

- Announcements:
 - Computer exam **next class**
- Questions?

Tomorrow's exam

For each problem, I'll specify the algorithm:

Shift Affine Vigenere Hill

and the attack:

Ciphertext only known plaintext

- You may use code that you wrote or that you got from the textbook.
- May require you to modify your code some on the fly
- Have your algorithms ready to run...

Can we generalize Fermat's little theorem to composite moduli?

The three-pass protocol is an application of Fermat's little theorem to key exchange

- How can Alice get a secret message to Bob without an established key?
- Can do it with locks.
- First 2 volunteers get to do a live demo

The three-pass protocol is an application of Fermat's little theorem to key exchange

- Situation: Alice wants to get a short message to Bob, but they don't have an established key to transmit it.
- Can do with locks:



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Note: it's always secured by one of their locks

In the three-pass protocol, the “locks” are random numbers that satisfy specific properties

- K: the secret message
- p: a large public prime number $> K$
- The two locks:
 - a: Alice's random #, $\gcd(a, p-1)=1$
 - b: Bob's random #, $\gcd(b, p-1)=1$
- To unlock their locks:
 - $a^{-1} \bmod (p-1)$
 - $b^{-1} \bmod (p-1)$

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Three-pass protocol:

Alice computes $K^a \pmod{p}$ and sends to Bob

Bob computes $(K^a)^b \pmod{p}$ and sends it back

Alice computes $((K^a)^b)^{a^{-1}} \pmod{p}$ and sends it back

Bob computes $((((K^a)^b)^{a^{-1}})^{b^{-1}} \pmod{p}$ and reads K

In the three-pass protocol, the “locks” are random numbers that satisfy specific properties

- 36 ● K: the secret message
- 59 ● p: a public prime number $> K$
- The two locks:
 - 17 ■ a: Alice's random #, $\gcd(a, p-1)=1$
 - 21 ■ b: Bob's random #, $\gcd(b, p-1)=1$
- To unlock their locks:
 - 41 ■ $a^{-1} \bmod (p-1)$
 - 47 ■ $b^{-1} \bmod (p-1)$

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Toy example:

$$36^{17} \pmod{59} = 12$$

$$12^{21} \pmod{59} = 45$$

$$45^{41} \pmod{59} = 48$$

$$48^{47} \pmod{59} = 36$$

Why does it work?

The basic principle relates the moduli of the expressions to the moduli in the exponents

● When dealing with numbers mod n , we can deal with their exponents mod _____

● So...

- Given integers a and b ,
- Since $aa^{-1} = bb^{-1} = 1 \pmod{p-1}$
- ...what's $K^{(aba^{-1}b^{-1})} \pmod{p}$?

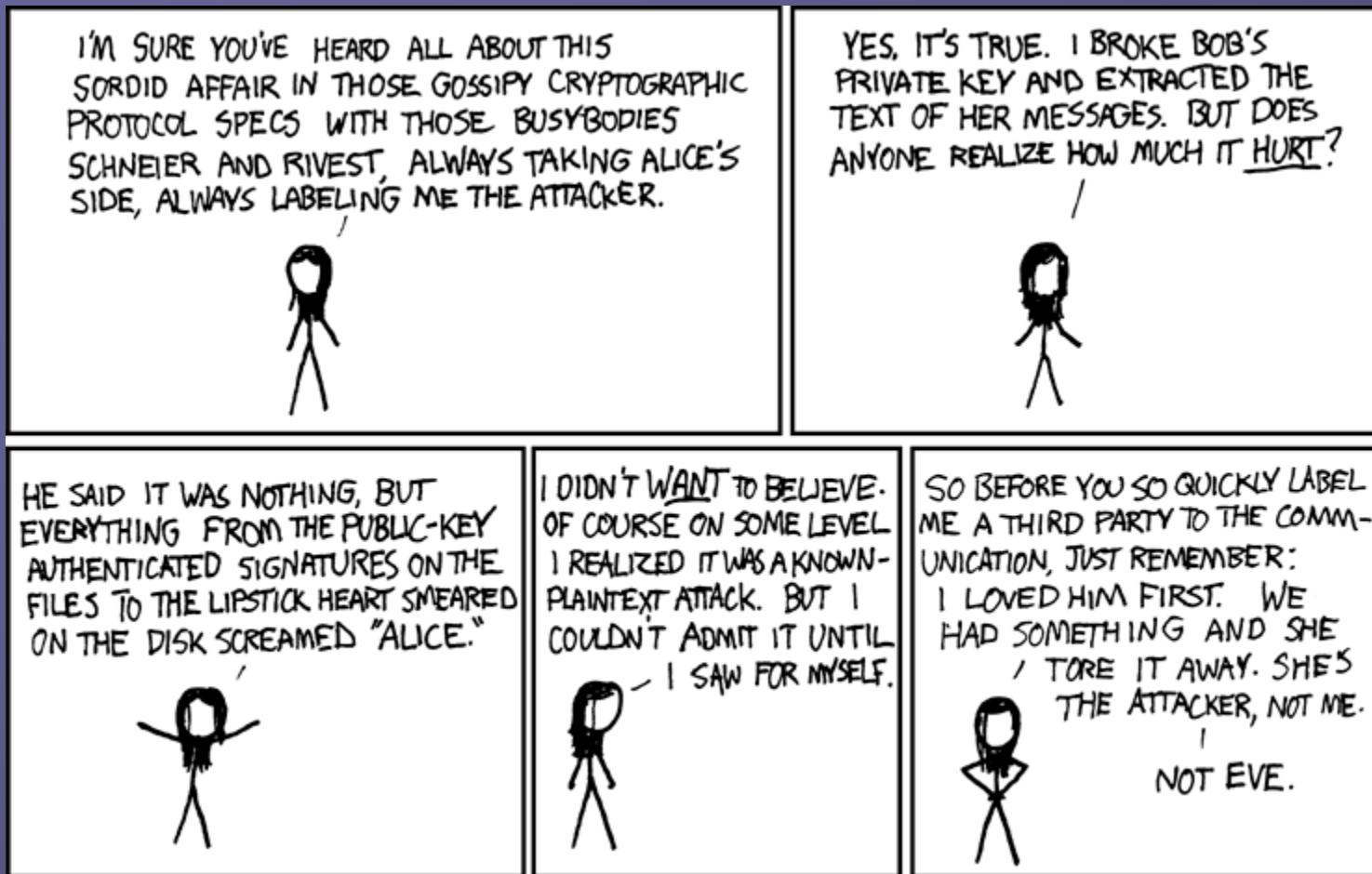
Why isn't this used in key exchange today?

- Trappe and Washington say that it's vulnerable to an "intruder-in-the-middle" attack. Think about this...

You are now prepared to read as much of the rest of chapter 3 as you like

- We'll revisit 3.7 (primitive roots) and 3.11 (fields) later
- The rest is more number theory fun.
- Tomorrow we start DES

Maybe Alice and Bob's exchange wasn't as secure as they thought...



<http://xkcd.com/c177.html>

Yet one more reason I'm barred from speaking at crypto conferences.