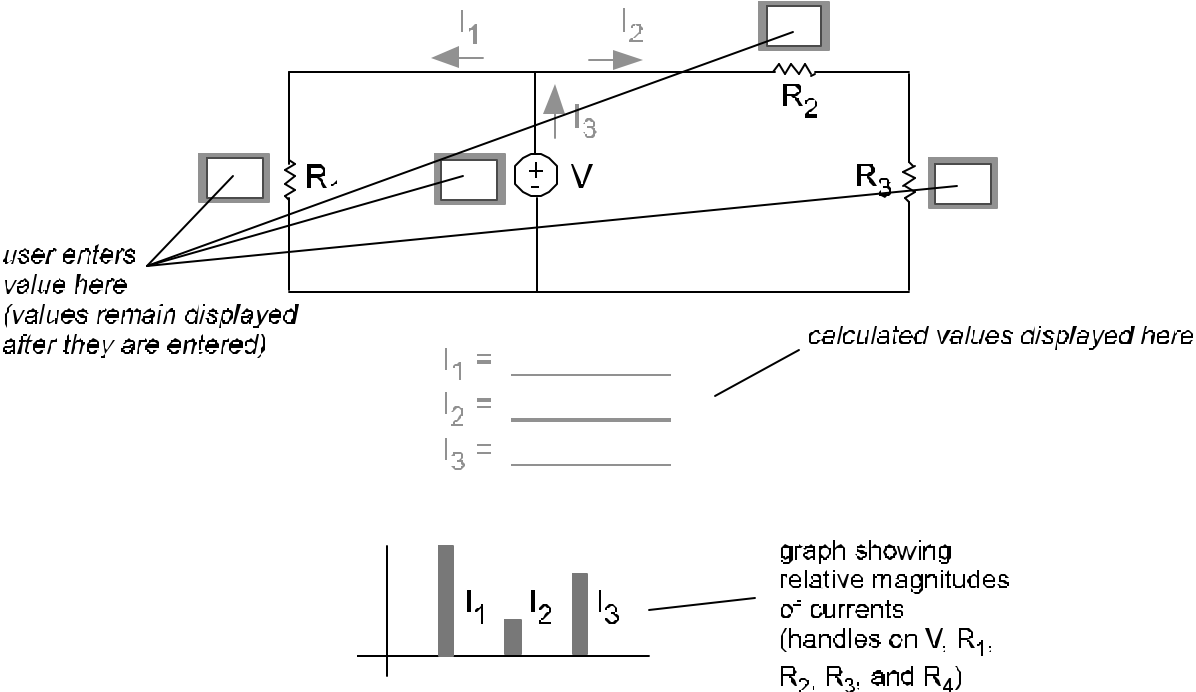
**Note 1:** In modules involving electric power, phasor voltages and currents are assumed RMS.

**Note 2:** Allowing parameters to have “handles” that allow them to be continuously varied will be useful in some of the modules, especially with those modules (1, 2, 5, 6, 7) having graphs showing relative variations.

\* *Not completed.*

**Module 1**

Module 1 is interactive and consists of a simple DC circuit. The user inputs values for the voltage source and resistances. The purpose of the module is to allow students to explore the consequences of elements (in this case resistances) in parallel and series.



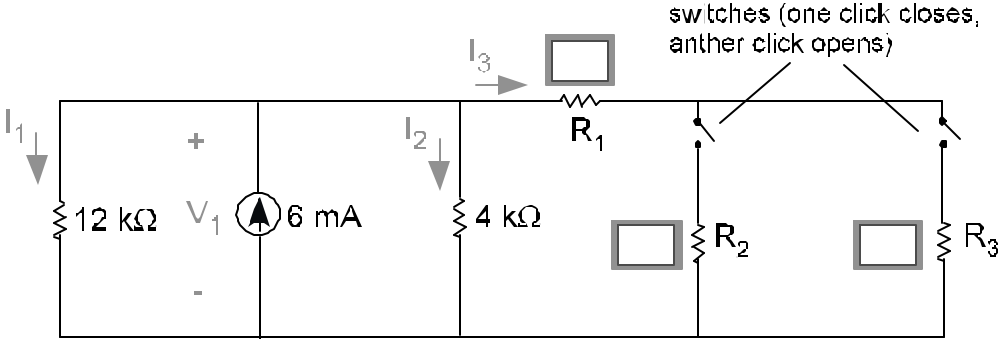
**True or false?**

1. If the resistance of  $R_3$  decreases, the power absorbed by  $R_2$  will decrease. *Why or why not?*
2. If the resistance of  $R_1$  decreases, the power absorbed by  $R_2$  will remain the same, and the power delivered by the voltage source will increase. *Why or why not?*

**Note:**  $R_1 = 0$  results in invalid model and does  $R_2 + R_3 = 0$ .

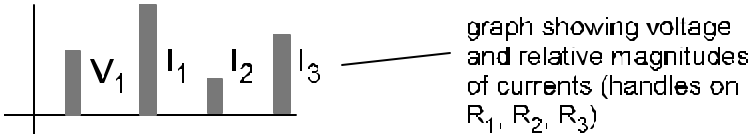
**Module 2**

Module 2 is interactive and consists of a simple DC circuit. The user inputs values for the resistances and can close and open the switches. The purpose of the module is to allow students to explore the consequences of elements (in this case resistances) in parallel and series.



$V_1 =$  \_\_\_\_\_       $I_1 =$  \_\_\_\_\_  
 $I_2 =$  \_\_\_\_\_  
 $I_3 =$  \_\_\_\_\_

*calculated values displayed here*



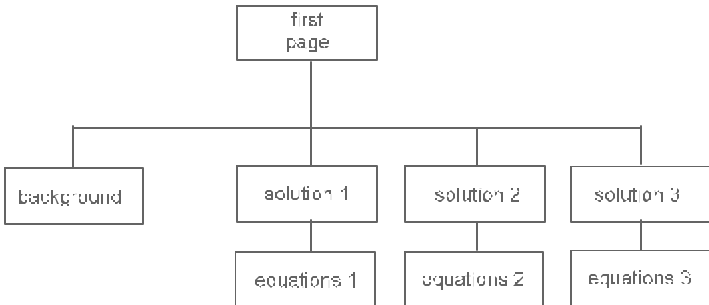
**True or false?**

1. If one or more of the switches are closed, the power delivered by the current source will increase from that delivered when all switches are open. *Why or why not?*
2. If one or more of the switches are closed,  $V_1$  will decrease from that present when all switches are open. *Why or why not?*
3. If one or more of the switches are closed, the current through  $I_1$  and  $I_2$  will increase from that present when all switches are open. *Why or why not?*

**Module 3**

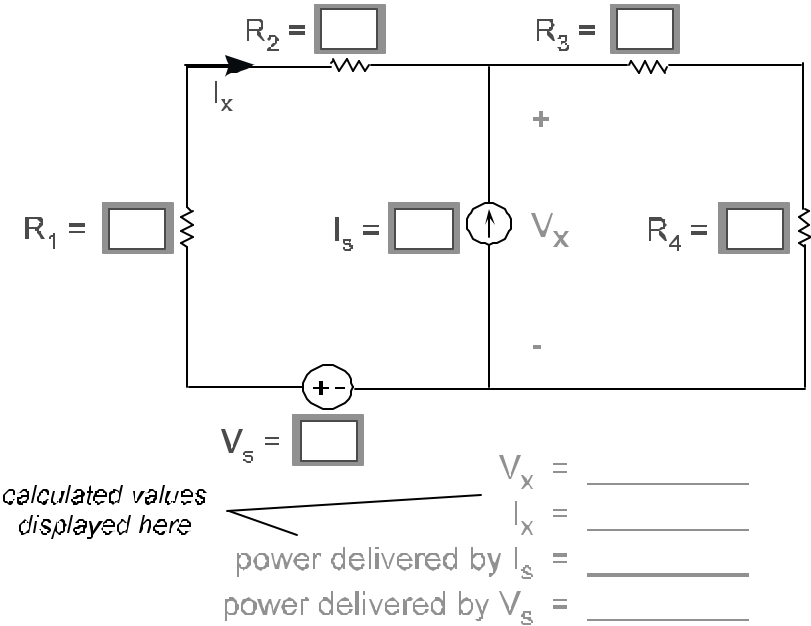
The purpose of module 3 is to illustrate the use of nodal analysis for a DC circuit. It is interactive with multiple pages. The first page will consist of an interactive unlabeled circuit. The user can work on this first page, entering numbers in for the circuit elements to check their own work.

Those just learning can click on **background**, **solution 1**, **solution 2**, or **solution 3**. **Background** is a static page describing the nodal technique. **Solution 1** page is an interactive circuit with a particular labeling. The user can click on **equations** to show the equations involved. **Solution 2** page is another interactive circuit with a particular labeling. The user can click on **equations** to show the equations involved. **Solution 3** page is another interactive circuit with a particular labeling. The user can click on **equations** to show the equations involved.



**first page**

- background
- solution 1
- solution 2
- solution 3



## background

first page

solution 1

solution 2

solution 3

### General

#### A. Writing node voltage equations

1. Choose a reference node.
2. Label node voltages.
3. If dependent sources are present, express control variables in terms of node voltages
4. Circle all voltage sources by supernodes
5. Write KCL equations for all nodes or supernodes not associated with the reference node.
6. Write element relations for voltage sources in terms of node voltages.

#### B. Solve for node voltages.

The result should be  $n-1$  independent equations in the  $n-1$  node voltages.

7. Solve equations for node voltages.

#### C. Use node voltages to find the variable in which you're interested.

Any quantity of interest can be expressed in terms of node voltages.

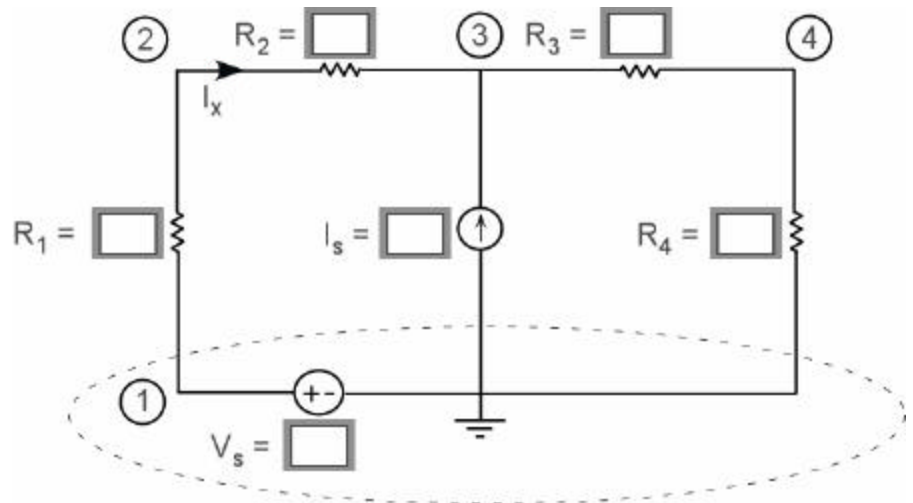
8. Express quantity of interest in terms of node voltages and solve.

### For this circuit

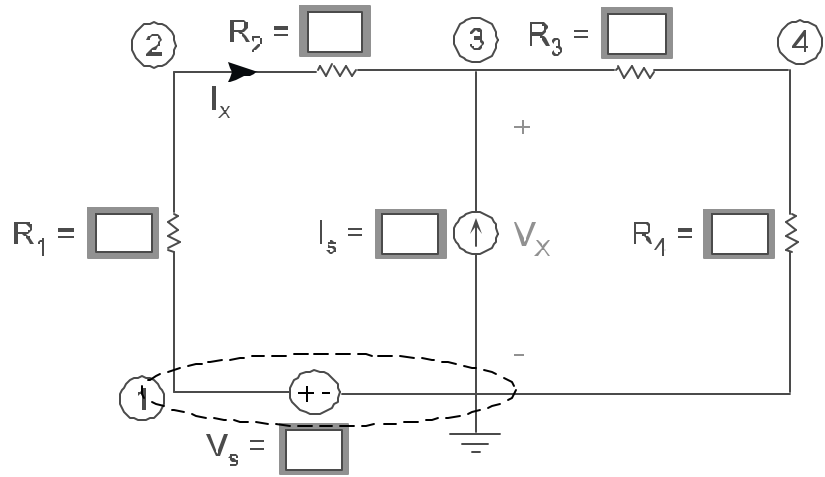
1. Assign Nodes. (one choice is shown)
2. Circle all voltage sources with supernodes.
3. Write KCL at all nodes except the reference and all supernodes that do not include the reference node.

In the circuit above the supernode includes the reference node; the supernode circles the reference node.

4. Write KCL at node 1, at node 2, and at node 3.
5. Write the element relation for  $V_s$ .
6. The result is 4 equations in 4 unknowns ( $V_1, V_2, V_3, V_4$ ).



**solution 1**



- |                            |       |         |       |
|----------------------------|-------|---------|-------|
| $V_x =$                    | _____ | $V_1 =$ | _____ |
| $I_x =$                    | _____ | $V_2 =$ | _____ |
| power delivered by $I_s =$ | _____ | $V_3 =$ | _____ |
| power delivered by $V_s =$ | _____ | $V_4 =$ | _____ |

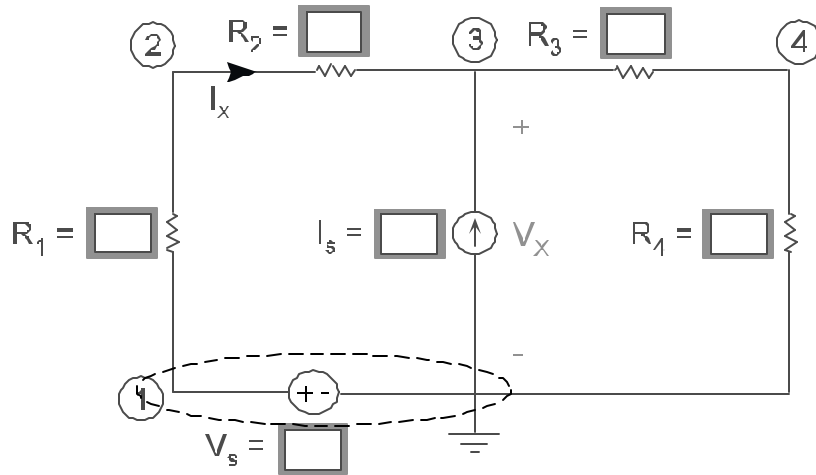
equations (from solution 1)

first page

solution 1

solution 2

solution 3



$$\begin{array}{ll}
 V_x = \underline{\hspace{2cm}} & V_1 = \underline{\hspace{2cm}} \\
 I_x = \underline{\hspace{2cm}} & V_2 = \underline{\hspace{2cm}} \\
 \text{power delivered by } I_s = \underline{\hspace{2cm}} & V_3 = \underline{\hspace{2cm}} \\
 \text{power delivered by } V_s = \underline{\hspace{2cm}} & V_4 = \underline{\hspace{2cm}}
 \end{array}$$

$$\text{node equations} \left\{ \begin{array}{l}
 V_1 = V_s \text{ (voltage source)} \\
 \frac{V_2 - V_1}{R_1} + \frac{V_2 - V_3}{R_2} = 0 \text{ (KCL at node 2)} \\
 \frac{V_3 - V_2}{R_2} + \frac{V_3 - V_4}{R_3} - I_s = 0 \text{ (KCL at node 3)} \\
 \frac{V_4 - V_3}{R_3} + \frac{V_4}{R_4} = 0 \text{ (KCL at node 4)}
 \end{array} \right.$$

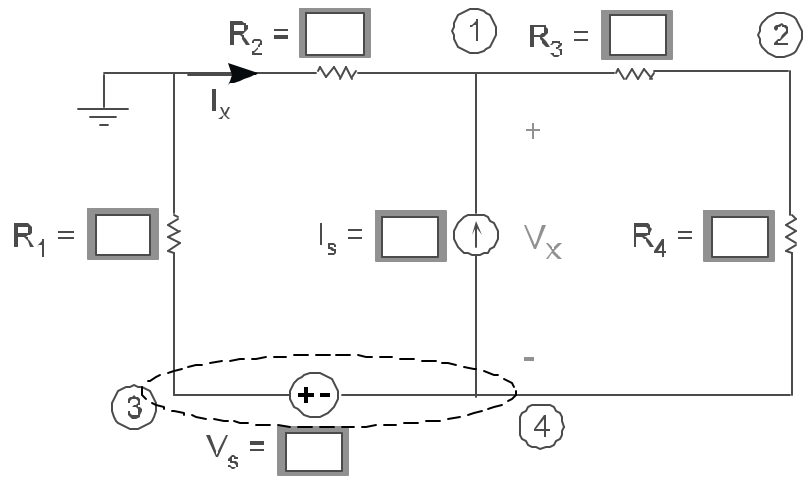
$$V_x = V_3$$

$$I_x = \frac{V_2 - V_3}{R_2}$$

$$\text{power delivered by current source} = V_3 I_s$$

$$\text{power delivered by voltage source} = V_s \frac{V_1 - V_2}{R_1}$$

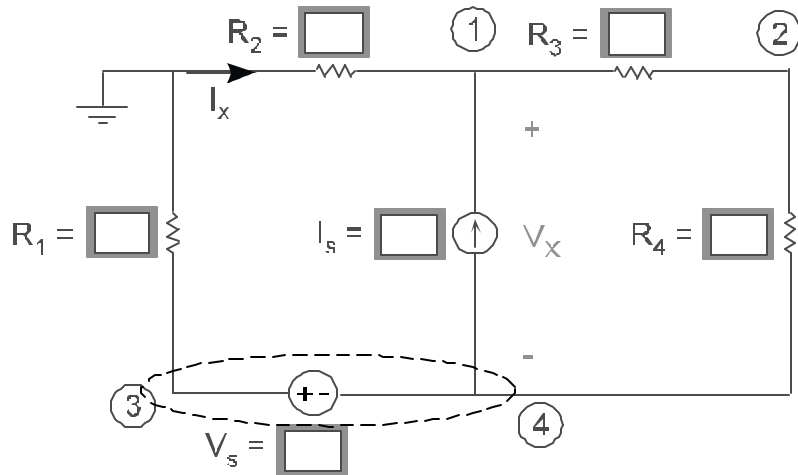
**solution 2**



$V_x =$	_____	$V_1 =$	_____
$I_x =$	_____	$V_2 =$	_____
power delivered by $I_s =$	_____	$V_3 =$	_____
power delivered by $V_s =$	_____	$V_4 =$	_____



equations (from solution 2)



$V_x =$	_____	$V_1 =$	_____
$I_x =$	_____	$V_2 =$	_____
power delivered by $I_s =$	_____	$V_3 =$	_____
power delivered by $V_s =$	_____	$V_4 =$	_____

$$\text{node equations } \left\{ \begin{array}{l} \frac{V_1}{R_2} + \frac{V_1 - V_2}{R_3} - I_s = 0 \quad (\text{KCL at node 1}) \\ \frac{V_2 - V_1}{R_3} + \frac{V_2 - V_4}{R_4} = 0 \quad (\text{KCL at node 2}) \\ V_3 - V_4 = V_s \quad (\text{voltage source}) \\ \frac{V_3}{R_1} + \frac{V_4 - V_2}{R_4} + I_s = 0 \quad (\text{KCL at supernode 3/4}) \end{array} \right.$$

$$V_x = V_1 - V_4$$

$$I_x = \frac{-V_1}{R_2}$$

$$\text{power delivered by current source} = (V_1 - V_4) I_s$$

$$\text{power delivered by voltage source} = V_s \frac{V_3}{R_1}$$

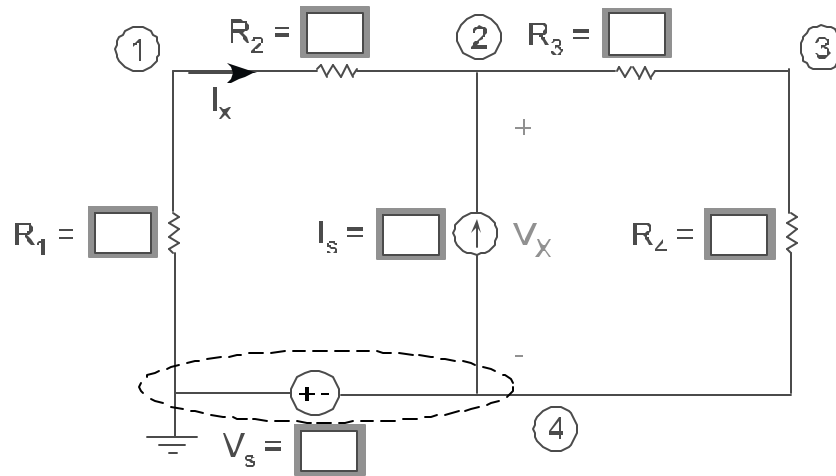
**solution 3**

first page

solution 1

solution 2

equations



$V_x =$  \_\_\_\_\_

$V_1 =$  \_\_\_\_\_

$I_x =$  \_\_\_\_\_

$V_2 =$  \_\_\_\_\_

power delivered by  $I_s =$  \_\_\_\_\_

$V_3 =$  \_\_\_\_\_

power delivered by  $V_s =$  \_\_\_\_\_

$V_4 =$  \_\_\_\_\_

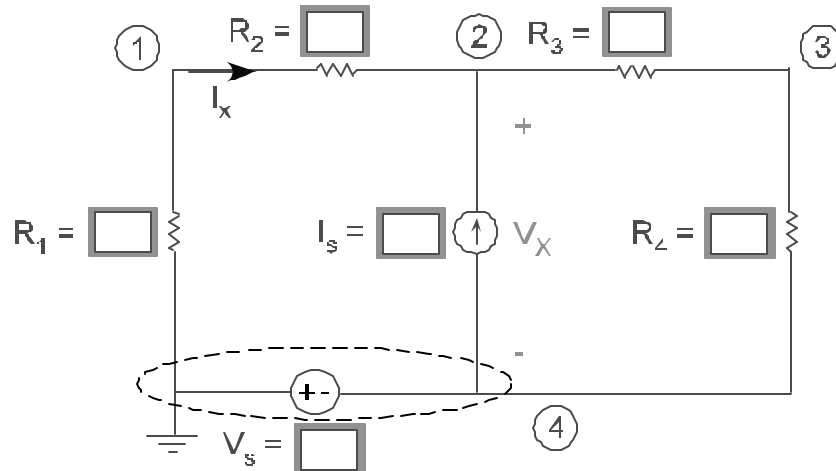
equations (from solution 3)

first page

solution 1

solution 2

solution 3



$$\begin{array}{ll}
 V_x = \underline{\hspace{2cm}} & V_1 = \underline{\hspace{2cm}} \\
 I_x = \underline{\hspace{2cm}} & V_2 = \underline{\hspace{2cm}} \\
 \text{power delivered by } I_s = \underline{\hspace{2cm}} & V_3 = \underline{\hspace{2cm}} \\
 \text{power delivered by } V_s = \underline{\hspace{2cm}} & V_4 = \underline{\hspace{2cm}}
 \end{array}$$

$$\text{node equations } \left\{ \begin{array}{l}
 \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} = 0 \quad (\text{KCL at node 1}) \\
 \frac{V_2 - V_1}{R_2} + \frac{V_2 - V_3}{R_3} - I_s = 0 \quad (\text{KCL at node 2}) \\
 \frac{V_3 - V_2}{R_3} + \frac{V_3 - V_4}{R_4} = 0 \quad (\text{KCL at node 3}) \\
 -V_4 = V_s \quad (\text{voltage source})
 \end{array} \right.$$

$$V_x = V_2 - V_4$$

$$I_x = \frac{-V_1}{R_1} = \frac{V_1 - V_2}{R_2}$$

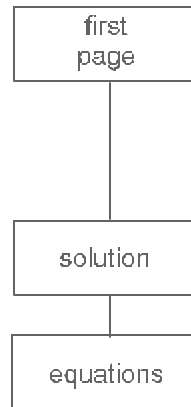
$$\text{power delivered by current source} = (V_2 - V_4) I_s$$

$$\text{power delivered by voltage source} = V_s \frac{-V_1}{R_1}$$

## Module 4

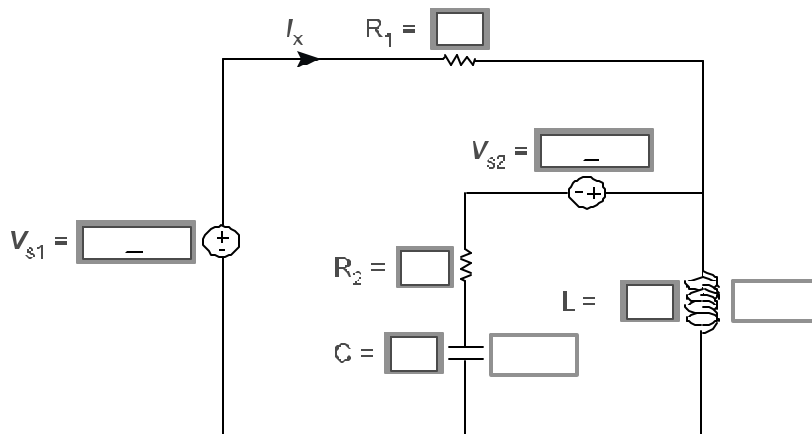
The purpose of module 4 is to illustrate the use of sinusoidal steady-state analysis using nodal analysis. The first page will consist of an interactive unlabeled phasor circuit. The user can work on this first page, entering numbers in for the circuit elements to check their own work.

Those learning can click on **solution** and can then click on **equations** to show the equations involved.



**first page**

**solution**



$m =$

RMS? *y/n*

current in polar form, magnitude (RMS for *y*, peak for *n*) and phase (in degrees)

$I_x =$  \_\_\_\_\_

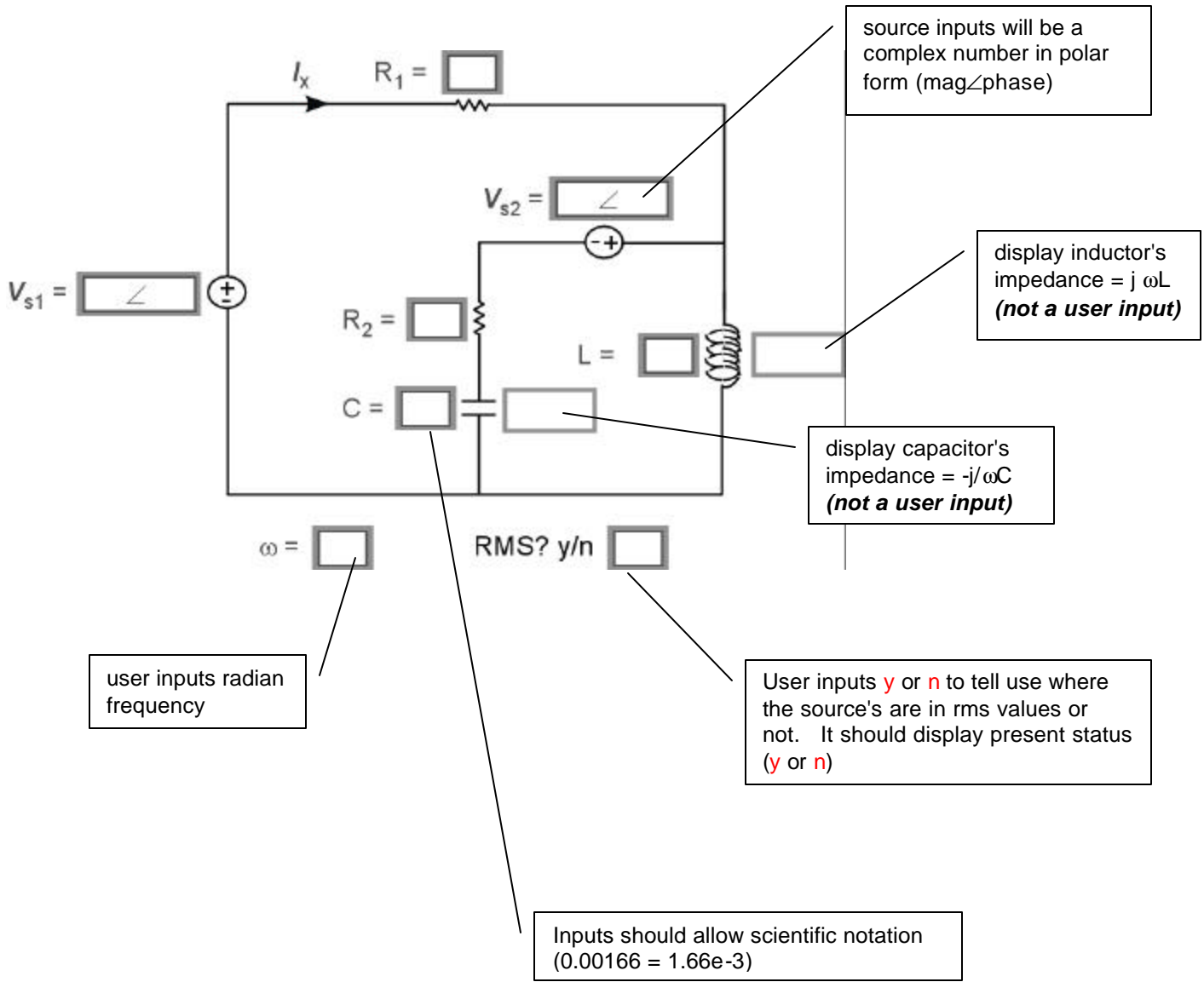
$S_{s1} =$  \_\_\_\_\_  $=$  \_\_\_\_\_

$S_{s2} =$  \_\_\_\_\_  $=$  \_\_\_\_\_

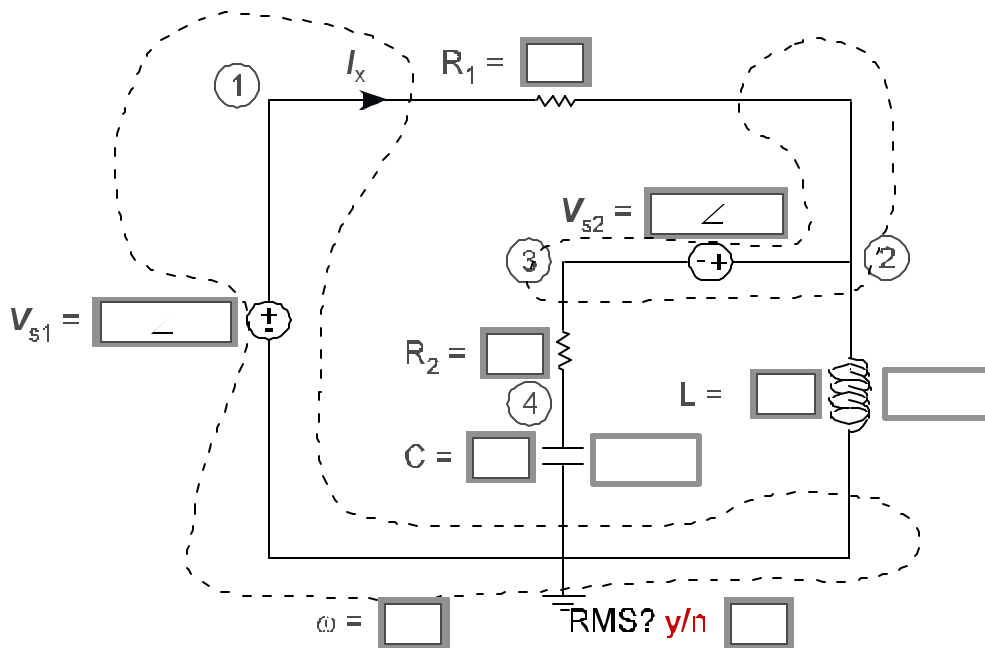
complex power absorbed by  $V_{s1}$  and  $V_{s2}$  respectively

In both rectangular and in polar form (phase in degrees)

Informational page to aid in module design



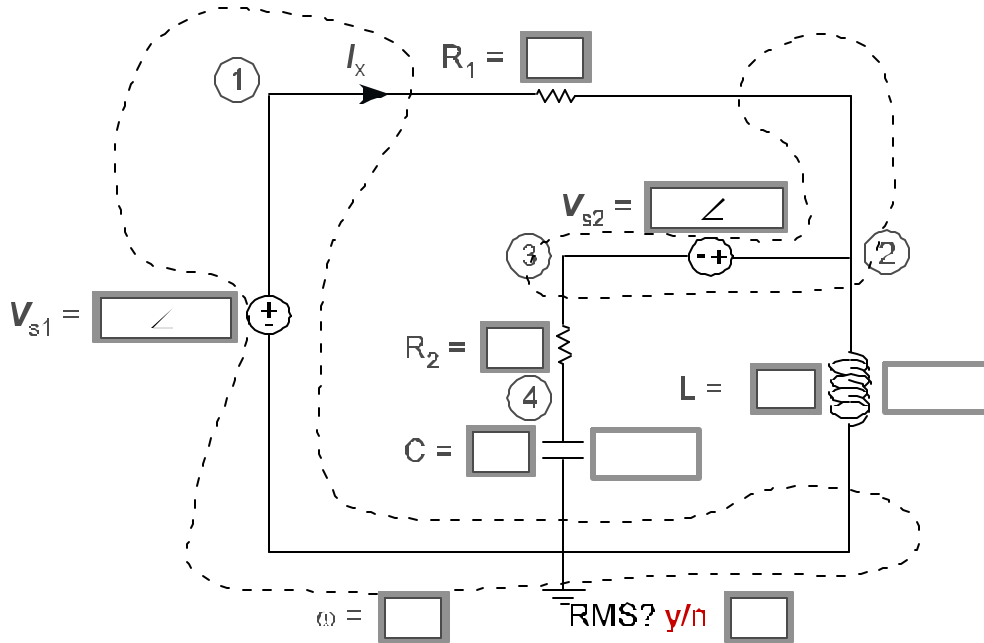
**solution**



$V_1 = \underline{\hspace{2cm}}$   
 $V_2 = \underline{\hspace{2cm}}$   
 $V_3 = \underline{\hspace{2cm}}$   
 $V_4 = \underline{\hspace{2cm}}$

$I_x = \underline{\hspace{2cm}}$   
 $S_{s1} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$   
 $S_{s2} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

equations



$V_1 =$  \_\_\_\_\_  
 $V_2 =$  \_\_\_\_\_  
 $V_3 =$  \_\_\_\_\_  
 $V_4 =$  \_\_\_\_\_

$I_x =$  \_\_\_\_\_  
 $S_{s1} =$  \_\_\_\_\_ = \_\_\_\_\_  
 $S_{s2} =$  \_\_\_\_\_ = \_\_\_\_\_

nodal equations

$$\left\{ \begin{array}{l} V_1 = V_{s1} \quad (\text{voltage source equation}) \\ V_2 - V_3 = V_{s2} \quad (\text{voltage source equation}) \\ \frac{V_4 - V_3}{R_2} + \frac{V_4}{-j/\omega C} = 0 \quad (\text{KCL at node 4}) \\ \frac{V_3 - V_4}{R_2} + \frac{V_2 - V_1}{R_1} + \frac{V_2}{j\omega L} = 0 \quad (\text{KCL at supernode 2/3}) \end{array} \right.$$

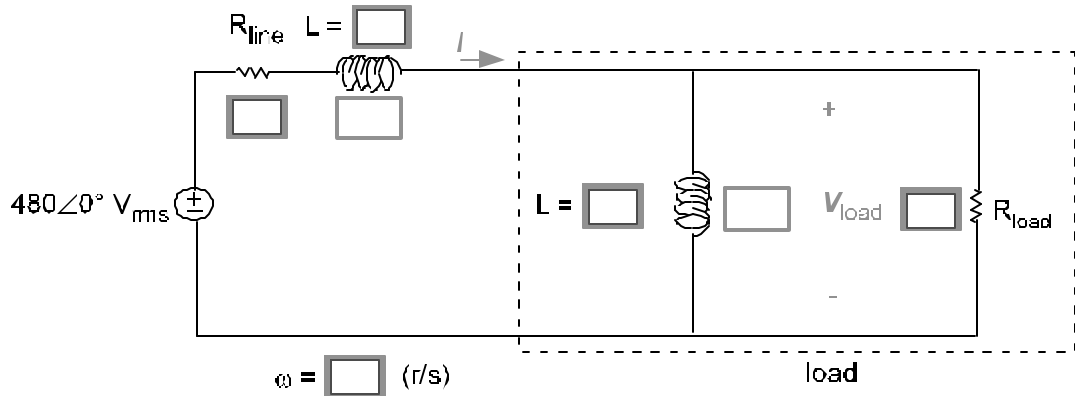
$$I_x = \frac{V_1 - V_2}{R_1}$$

$$S_{s1} = V_{s1} \left( \frac{V_1 - V_2}{R_1} \right)^* \quad \left[ = \frac{1}{2} V_{s1} \left( \frac{V_1 - V_2}{R_1} \right)^* \quad (\text{using peak values}) \right]$$

$$S_{s2} = V_{s2} \left( \frac{V_4 - V_3}{R_2} \right)^* \quad \left[ = \frac{1}{2} V_{s2} \left( \frac{V_4 - V_3}{R_2} \right)^* \quad (\text{using peak values}) \right]$$

## Module 5

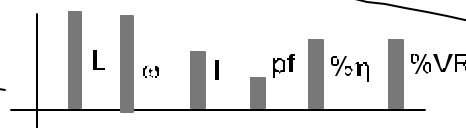
Module 5 is intended to demonstrate the effect of load inductance on voltage regulation and efficiency in power distribution systems.



$I = \text{[ ]}$   
 $V_{\text{load}} = \text{[ ]}$   
 $V_{\text{line}} = \text{[ ]}$

%VR =  $\text{[ ]}$   
 % $\eta$  =  $\text{[ ]}$   
 load PF =  $\text{[ ]}$

graph showing relative variations of L, I (current magnitude), power factor, percent efficiency, and percent voltage regulation (handles on R, L, and  $\omega$ )



calculated values displayed here

- ~ phasor current and voltages in RMS and polar form -- magnitude and phase (in degrees)
- ~ %VR = %voltage regulation =  $(480 - |V_{\text{load}}|) / |V_{\text{load}}|$
- ~ % $\eta$  = average power delivered to load / average power delivered by source  
 $= R_{\text{c}}(V_{\text{load}} I^*) / R_{\text{c}}(\angle 80 I^*)$
- ~ load pf =  $\cos(\theta_{\text{load voltage}} - \theta_{\text{current}})$

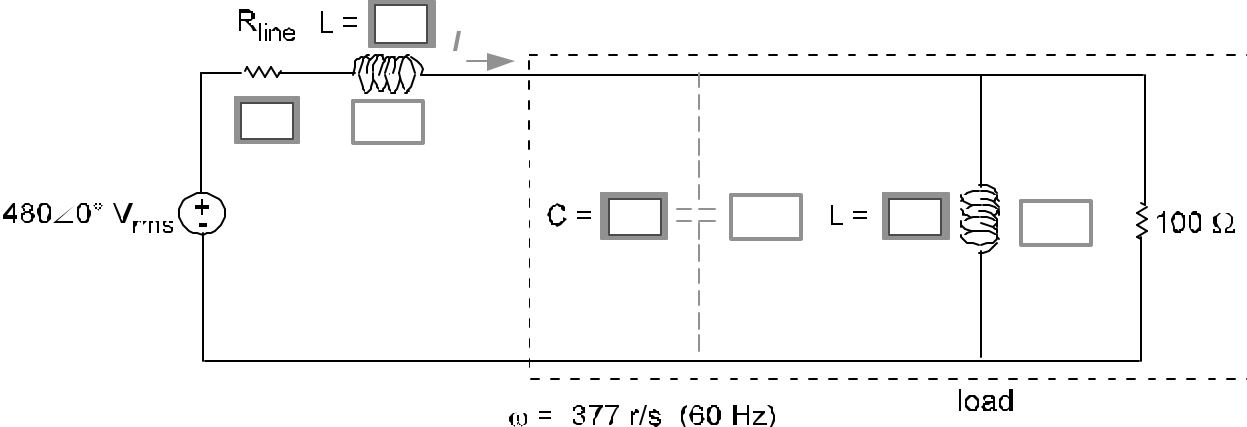
### True or false?

1. As L increases, the power absorbed by the 2  $\Omega$  line resistance will decrease.  
*Why or why not?*
2. As  $R_{\text{load}}$  increases, the pf will increase which will cause the efficiency to increase.  
*Why or why not?*
3. For a given  $R_{\text{line}}$  and  $R_{\text{load}}$ ,  $\eta$  will be the maximum possible when  $L=0$ .  
*Why or why not?*



**Module 6**

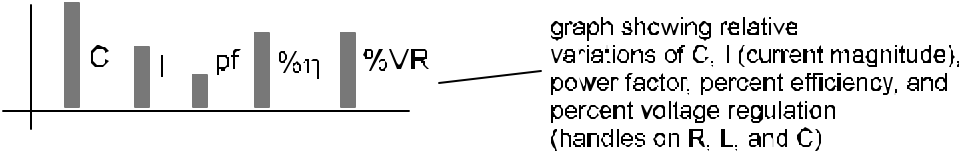
Module 6 is intended to demonstrate the effect of power factor correction on voltage regulation and efficiency in power distribution systems.



**note:** to avoid overcorrection,  $\omega L$  should be less than  $1/\omega C$

$I = \underline{\hspace{2cm}}$

$\%VR$	=	<u>                    </u>
$\%\eta$	=	<u>                    </u>
load PF	=	<u>                    </u>

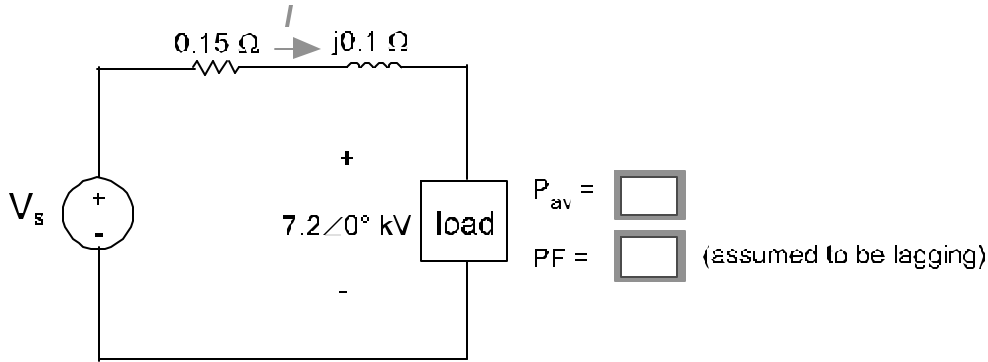


**True or false?**

1. The required C, in Farads, increases as the inductance, in Henries, increases.  
*Why or why not?*
  
2. As the pf is increased toward 1,  $|I|$  will decrease which will increase the %VR.  
*Why or why not?*

## Module 7

Module 7 is intended to demonstrate the effect of using transformers on voltage regulation and efficiency.

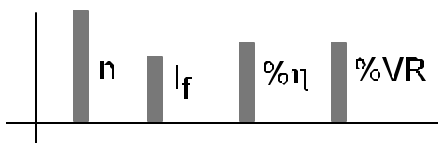
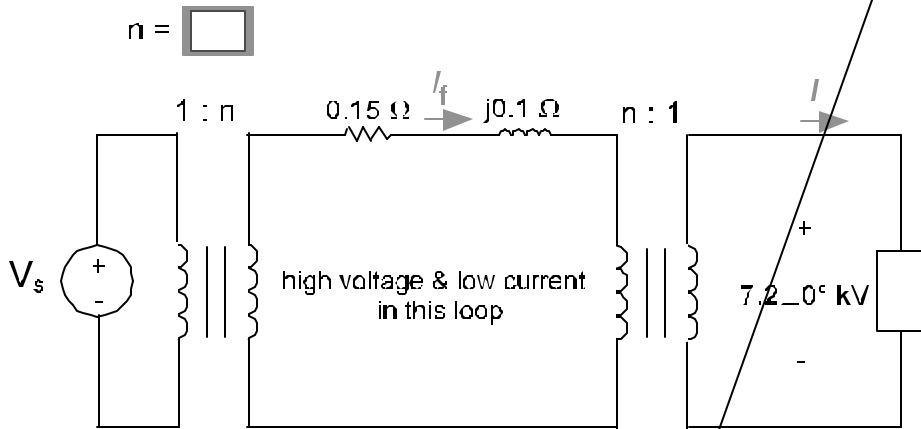


%VR = \_\_\_\_\_

% $\eta$  = \_\_\_\_\_

$I$  = \_\_\_\_\_

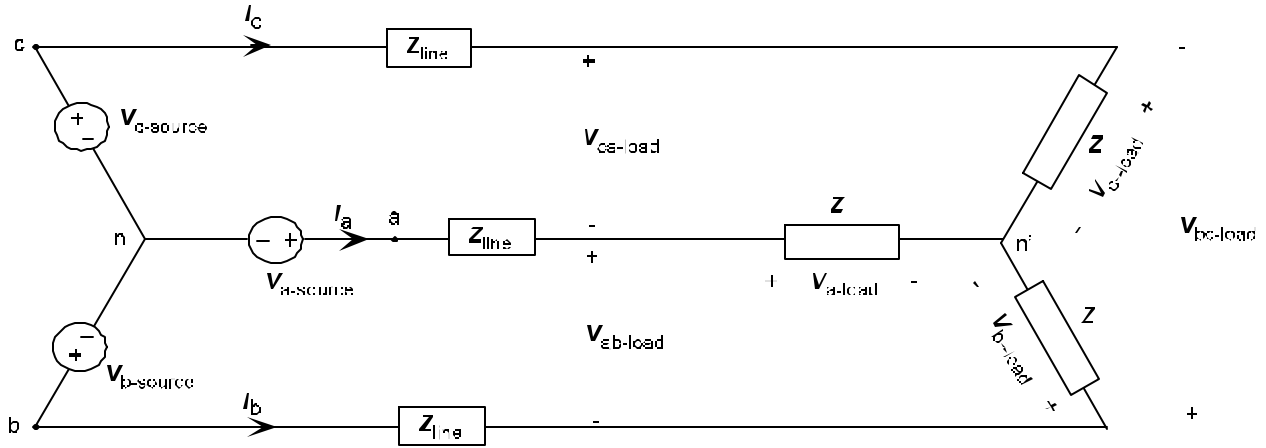
phasor currents given in polar form—magnitude (in RMS) and phase (in degrees)



graph showing relative variations of  $n$ ,  $I_f$  (current magnitude), percent efficiency, and percent voltage regulation (handles on  $n$ ,  $P_{av}$ , pf)

## Module 8

Module 8 is intended to allow the user to explore the relationships involved in a balanced 3 $\phi$  Y-Y system.



$V_{a-load}$  =  /  At load end. In polar form, magnitude in RMS, phase in degrees

$Z_{line}$  =  /

%VR = \_\_\_\_\_

% $\eta$  = \_\_\_\_\_

$Z$  =  /

at source

$V_a$	=	_____
$V_b$	=	_____
$V_c$	=	_____
$S_{delivered}$	=	_____
	=	_____

line currents

$I_a$  = \_\_\_\_\_

$I_b$  = \_\_\_\_\_

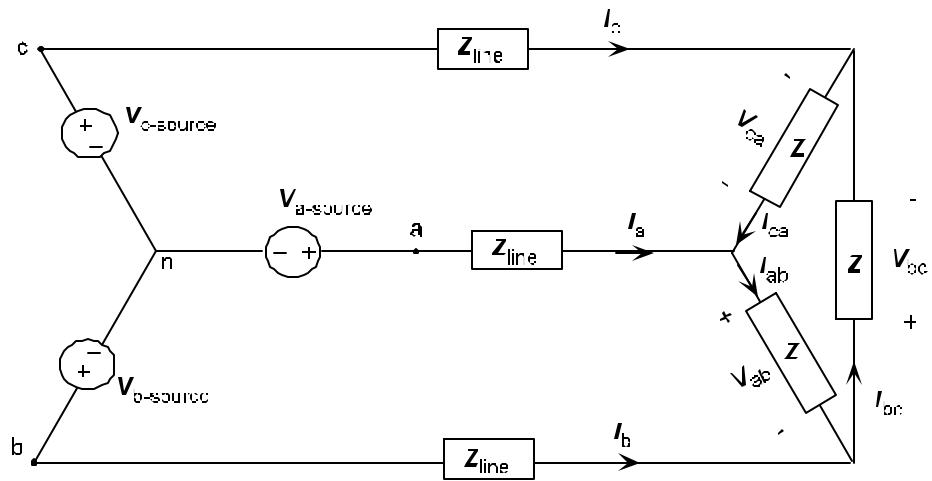
$I_c$  = \_\_\_\_\_

at load

$V_a$	=	_____
$V_b$	=	_____
$V_c$	=	_____
$V_{ab}$	=	_____
$V_{bc}$	=	_____
$V_{ca}$	=	_____
$I_{ab}$	=	_____
$I_{bc}$	=	_____
$I_{ca}$	=	_____
$S_{absorbed}$	=	_____
	=	_____

## Module 9

Module 9 is intended to allow the user to explore the relationships involved in a balanced 3 $\phi$  Y- $\Delta$  system.



$V_{ab} =$    $\angle$   *At load end. In polar form, magnitude in RMS, phase in degrees*

$Z_{line} =$    $\angle$

%VR = \_\_\_\_\_

% $\eta$  = \_\_\_\_\_

$Z =$    $\angle$

at source

$V_a =$	_____
$V_b =$	_____
$V_c =$	_____
$S_{delivered} =$	_____
	_____

line currents

$I_a =$  \_\_\_\_\_

$I_b =$  \_\_\_\_\_

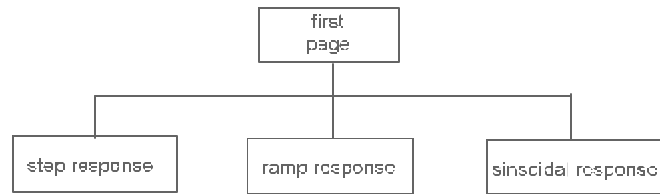
$I_c =$  \_\_\_\_\_

at load

$V_{ab} =$	_____
$V_{bc} =$	_____
$V_{ca} =$	_____
$I_{ab} =$	_____
$I_{bc} =$	_____
$I_{ca} =$	_____
$S_{absorbed} =$	_____
	_____

## Module 10

The purpose of module 10 is to allow the user to explore first-order systems via their step, ramp, and sinusoidal responses.



---

*first page*

step input

ramp input

sinusoidal input

$$t \frac{dx}{dt} + x = K f(t)$$
$$\left\{ \begin{array}{l} f(t) = \text{system input (excitation)} \\ x = \text{system output (response)} \\ t = \text{time constant} \\ K = \text{static gain coefficient} \end{array} \right.$$

**step input**

$$t \frac{dx}{dt} + x = K f(t) \quad \left\{ \begin{array}{l} f(t) = \text{system input (excitation)} \\ x = \text{system output (response)} \\ t = \text{time constant} \\ K = \text{static gain coefficient} \end{array} \right.$$

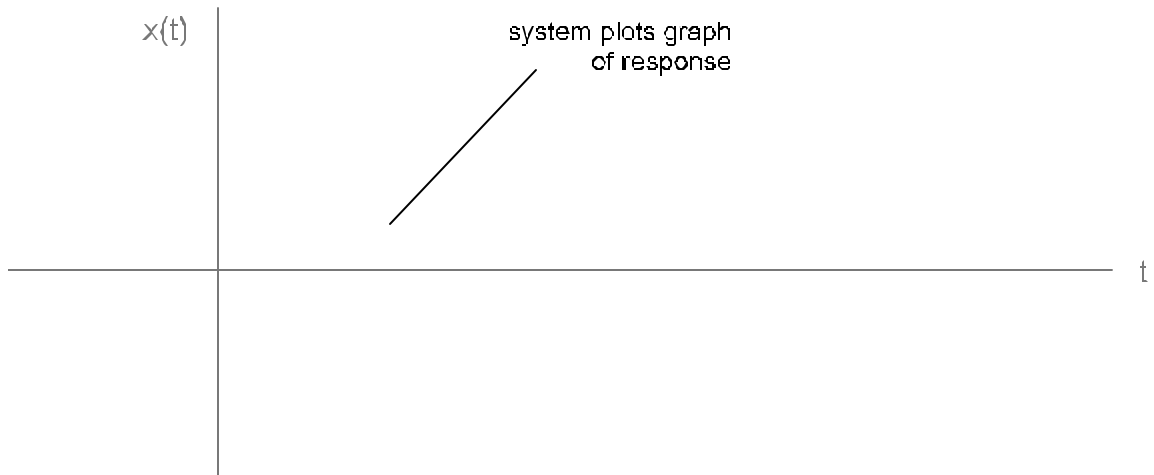
response for  $f(t) = A u(t-t_0)$  and  $x(t)|_{t=t_0} = x_0$ ,

$$x(t) = KA - (KA - x_0)e^{-(t-t_0)/\tau}$$

$K =$       $\tau =$       $t_0 =$       $x_0 =$

system gives analytic form of response

$x(t) =$  \_\_\_\_\_  $(t > t_0)$



**ramp input**

step input    first page    sinusoidal input

$$t \frac{dx}{dt} + x = K f(t) \quad \left\{ \begin{array}{l} f(t) = \text{system input (excitation)} \\ x = \text{system output (response)} \\ t = \text{time constant} \\ K = \text{static gain coefficient} \end{array} \right.$$

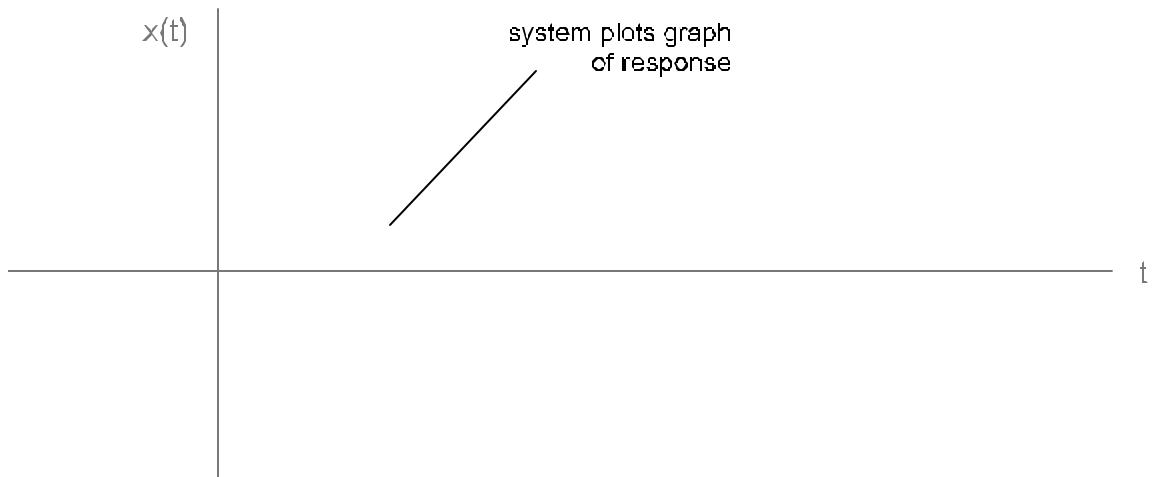
response for  $f(t) = A r(t-t_0)$  and  $x(t)|_{t=t_0} = x_0$ ,

$$x(t) = KA [(t - t_0) - \tau] - \{KA [-t_0 - \tau] - x_0\} e^{-\frac{(t-t_0)}{\tau}}$$

$K =$       $\tau =$       $t_0 =$       $x_0 =$

system gives analytic form of response

$x(t) =$  \_\_\_\_\_  $(t > t_0)$



**sinusoidal input**

$$t \frac{dx}{dt} + x = K f(t) \quad \left\{ \begin{array}{l} f(t) = \text{system input (excitation)} \\ x = \text{system output (response)} \\ \tau = \text{time constant} \\ K = \text{static gain coefficient} \end{array} \right.$$

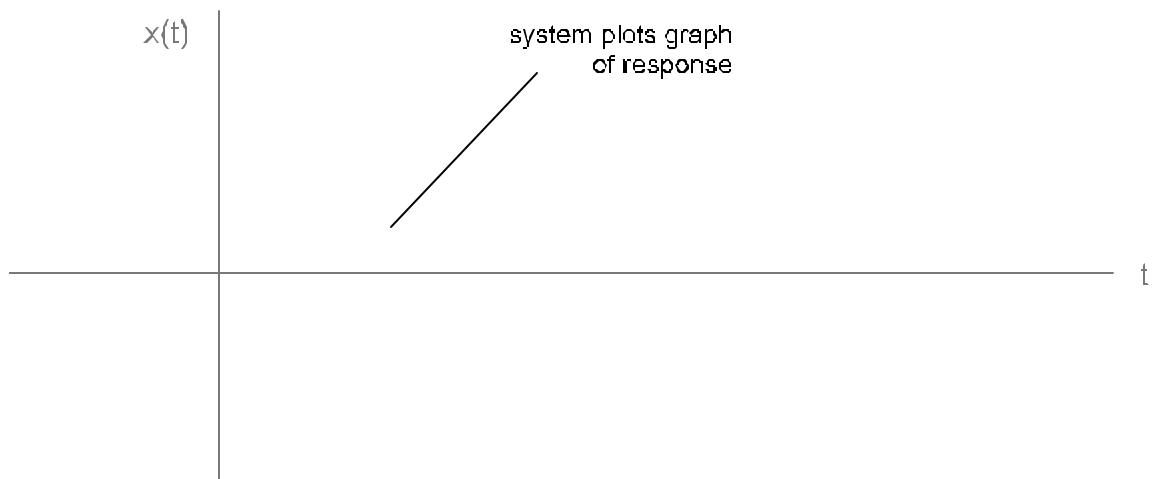
response for  $f(t) = A \cos(\omega t + \phi)$  and  $x(t)|_{t=t_0} = x_0$ ,

$$x(t) = \frac{KA}{\sqrt{1 + (\omega\tau)^2}} \cos[\omega t + \phi - \tan^{-1}(\omega\tau)] - \left\{ \frac{KA}{\sqrt{1 + (\omega\tau)^2}} \cos[\phi - \tan^{-1}(\omega\tau)] - x_0 \right\} e^{-\frac{(t-t_0)}{\tau}}$$

$K =$       $\tau =$       $t_0 =$       $x_0 =$

system gives analytic form of response

$x(t) =$  \_\_\_\_\_ ( $t > t_0$ )





**Module 11** (to follow)  
**Module 12** (to follow)

## Module 13

Module 13 is intended to help the user sketch Bode plots by hand. Understanding how Bode plots are sketched help students develop a deeper understanding of frequency domain analysis.

In the module, the user will enter a transfer function and the module provides

- 1) Factored transfer function in Bode form,
- 2) Straight line approximations for all individual factors (dotted lines) and the overall straight line approximation for the transfer function (solid line) and the actual magnitude and phase curves

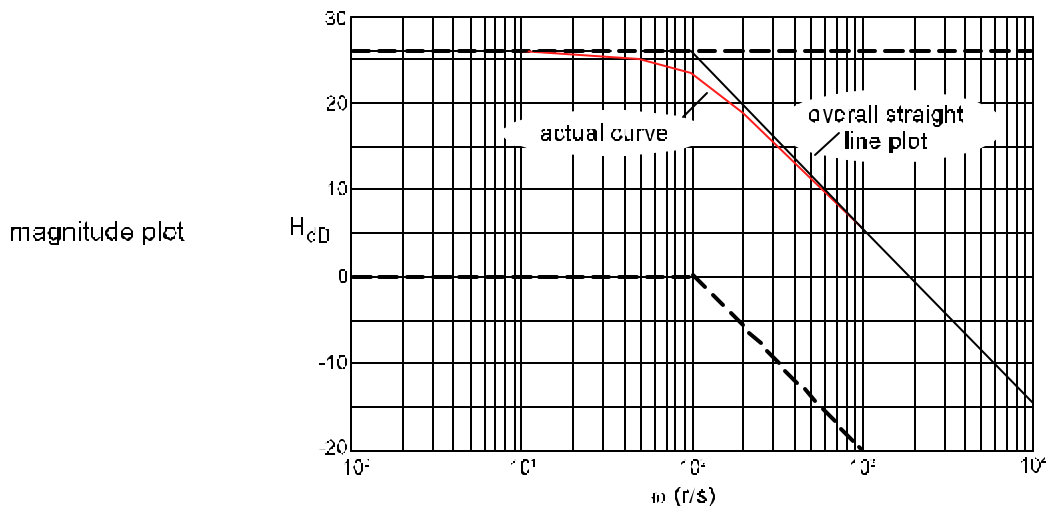
Example:

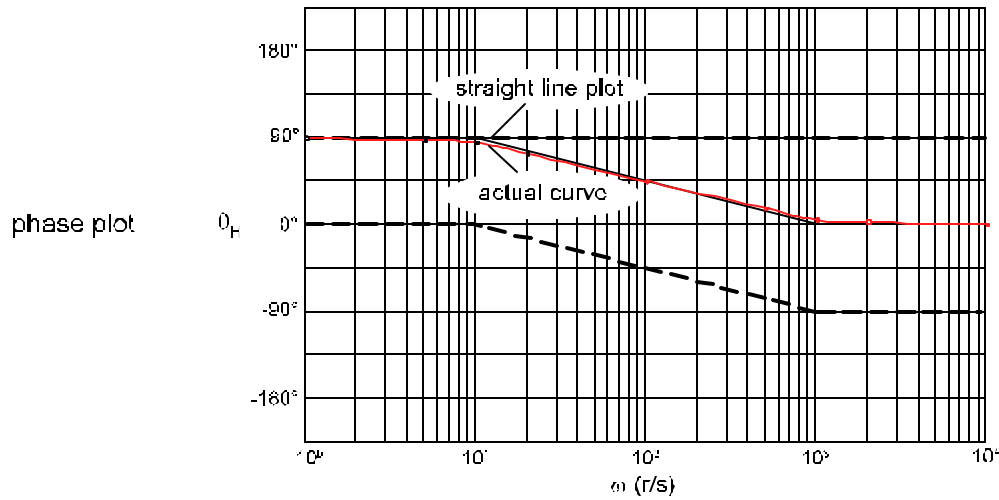
The user enters  $H(s) = \frac{2000s}{s + 100}$

- 1) The system responds by providing the transfer function in factored Bode form.

$$H = 20 \frac{s}{\left( \frac{s}{100} + 1 \right)}$$

- 2) Magnitude and phase plots





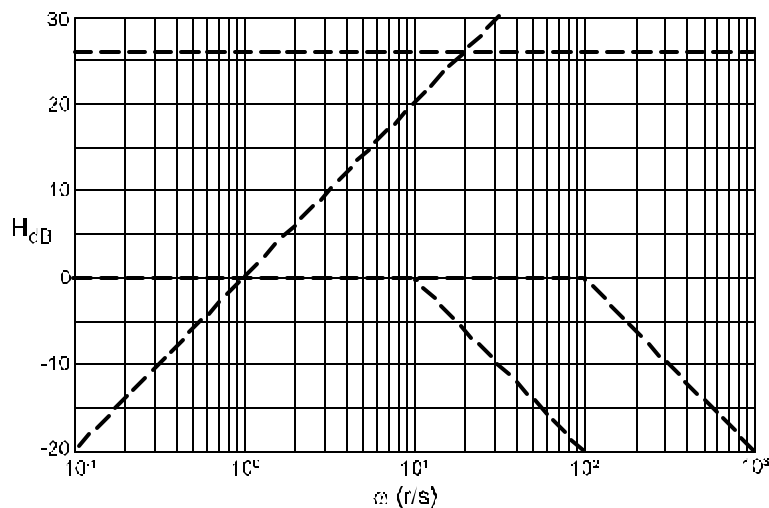
If it's not too difficult, a meaningful enhancement would be to allow the user to choose only factors or only overall curves.

Below is an example, using another transfer function, to partially illustrate this for the magnitude curves.

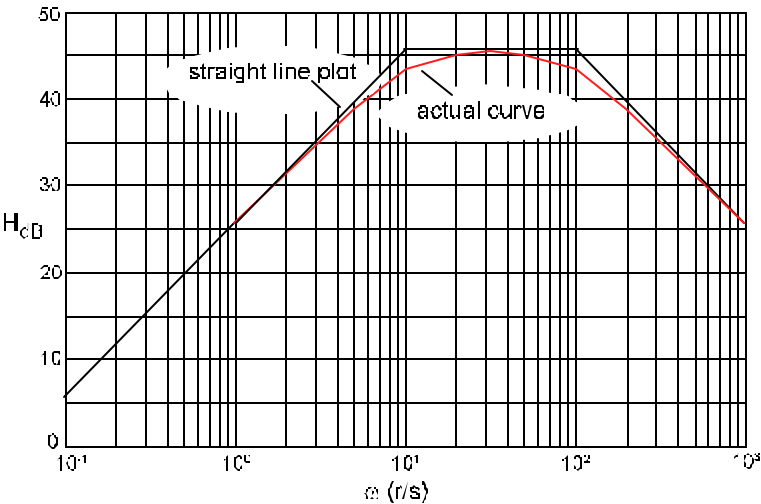
User enters  $H(s) = \frac{2(10^4)s}{s^2 + 110s + 10^3}$

System responds with  $H(s)$  in factored Bode form.  $H(s) = \frac{20s}{\left(\frac{s}{10} + 1\right)\left(\frac{s}{100} + 1\right)}$

User can choose a plot with only individual factors.



Or only the overall curves



**Module 14** (to follow)