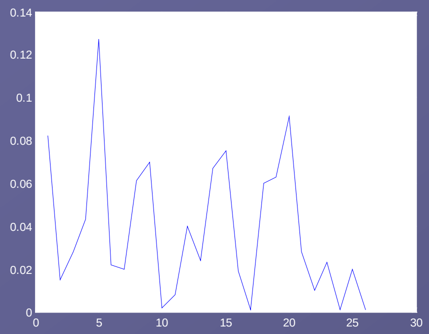
DTTF/NB479: Dszquphsbqiz Day 6

- Announcements:
  - Programming exam next Thursday on breaking codes from chapter 2
  - Written exam at start of week 4 on concepts from chapter 2
- Questions?
- This week: see schedule page
  - 2 days of chapter 3, then back to Hill cipher

Vigenere is more secure than affine cipher, but still breakable



- You should be able to answer:
    1. What makes a Vigenere cipher more secure than a shift cipher?
    2. Why does the max of dot(A_0,A_i) occur when i==0?
    3. What are the advantages and disadvantages of using the dot product method (method 2) vs. max is 'e' (method 1) to decrypt the key?
    4. How do we find the key length?

Vigenere can be made secure with appropriate precautions

- From http://sharkysoft.com/misc/vigenere/
  - Key must be as long as the plaintext message.
  - Build the key from random characters.
  - Never use the key again.
  - Don't use text decorations (spaces, punctuations, capitalization).
  - Protect the key.

Vigenere trivia (if time at end)

- Consider *Gadsby* by Ernest Vincent Wright, February 1939:
  - http://www.spinelessbooks.com/gadsby/01.html
- What do you notice about it?

The Extended Euclidean algorithm is very important

- Why? A means to find GCD(a,b)
- How does the algorithm?
- How fast is it?
- Why does it work?
- How does it help?
  - Used to find inverses of very large numbers (mod n).

Back to Basics: 3. GCD

- gcd(a,b)=$\max_j$ (j|a and j|b).
- Def.: a and b are relatively prime iff gcd(a,b)=1
- gcd(14,21) easy…
- What about gcd(1856, 5862)?
  - Or gcd(500267500347832384769, 12092834543475893256574665)?

- Do you really want to factor each one?
- What's our alternative?

Euclid’s Algorithm

```
gcd(a,b) {
   if (a < b) swap (a,b)
   // a > b
   r = a % b
   while (r ~= 0) {
      a = b
      b = r
      r = a % b
   }
   gcd = b // last r ~= 0
}
```

Calculate gcd(1856, 5862)

=2

Euclid's Algorithm

```
gcd(a,b) {
    if (a > b) swap (a,b)
    // a > b
    r = a % b
    while (r ~= 0) {
        a = b
        b = r
        r = a % b
    }
    gcd = b // last r ~= 0
}
```

Assume a > b

Let q_i and r_i be the series of quotients and remainders, respectively, found along the way.

$a = q_1 b + r_1$

$b = q_2 r_1 + r_2$

$r_1 = q_3 r_2 + r_3$

...

$r_{i-2} = q_i r_{i-1} + r_i$

...

$r_{k-2} = q_k r_{k-1} + r_k$ r_k is gcd(a,b)

$r_{k-1} = q_{k+1} r_k$

You'll prove this computes the gcd in Homework 3 (by induction)…

Fundamental result:
If $d = gcd(a,b)$ then $ax + by = d$

- For *some* integers x and y.
- ***<u>These ints are just a by-product of the Euclidean algorithm!</u>***
- Allows us to find a^{-1} *(mod n)* very quickly…
  - Choose $b = n$ and $d = 1$.
  - If $gcd(a,n) = 1$, then $ax + ny = 1$
  - $ax \equiv 1 \pmod{n}$ because it differs from 1 by a multiple of n
  - Therefore, $x \equiv a^{-1} \pmod{n}$.
- Why does the result hold?
- How do we find x and y?

Why does this work?

Given a,b ints, not both 0, and gcd(a,b) = d.

Prove ax + by = d

Recall gcd(a,b,)=d = r_k is the last non-zero remainder found via Euclid.

We'll show the property true for all remainders r_j (by strong induction)

Assume a > b

Let q_i and r_i be the series of quotients and remainders, respectively, found along the way.

$$a = q_1 b + r_1$$
$$b = q_2 r_1 + r_2$$
$$r_1 = q_3 r_2 + r_3$$
$$\ldots$$
$$r_{i-2} = q_i r_{i-1} + r_i$$
$$\ldots$$
$$r_{k-2} = q_k r_{k-1} + r_k$$
$$r_{k-1} = q_{k+1} r_k$$

How to find x and y?

x and y swapped from book, which assumes that a < b on p. 69

To find x, take

$x_0 = 1$, $x_1 = 0$,

$x_j = x_{j-2} - q_{j-1}x_{j-1}$

To find y, take

$y_0 = 0$, $y_1 = 1$,

$y_j = y_{j-2} - q_{j-1}y_{j-1}$

Use to calculate x_k and y_k (the desired result)

Example:

gcd(1856,5862)=2

Yields x = -101, y = 319

Assume a > b

Let q_i and r_i be the series of quotients and remainders, respectively, found along the way.

i	q_i	r_i	x_i
0	–	–	1
1	3	294	0
2	6	92	= 1 – 3(0)=1
3	3	18	…
4	5	2	
5	9	0	

Check: 5862(-101) + 1856(319) = 2?

This gives us a way to find a^{-1} (mod n)

- Solve $ax + ny \equiv 1$ using extended Euclid.
- $a^{-1} \equiv n$

- If time,

  demo 89734^{-1} mod(524287)

  using [g,x,y] = gcd(a,n)