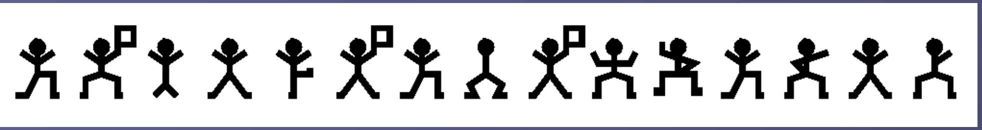
DTTF/NB479: Dszquphsbqiz Day 2

- Announcements:
  - Subscribe to piazza and start HW1
- Questions?
- Roll Call
- Today: affine ciphers

Sherlock Holmes, *The Adventure of the Dancing Men* (1898)
Who got it?

In a letter:

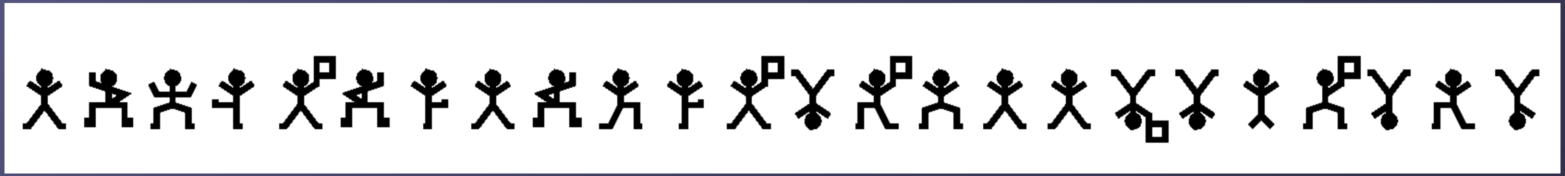


2 weeks later:

2 mornings later:

3 days later:

4 days later:



Affine ciphers

Somewhat stronger since **scale, then shift**:

$x \rightarrow \alpha x + \beta \pmod{26}$

Say y = 5x + 3; x = ‘hellothere’;

Then y = ‘mxggv…’

Affine ciphers: $x \rightarrow \alpha x + \beta \pmod{26}$

Consider the 4 attacks:

1. How many possibilities must we consider in brute force attack?

Restrictions on α

Consider $y = 2x$, $y = 4x$,   or $y = 13x$

What happens?

Basics 1: Divisibility

Definition:

$$\text{Given } a, b \in Z, a \neq 0.$$

$$a \mid b \text{ means } \exists k \in Z \text{ s.t. } b = ka$$

Property 1:

$$\forall a \neq 0, \quad a \mid 0, \quad a \mid a, \quad 1 \mid a$$

Property 2 (transitive):

$$a \mid b \text{ and } b \mid c \Rightarrow a \mid c$$

Property 3 (linear combinations):

$$a \mid b \text{ and } a \mid c \Rightarrow a \mid (sb + tc) \forall s, t \in Z$$

Basics 2: Primes

- Any integer p > 1 divisible by only p and 1.
- How many are there?
- Prime number theorem:
  - Let $\pi(x)$ be the number of primes less than x.
  - Then $$\lim_{x\to\infty} \pi(x) = \frac{x}{\ln(x)}$$
  - Application: how many 319-digit primes are there?
- Every positive integer is a unique product of primes.

Basics: 3. GCD

- $gcd(a,b)=max_j\ (j|a \text{ and } j|b)$.
- Def.: a and b are relatively prime iff gcd(a,b)=1
- gcd(14,21) easy…

Basics 4: Congruences

- Def: a≡b (mod n) iff (a-b) = nk for some int k
- Properties

$$\begin{aligned}
&\text{Consider } a,b,c,d \in Z, n \neq 0 \\
&a \equiv b \pmod{n} \text{ if } \exists k \in Z \text{ s.t. } a = b + nk \\
&a \equiv 0 \pmod{n} \text{ iff } n \mid a \\
&a \equiv a \pmod{n} \\
&a \equiv b \pmod{n} \text{ iff } b \equiv a \pmod{n} \\
&a \equiv b, b \equiv c \pmod{n} \Rightarrow a \equiv c \pmod{n}
\end{aligned}$$

$$\begin{aligned}
&\text{If } a \equiv b, c \equiv d \pmod{n}, \text{then} \\
&(a+c) \equiv (b+d) \pmod{n} \\
&(a-c) \equiv (b-d) \pmod{n} \\
&ac \equiv bd \pmod{n} \\
&\text{If } \gcd(a,n) = 1 \text{ and } ab \equiv ac \pmod{n}, \text{then} \\
&b \equiv c \pmod{n}
\end{aligned}$$

- You can easily solve congruences ax≡b (mod n) if gcd(a,n) = 1 and the numbers are small.
  - Example: 3x+ 6 ≡ 1 (mod 7)
- If gcd(a,n) isn’t 1, there are multiple solutions (next week)

Restrictions on α

Consider y= 2x,   y = 4x,   or   y = 13x

The problem is that $\gcd(\alpha, 26) != 1$.

The function has no inverse.

Finding the decryption key

- You need the inverse of $y = 5x + 3$
- In *Integer (mod 26) World*, of course…
- $y \equiv 5x + 3 \pmod{26}$

Affine ciphers: x → ax + b (mod 26)

- Consider the 4 attacks:
  1. Ciphertext only:
     - How long is brute force?
  2. Known plaintext
     - How many characters do we need?
  3. Chosen plaintext
     - Wow, this is easy. Which plaintext easiest?
  4. Chosen ciphertext
     - Also easy: which ciphertext?

Vigenere Ciphers

- Idea: the key is a *vector* of shifts
  - The key and its length are unknown to Eve
  - Ex. Use a word like *hidden (7 8 3 3 4 13)*.
  - Example:

Key

```
The recent development of various methods of
 7 8 3   3 413 7 8 3   3 413 7 8 3 3 413 7 8   3 3   413 7 8 3 3 4  13 7 8 3 3 413   7 8
 015 7  20 815112122   6 8 811191718161720 1  17 8  25132416172322  2511 11017 7 5  2113
aph uiplvw giiltrsqrub ri znyqrxw zlbkrhf vn
```

- Encryption:
  - Repeat the vector as many times as needed to get the same length as the plaintext
  - Add this repeated vector to the plaintext.