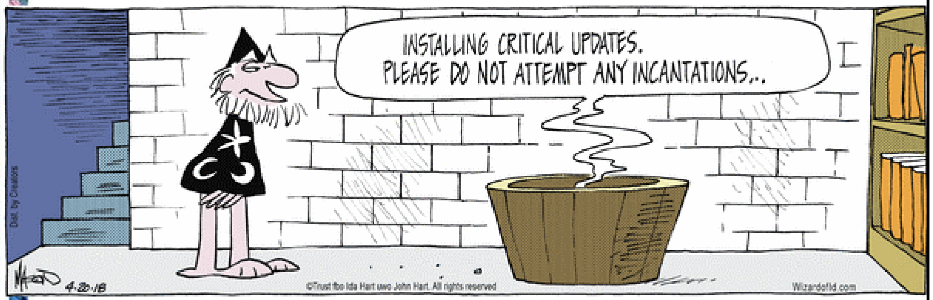
MA/CSSE 474
Theory of Computation

CFL Hierarchy
CFL Decision Problems

Your Questions?

- Previous class days' material
- Reading Assignments
- HW 12 or 13 problems
- Anything else

I have included some slides online that we will not have time to do in class, but may be helpful to you anyway.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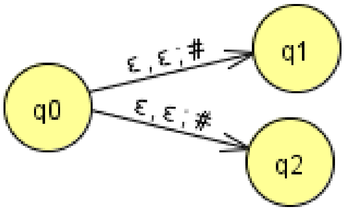
{*xcy* : *x*, *y* ∈ {0, 1}* and *x* ≠ *y*}

- SURPRISINGLY, it is Context-free! HW 13. Here is the beginning of a proof:
- We can build a PDA *M* to accept *L*. All *M* has to do is to find one way in which *x* and *y* differ.
- *M* starts by pushing a bottom of stack marker # onto the stack.
- Then it nondeterministically chooses to go to state 1 or 2.



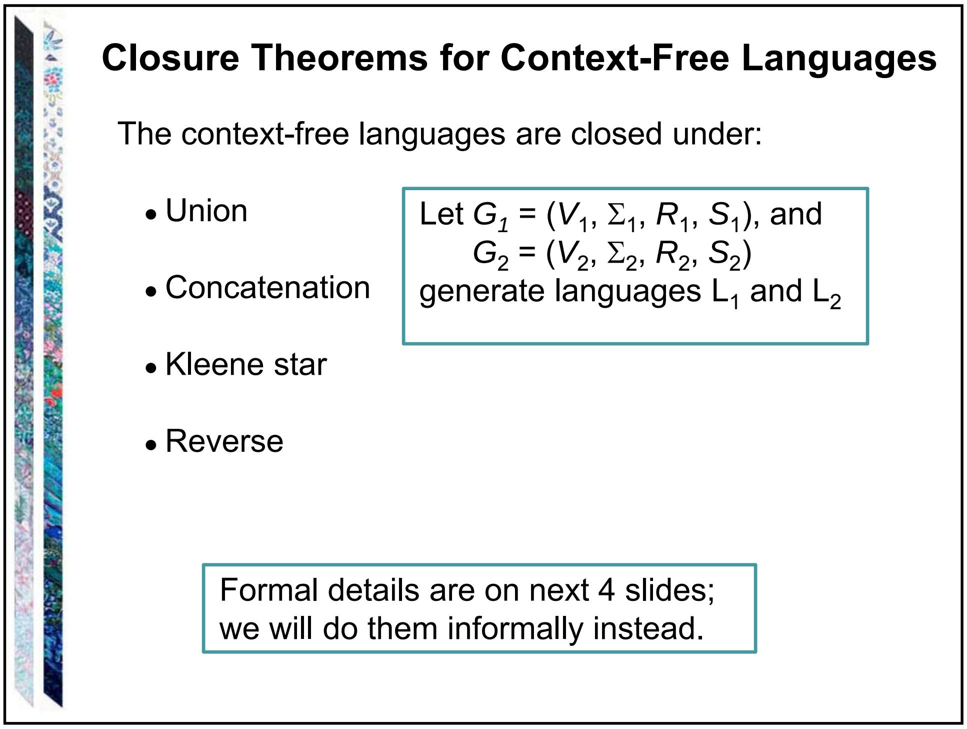
PDA Variations?

- In HW12, we see that acceptance by "accepting state only" is equivalent to acceptance by empty stack and accepting state.

  Equivalent In this sense: Given a language L, there is a PDA that accepts L by accepting state and empty stack iff there is a PDA that accepts L by accepting state only.

- FSM plus two stacks?

- FSM plus FIFO queue (instead of stack)?



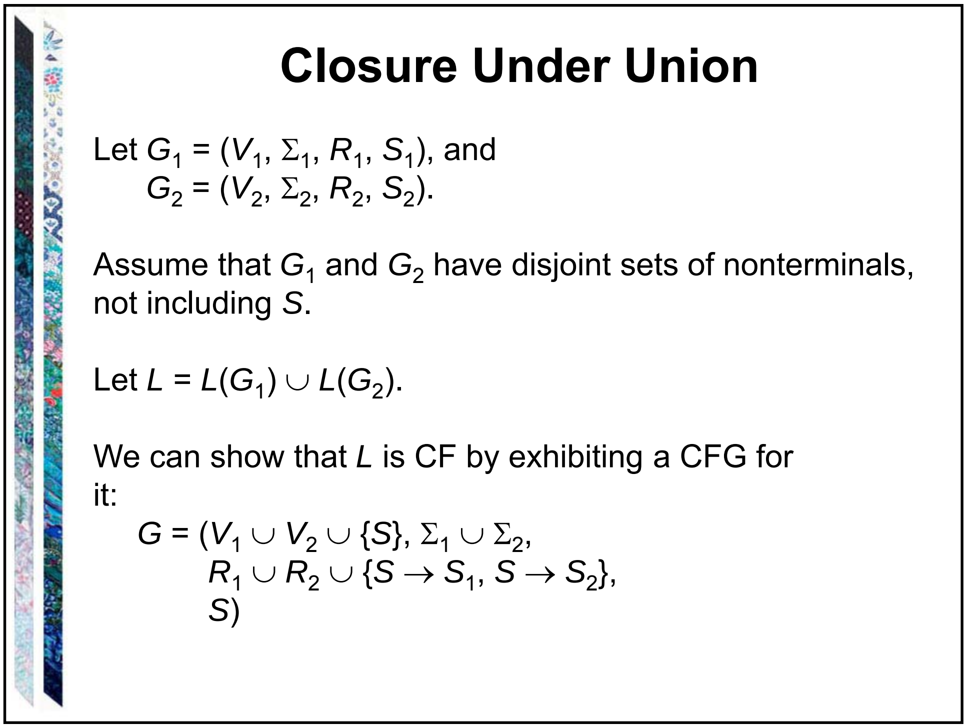
Closure Theorems for Context-Free Languages

The context-free languages are closed under:

- Union
- Concatenation
- Kleene star
- Reverse

Let $G_1 = (V_1, \Sigma_1, R_1, S_1)$, and
$G_2 = (V_2, \Sigma_2, R_2, S_2)$
generate languages L_1 and L_2

Formal details are on next 4 slides;
we will do them informally instead.



Closure Under Union

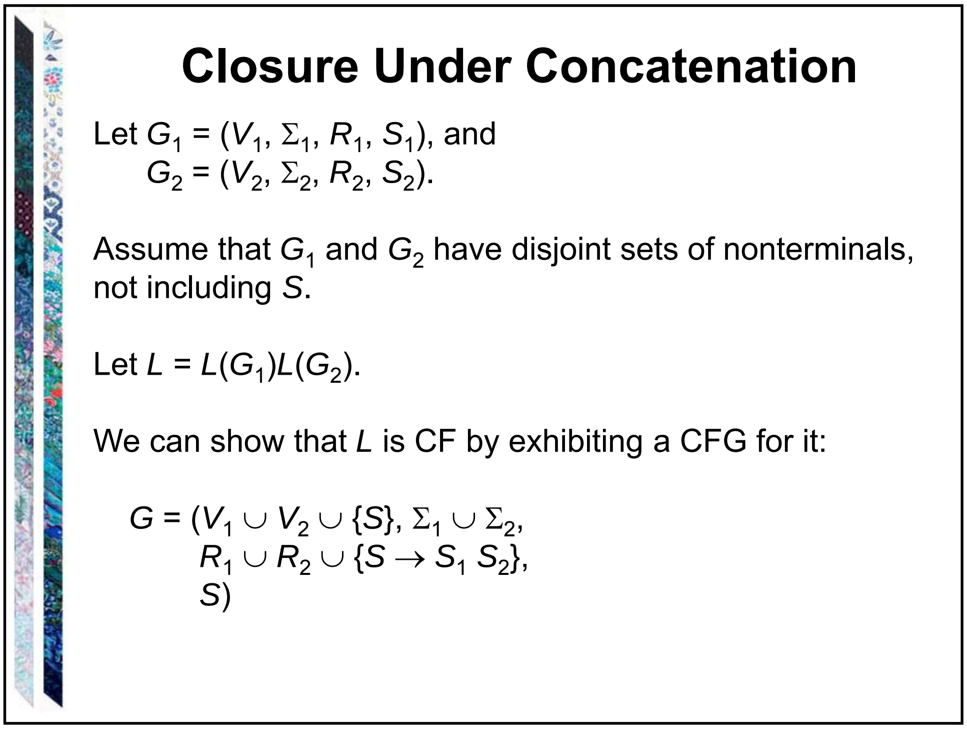
Let $G_1 = (V_1, \Sigma_1, R_1, S_1)$, and
$G_2 = (V_2, \Sigma_2, R_2, S_2)$.

Assume that G_1 and G_2 have disjoint sets of nonterminals, not including S.

Let $L = L(G_1) \cup L(G_2)$.

We can show that L is CF by exhibiting a CFG for it:

$G = (V_1 \cup V_2 \cup \{S\}, \Sigma_1 \cup \Sigma_2,$
$R_1 \cup R_2 \cup \{S \rightarrow S_1, S \rightarrow S_2\},$
$S)$



Closure Under Concatenation

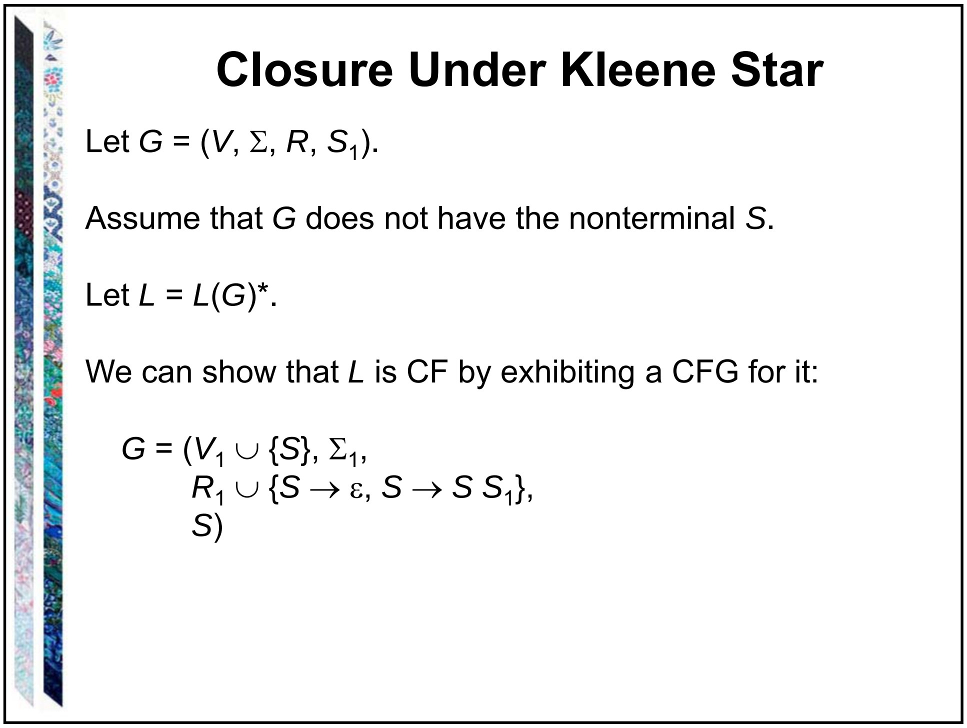
Let $G_1 = (V_1, \Sigma_1, R_1, S_1)$, and
$G_2 = (V_2, \Sigma_2, R_2, S_2)$.

Assume that G_1 and G_2 have disjoint sets of nonterminals, not including S.

Let $L = L(G_1)L(G_2)$.

We can show that L is CF by exhibiting a CFG for it:

$$G = (V_1 \cup V_2 \cup \{S\}, \Sigma_1 \cup \Sigma_2,\\ R_1 \cup R_2 \cup \{S \rightarrow S_1\ S_2\},\\ S)$$



Closure Under Kleene Star

Let $G = (V, \Sigma, R, S_1)$.

Assume that G does not have the nonterminal S.

Let $L = L(G)^*$.

We can show that L is CF by exhibiting a CFG for it:

$$G = (V_1 \cup \{S\}, \Sigma_1,\\ R_1 \cup \{S \rightarrow \varepsilon, S \rightarrow S\ S_1\},\\ S)$$

Closure Under Reverse

$L^R = \{w \in \Sigma^* : w = x^R \text{ for some } x \in L\}$.

Let $G = (V, \Sigma, R, S)$ be in Chomsky normal form.

Every rule in G is of the form $X \rightarrow BC$ or $X \rightarrow a$, where X, B, and C are elements of $V - \Sigma$ and $a \in \Sigma$.

- $X \rightarrow a$: $L(X) = \{a\}$. $\{a\}^R = \{a\}$.
- $X \rightarrow BC$: $L(X) = L(B)L(C)$. $(L(B)L(C))^R = L(C)^R L(B)^R$.

Construct, from G, a new grammar G', such that $L(G') = L^R$:
$G' = (V_G, \Sigma_G, R', S_G)$, where R' is constructed as follows:

- For every rule in G of the form $X \rightarrow BC$, add to R' the rule $X \rightarrow CB$.
- For every rule in G of the form $X \rightarrow a$, add to R' the rule $X \rightarrow a$.

Closure Under Intersection

The context-free languages are not closed under intersection:

The proof is by counterexample. Let:

$L_1 = \{a^n b^n c^m : n, m \geq 0\}$ /* equal a's and b's.
$L_2 = \{a^m b^n c^n : n, m \geq 0\}$ /* equal b's and c's.

Both L_1 and L_2 are context-free, since there exist straightforward context-free grammars for them.

But now consider:
$L = L_1 \cap L_2$
$= \{a^n b^n c^n : n \geq 0\}$

Recall: Closed under union but not closed under intersection implies not closed under complement.
And we saw a specific example of a CFL whose complement was not CF.

Closure Under Complement

$L_1 \cap L_2 = \neg(\neg L_1 \cup \neg L_2)$

The context-free languages are closed under union, so if they were closed under complement, they would be closed under intersection (which they are not).

Alternative approach:
In a previous class, we demonstrated that the complement of L = $A^nB^nC^n$ is context-free, while L itself is not context-free,

The Intersection of a Context-Free Language and a Regular Language is Context-Free

$L = L(M_1)$, a PDA = $(K_1, \Sigma, \Gamma_1, \Delta_1, s_1, A_1)$.
$R = L(M_2)$, a deterministic FSM = $(K_2, \Sigma, \delta, s_2, A_2)$.

We construct a new PDA, M_3, that accepts $L \cap R$ by simulating the parallel execution of M_1 and M_2.

$M = (K_1 \times K_2, \Sigma, \Gamma_1, \Delta, [s_1, s_2], A_1 \times A_2)$.

Insert into Δ:

For each rule $((q_1, a, \beta), (p_1, \gamma))$ in Δ_1,
and each rule (q_2, a, p_2) in δ,
Δ contains $(([q_1, q_2]\ a, \beta), ([p_1, p_2], \gamma))$.

For each rule $((q_1, \varepsilon, \beta), (p_1, \gamma)$ in Δ_1,
and each state q_2 in K_2,
Δ contains $(([q_1, q_2], \varepsilon, \beta), ([p_1, q_2], \gamma))$.

I use square brackets for ordered pairs of states from $K_1 \times K_2$, to distinguish them from the tuples that are part of the notations for transitions in M_1, M_2, and M.

This works because: we can get away with only one stack.

The Difference between a Context-Free Language and a Regular Language is Context-Free

Theorem: The difference ($L_1 - L_2$) between a context-free language L_1 and a regular language L_2 is context-free.

Proof: $L_1 - L_2 = L_1 \cap \neg L_2$.

If L_2 is regular then so is $\neg L_2$.

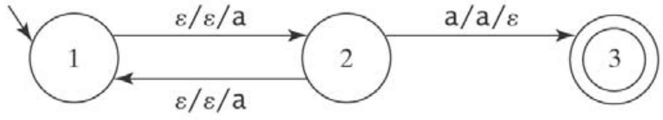
If L_1 is context-free, so is $L_1 \cap \neg L_2$.

Halting

It is possible that a PDA may

- not halt,
- never finish reading its input.

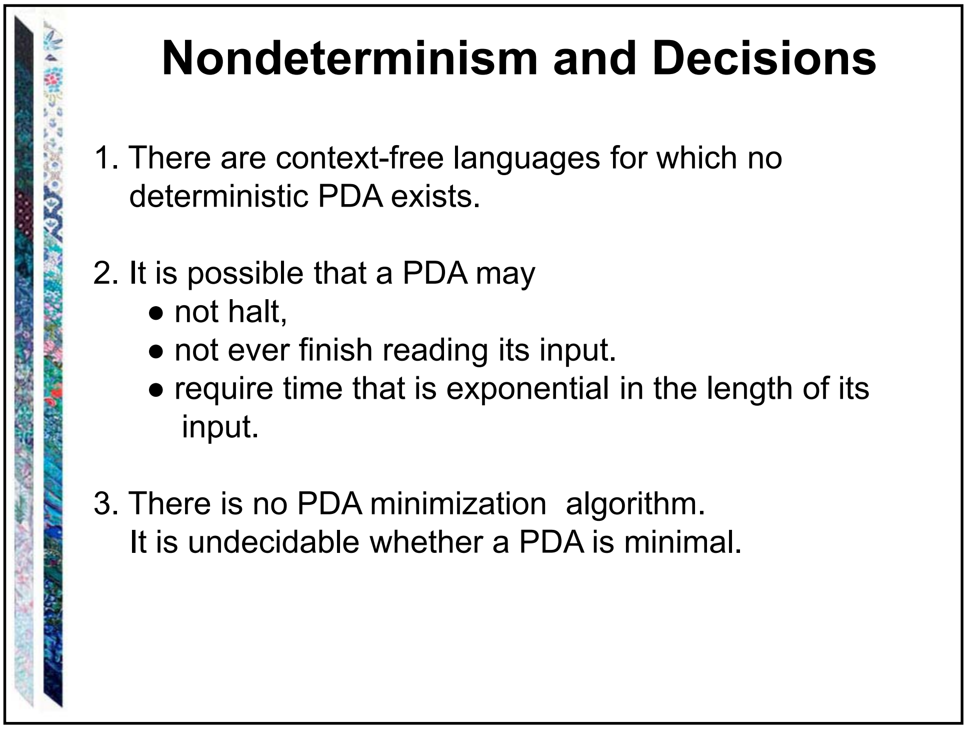
Let $\Sigma = \{a\}$ and consider M =



$L(M) = \{a\}$: $(1, a, \varepsilon) \vdash (2, a, a) \vdash (3, \varepsilon, \varepsilon)$

On any other input except a:

- *M* will never halt, or
- *M* will never finish reading its input unless its input is ε.

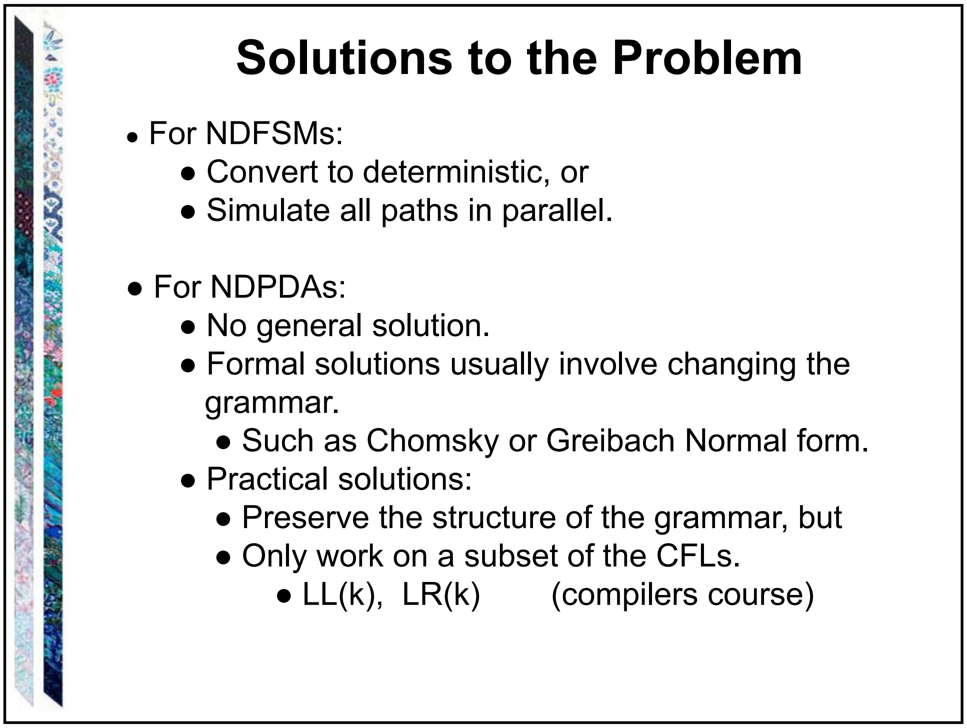


Nondeterminism and Decisions

1. There are context-free languages for which no deterministic PDA exists.

2. It is possible that a PDA may
   - not halt,
   - not ever finish reading its input.
   - require time that is exponential in the length of its input.

3. There is no PDA minimization algorithm.
   It is undecidable whether a PDA is minimal.



Solutions to the Problem

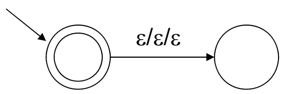
- For NDFSMs:
  - Convert to deterministic, or
  - Simulate all paths in parallel.

- For NDPDAs:
  - No general solution.
  - Formal solutions usually involve changing the grammar.
    - Such as Chomsky or Greibach Normal form.
  - Practical solutions:
    - Preserve the structure of the grammar, but
    - Only work on a subset of the CFLs.
      - LL(k), LR(k) (compilers course)

Deterministic PDAs

A PDA *M* is ***deterministic*** iff:

- Δ_M contains no pairs of transitions that compete with each other, and
- Whenever *M* is in an accepting configuration it has no available moves.



M can choose between accepting and taking the ε-transition, so it is not deterministic.

Deterministic CFLs (very quick overview without many details)

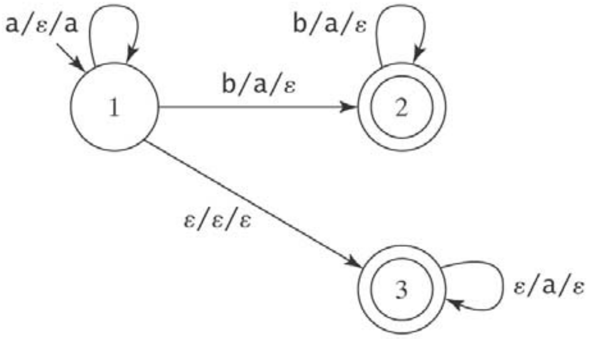
A language *L* is ***deterministic context-free*** iff *L*\$ can be accepted by some deterministic PDA.

Why \$?

Let $L = \text{a}^* \cup \{\text{a}^n\text{b}^n : n > 0\}$.

An NDPDA for *L*

$L = \text{a}^* \cup \{\text{a}^n\text{b}^n : n > 0\}$.



A DPDA for *L*$

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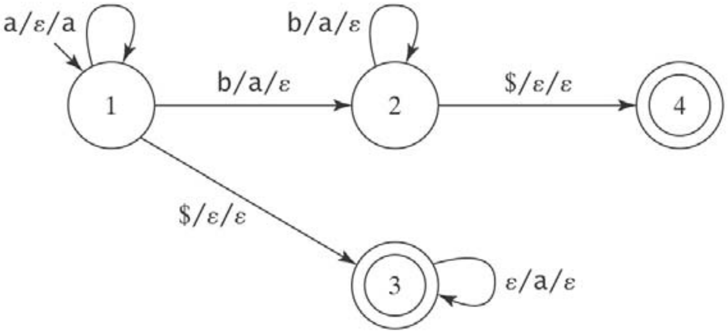


DCFL Properties (skip the details)

.

The Deterministic CF Languages are closed under complement.

The Deterministic CF Languages are not closed under intersection or union.

Nondeterministic CFLs

Theorem: There exist CLFs that are not deterministic.

Proof: By example. Let $L = \{a^ib^jc^k, i \neq j \text{ or } j \neq k\}$. L is CF. If L is DCF then so is:

$$\begin{aligned} L' &= \neg L. \\ &= \{a^ib^jc^k, i, j, k \geq 0 \text{ and } i = j = k\} \cup \\ &\quad \{w \in \{a, b, c\}^* : \text{the letters are out of order}\}. \end{aligned}$$

But then so is:

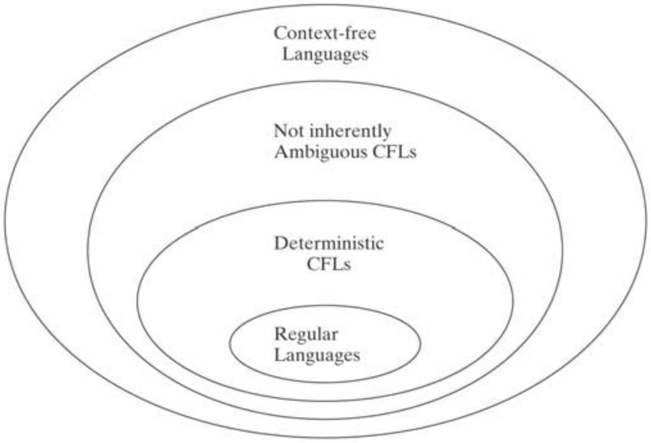
$$\begin{aligned} L'' &= L' \cap a^*b^*c^*. \\ &= \{a^nb^nc^n, n \geq 0\}. \end{aligned}$$

But it isn't. So L is CF but not DCF.

This simple fact poses a real problem for the designers of efficient context-free parsers.

Solution: design a language that is deterministic. LL(k) or LR(k).

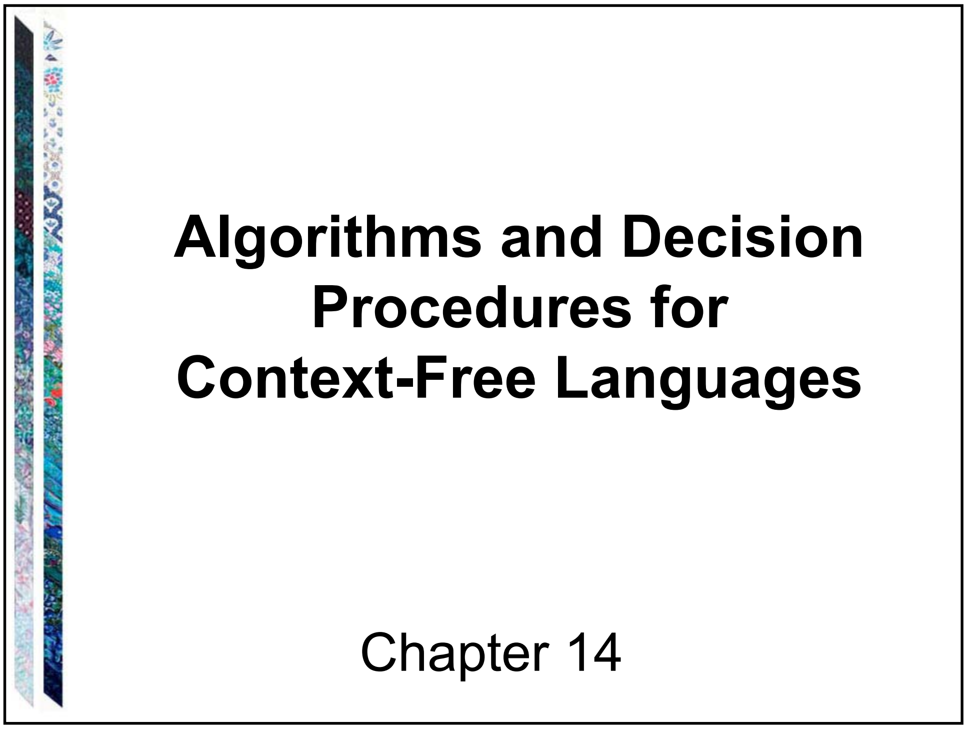
The CFL Hierarchy



Context-Free Languages Over a Single-Letter Alphabet

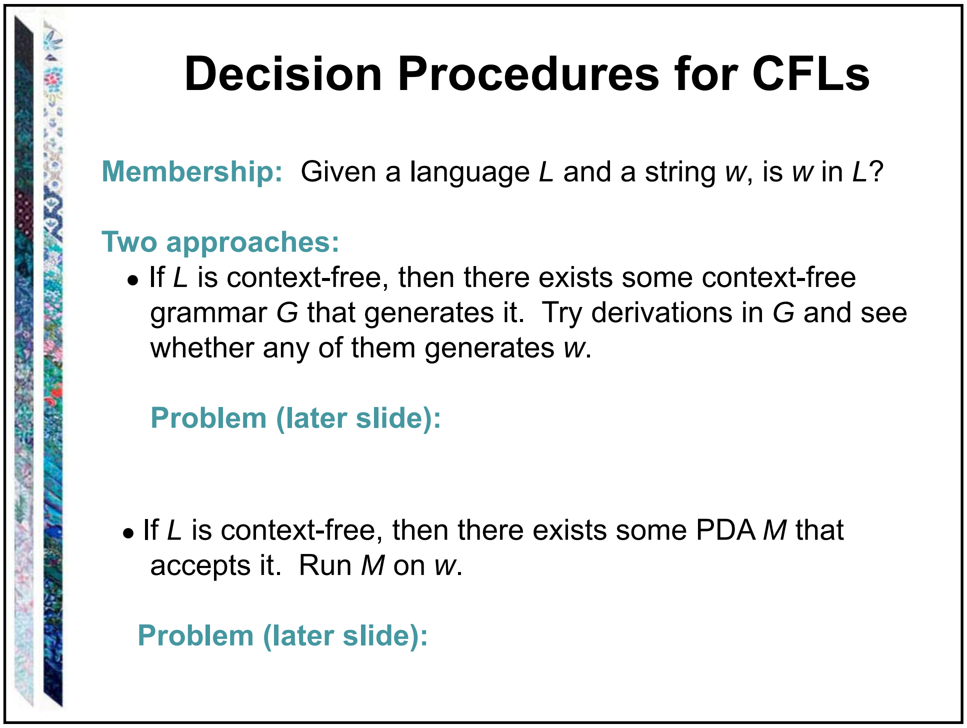
***Theorem**:* Any context-free language over a single-letter alphabet is regular.

***Proof**:* Requires Parikh's Theorem, which we are skipping



Algorithms and Decision Procedures for Context-Free Languages

Chapter 14



Decision Procedures for CFLs

Membership: Given a language *L* and a string *w*, is *w* in *L*?

Two approaches:

- If *L* is context-free, then there exists some context-free grammar *G* that generates it. Try derivations in *G* and see whether any of them generates *w*.

  Problem (later slide):

- If *L* is context-free, then there exists some PDA *M* that accepts it. Run *M* on *w*.

  Problem (later slide):

Decision Procedures for CFLs

Membership: Given a language *L* and a string *w*, is *w* in *L*?

Two approaches:

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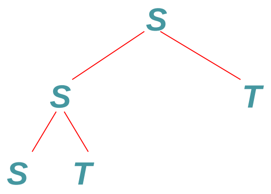
S $\rightarrow$ ***S T* | a** **Try to derive aaa**



S

S *T*

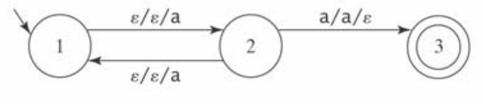
S *T*

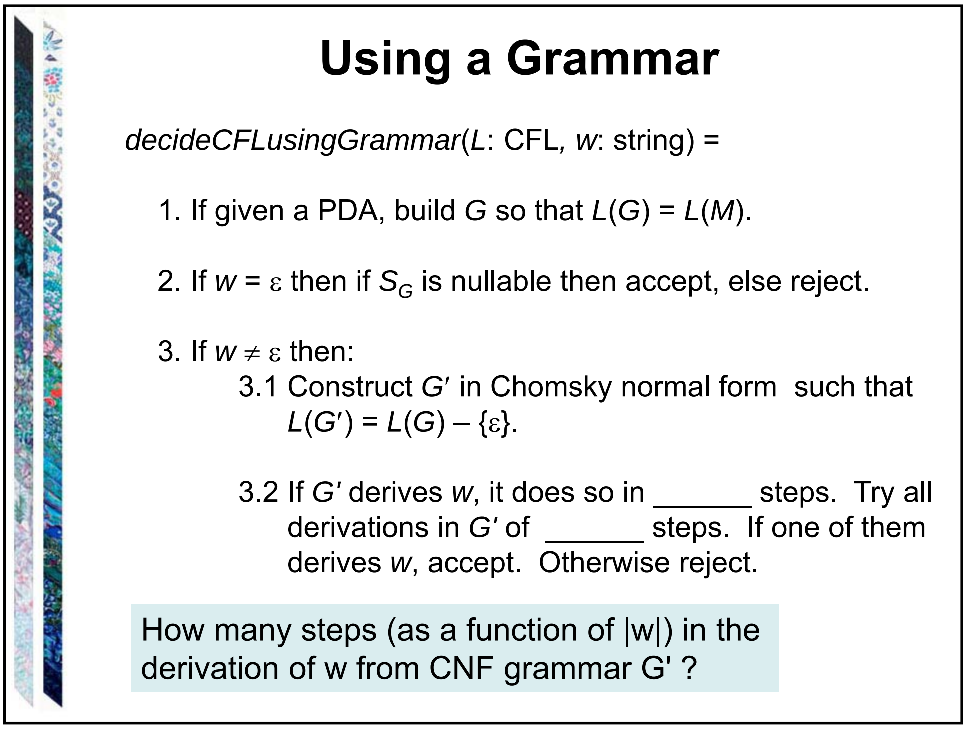
Decision Procedures for CFLs

Membership: Given a language *L* and a string *w*, is *w* in *L*?

- If *L* is context-free, then there exists some PDA *M* that accepts it. Run *M* on *w*.

Problem:

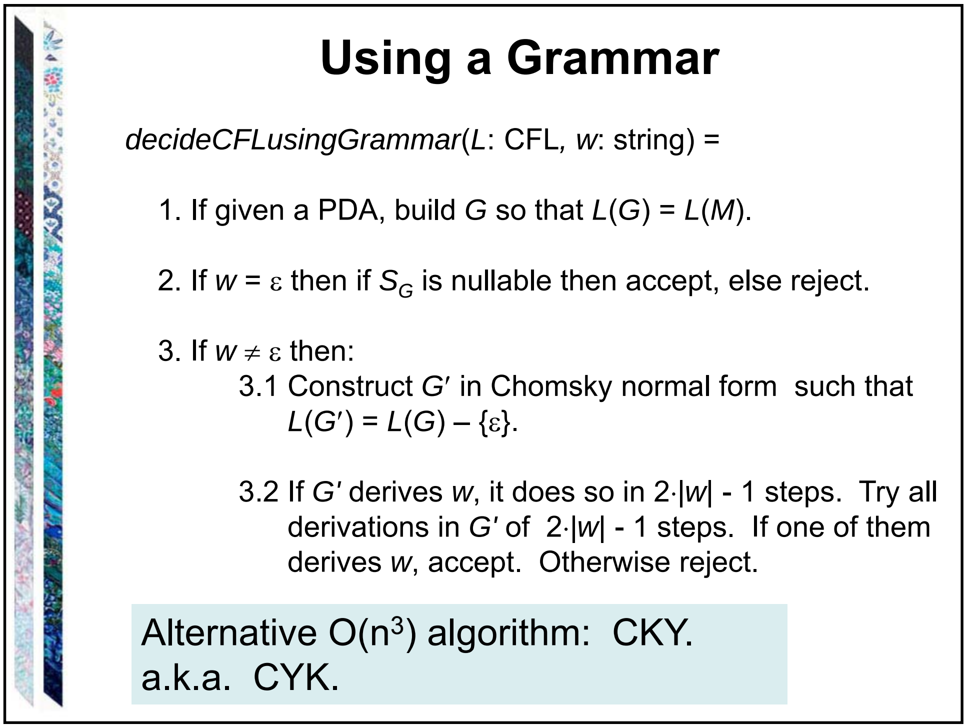




Using a Grammar

```
decideCFLusingGrammar(L: CFL, w: string) =
    1. If given a PDA, build G so that $L(G) = L(M)$.
    2. If $w = \varepsilon$ then if $S_G$ is nullable then accept, else reject.
    3. If $w \neq \varepsilon$ then:
        3.1 Construct $G'$ in Chomsky normal form such that
            $L(G') = L(G) - \{\varepsilon\}$.
        3.2 If $G'$ derives w, it does so in ______ steps. Try all
            derivations in $G'$ of ______ steps. If one of them
            derives w, accept. Otherwise reject.
```

How many steps (as a function of |w|) in the derivation of w from CNF grammar G' ?



Using a Grammar

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    3. If $w \neq \varepsilon$ then:
        3.1 Construct $G'$ in Chomsky normal form such that
            $L(G') = L(G) - \{\varepsilon\}$.
        3.2 If $G'$ derives w, it does so in $2 \cdot |w| - 1$ steps. Try all
            derivations in $G'$ of $2 \cdot |w| - 1$ steps. If one of them
            derives w, accept. Otherwise reject.
```

Alternative $O(n^3)$ algorithm: CKY.
a.k.a. CYK.

Emptiness

Given a context-free language *L*, is *L* = ∅?

decideCFLempty(*G*: context-free grammar) =

1. Let *G′* = *removeunproductive*(*G*).
2. If *S* is not present in *G′* then return *True*
   else return *False*.

Finiteness

Given a context-free language *L*, is *L* infinite?

decideCFLinfinite(*G*: context-free grammar) =

1. Lexicographically enumerate all strings in Σ^* of length greater than b^n and less than or equal to $b^{n+1} + b^n$.
2. If, for any such string *w*, *decideCFL*(*L, w*) returns *True* then return *True*. *L* is infinite.
3. If, for all such strings *w*, *decideCFL*(*L, w*) returns *False* then return *False*. *L* is not infinite.

Why these bounds?

Some Undecidable Questions about CFLs

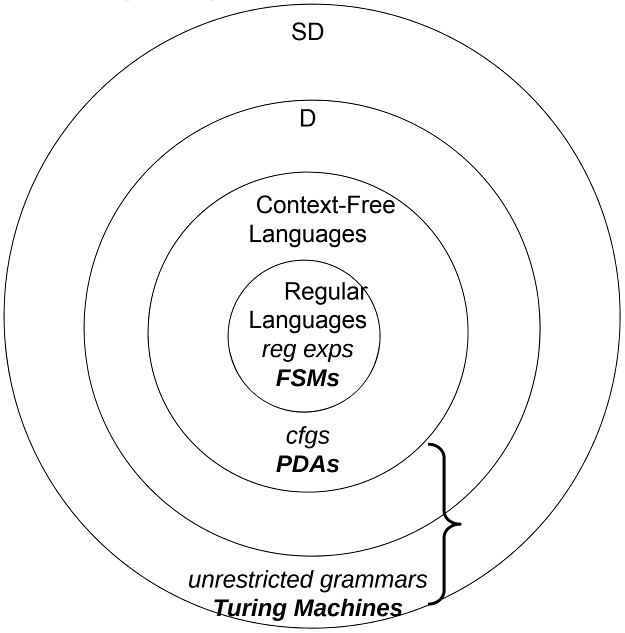
- Is $L = \Sigma^*$?
- Is the complement of L context-free?
- Is L regular?
- Is $L_1 = L_2$?
- Is $L_1 \subseteq L_2$?
- Is $L_1 \cap L_2 = \varnothing$?
- Is L inherently ambiguous?
- Is G ambiguous?

Regular and CF Languages

Regular Languages	**Context-Free Languages**
● regular exprs.	● context-free grammars
● or	
● regular grammars	
● = DFSMs	● = NDPDAs
● recognize	● parse
● minimize FSMs	● try to find unambiguous grammars
	● try to reduce nondeterminism in PDAs
	● find efficient parsers
● closed under:	● closed under:
♦ concatenation	♦ concatenation
♦ union	♦ union
♦ Kleene star	♦ Kleene star
♦ complement	
♦ intersection	♦ intersection w/ reg. langs
● pumping theorem	● pumping theorem
● D = ND	● D ≠ ND

TURING MACHINE INTRO

Languages and Machines



SD

D

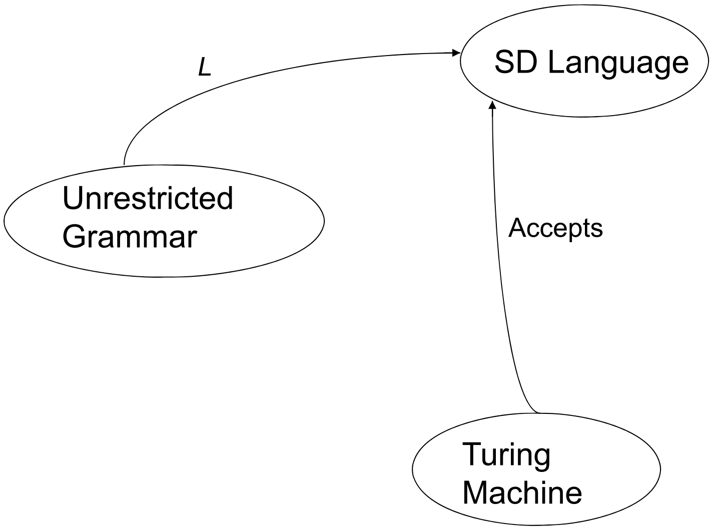
Context-Free Languages

Regular Languages *reg exps* ***FSMs***

cfgs ***PDAs***

unrestricted grammars ***Turing Machines***

Grammars, SD Languages, and Turing Machines



Turing Machines (TMs)

We want a new kind of automaton:

- powerful enough to describe all computable things,

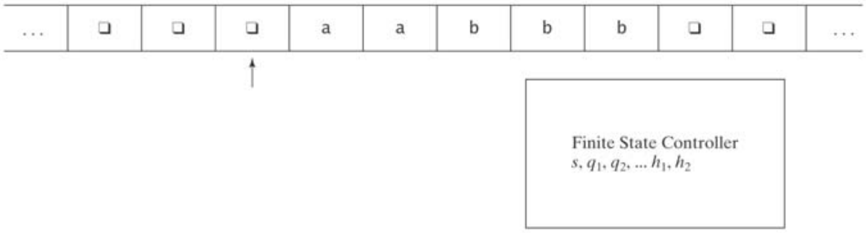
  unlike FSMs and PDAs.

- simple enough that we can reason formally about it

  like FSMs and PDAs,
  unlike real computers.

Goal: Be able to prove things about what can and cannot be computed.

Turing Machines



At each step, the machine must:

- choose its next state,
- write on the current square, and
- move left or right.

A Formal Definition

A (deterministic) Turing machine *M* is (K, Σ, Γ, δ, s, H):

- *K* is a finite set of states;
- Σ is the input alphabet, which does not contain Æ ;
- Γ is the tape alphabet,
  which must contain Æ and have Σ as a subset.
- $s \in K$ is the initial state;
- $H \subseteq K$ is the set of halting states;
- δ is the transition function:

$(K - H) \quad \times \quad \Gamma$ to $K \quad \times \quad \Gamma \quad \times \quad \{\rightarrow, \leftarrow\}$

non-halting state × tape char → state × tape char × direction to move (R or L)

Notes on the Definition

1. The input tape is infinite in both directions.

2. δ is a function, not a relation. So this is a definition for deterministic Turing machines.

3. δ must be defined for all (state, tape symbol) pairs unless the state is a halting state.

4. Turing machines do not necessarily halt (unlike FSM's and most PDAs). Why? To halt, they must enter a halting state. Otherwise they loop.

5. Turing machines generate output, so they can compute functions.

An Example

M takes as input a string in the language:

$\{a^i b^j, 0 \leq j \leq i\}$,

and adds b's as required to make the number of b's equal the number of a's.

The input to *M* will look like this:

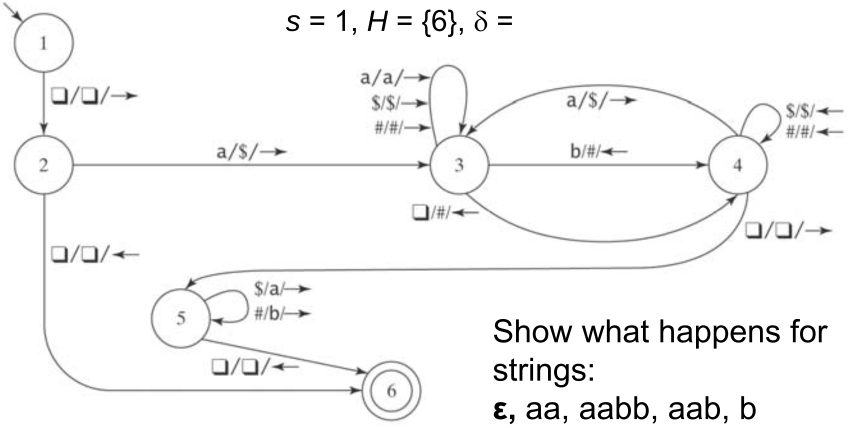
...	❑	a	a	a	b	❑	❑	❑	...
	↑								

The output should be:

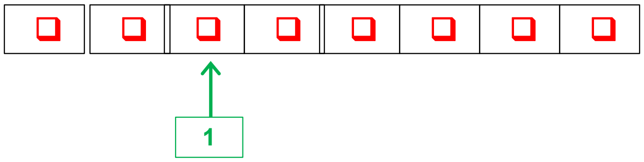
...	❑	a	a	a	b	b	b	❑	...
							↑		

The Details (ε)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1, *H* = {6}, δ =



Show what happens for strings:
ε, aa, aabb, aab, b



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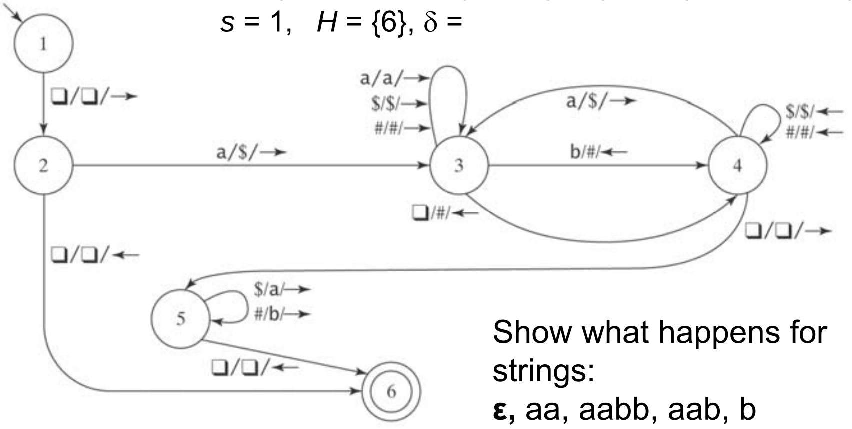

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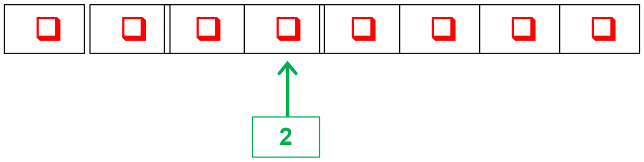

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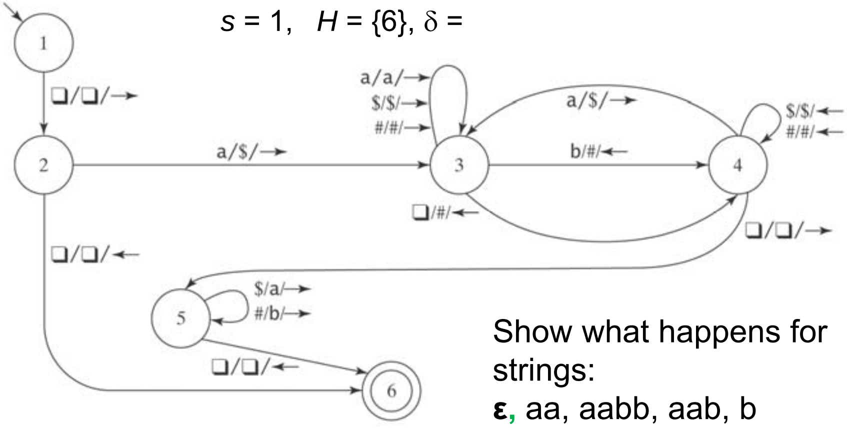


Show what happens for strings:
ε, aa, aabb, aab, b

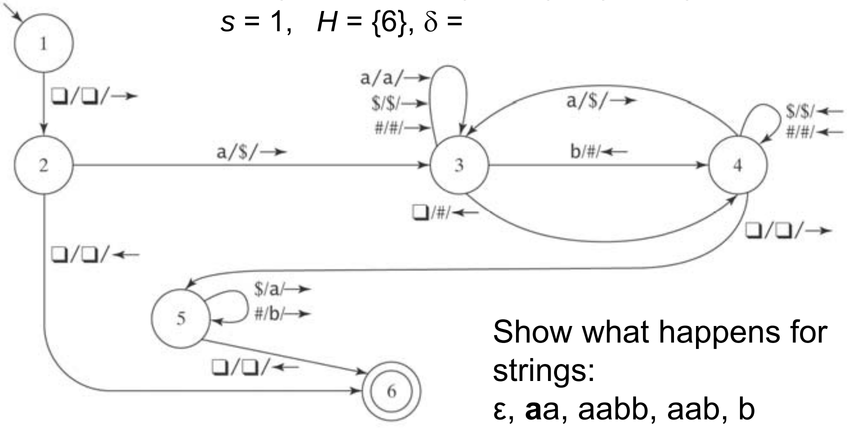
❑	❑	❑	❑	❑	❑	❑	❑

↑

6

The Details(aa)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1,   *H* = {6}, δ =



Show what happens for strings:
ε, **a**a, aabb, aab, b

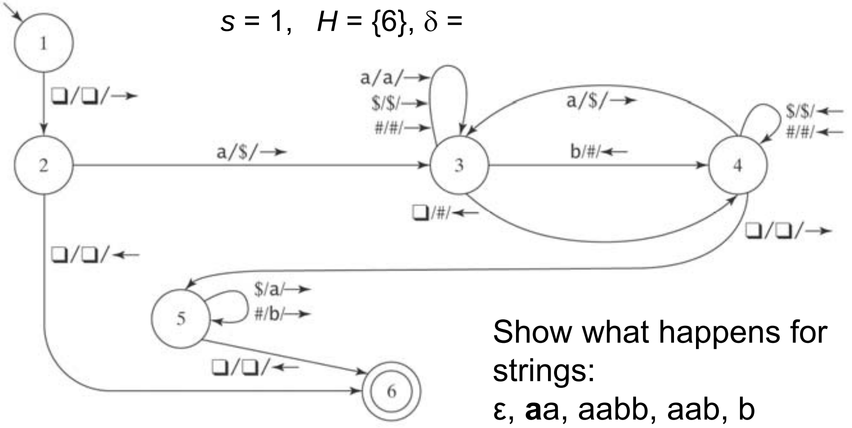
❑	❑	❑	a	a	❑	❑	❑

↑

1

The Details(aa)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1,   *H* = {6}, δ =



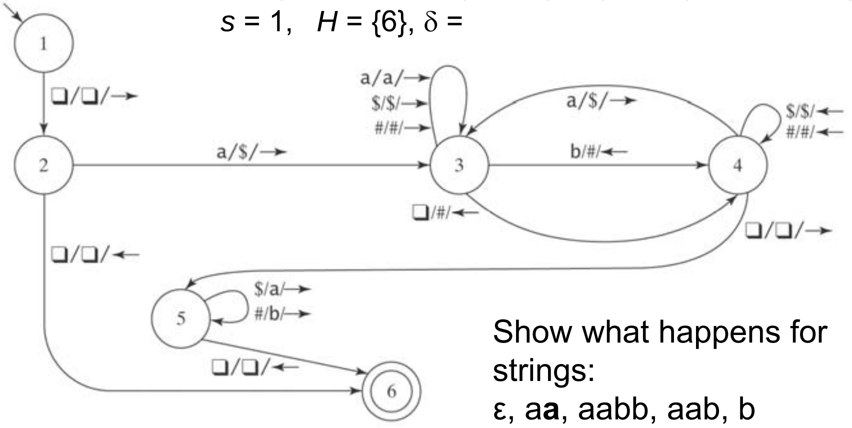
Show what happens for strings:
ε, **a**a, aabb, aab, b

❑	❑	❑	a	a	❑	❑	❑

↑
2

The Details(aa)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1,   *H* = {6}, δ =



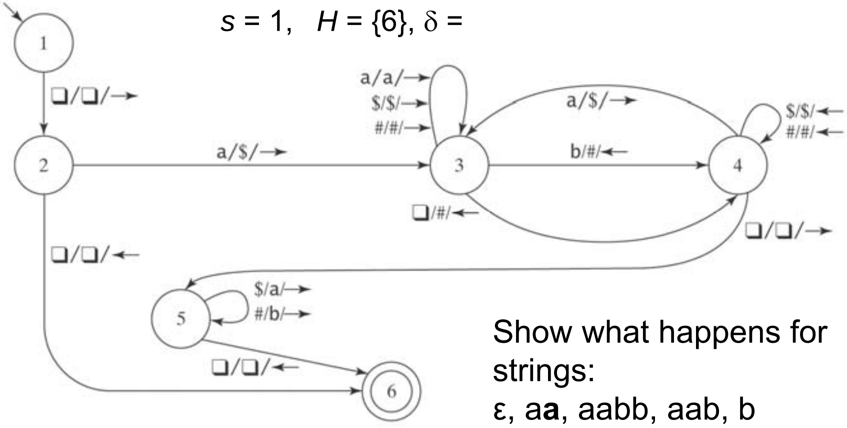
Show what happens for strings:
ε, a**a**, aabb, aab, b

❑	❑	❑	$	a	❑	❑	❑

↑
3

The Details(aa)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1,   *H* = {6}, δ =



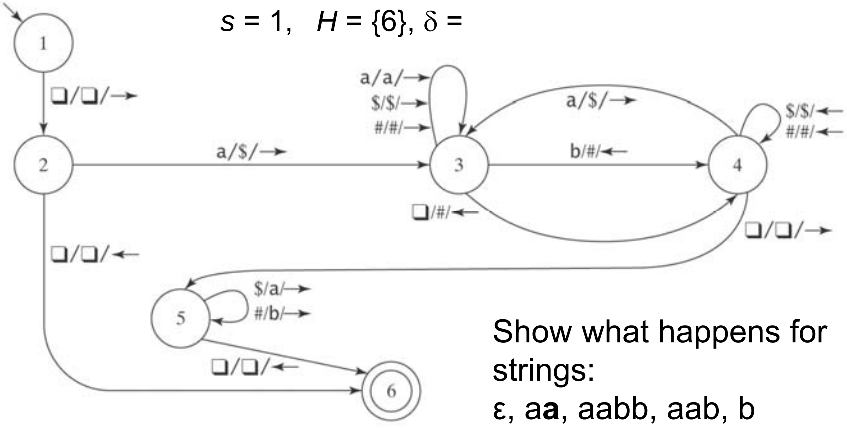
Show what happens for strings:
ε, a**a**, aabb, aab, b

❑	❑	❑	$	a	❑	❑	❑

↑
3

The Details(aa)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1,   *H* = {6}, δ =



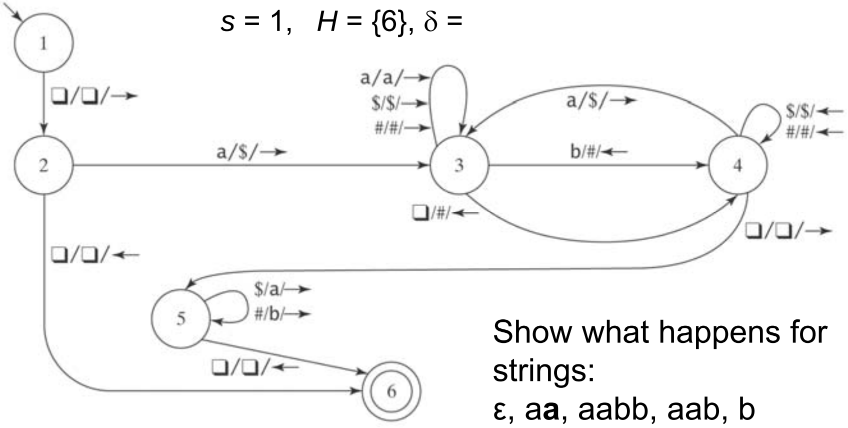
Show what happens for strings:
ε, a**a**, aabb, aab, b

❑	❑	❑	$	a	#	❑	❑

↑
4

The Details(aa)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1,   *H* = {6}, δ =



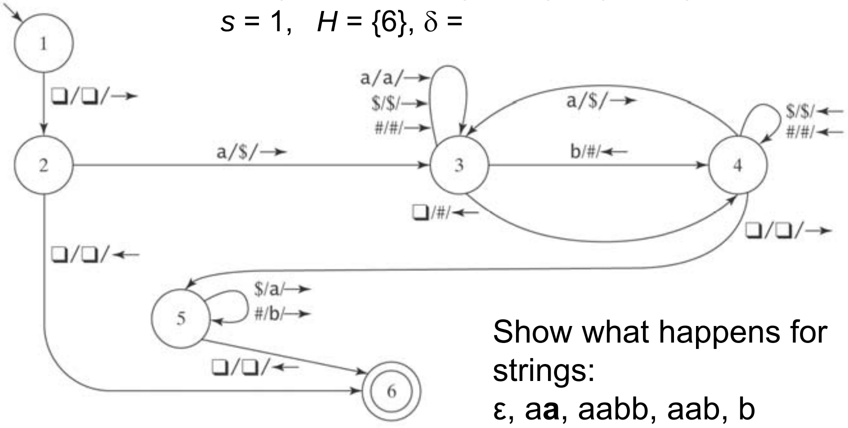
Show what happens for strings:
ε, a**a**, aabb, aab, b

❑	❑	❑	$	$	#	❑	❑

↑
3

The Details(aa)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1,   *H* = {6}, δ =



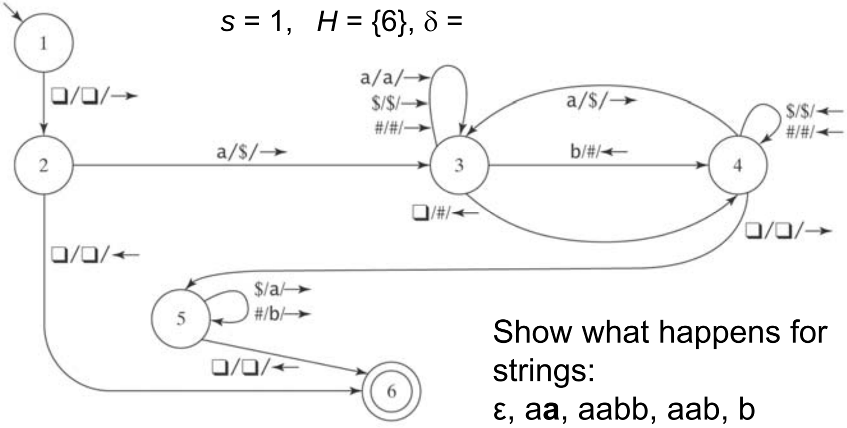
Show what happens for strings:
ε, a**a**, aabb, aab, b

❑	❑	❑	$	$	#	❑	❑

↑
3

The Details(aa)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1,   *H* = {6}, δ =



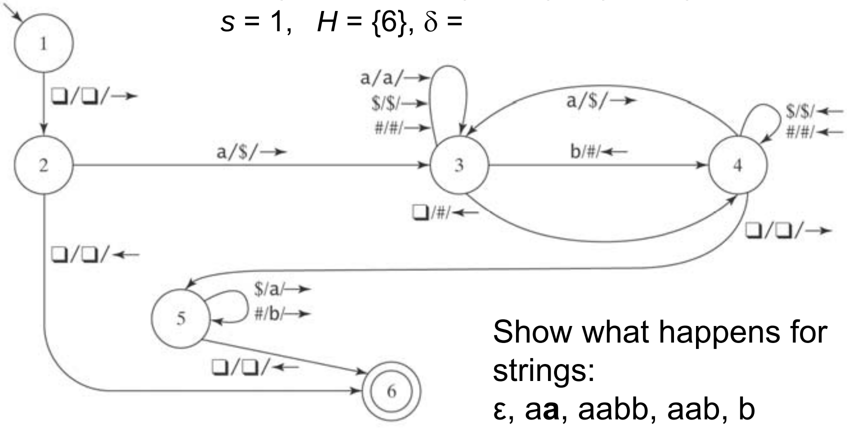
Show what happens for strings:
ε, a**a**, aabb, aab, b

❑	❑	❑	$	$	#	❑	❑

3

The Details(aa)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1,   *H* = {6}, δ =



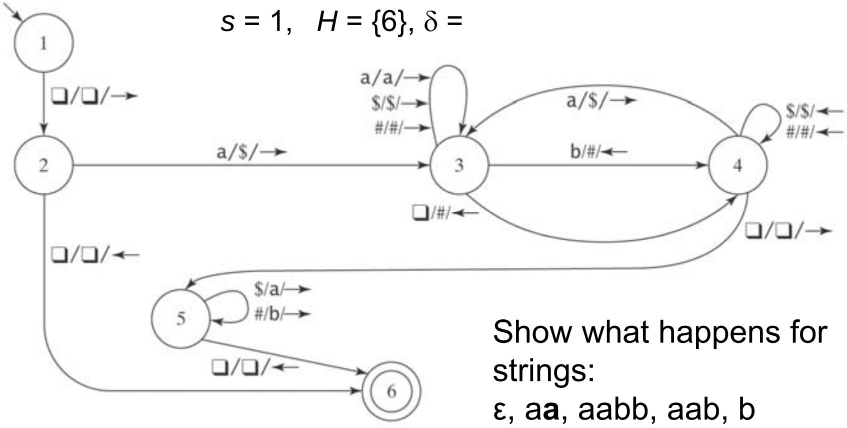
Show what happens for strings:
ε, a**a**, aabb, aab, b

❑	❑	❑	$	$	#	#	❑

4

The Details(aa)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1,   *H* = {6}, δ =



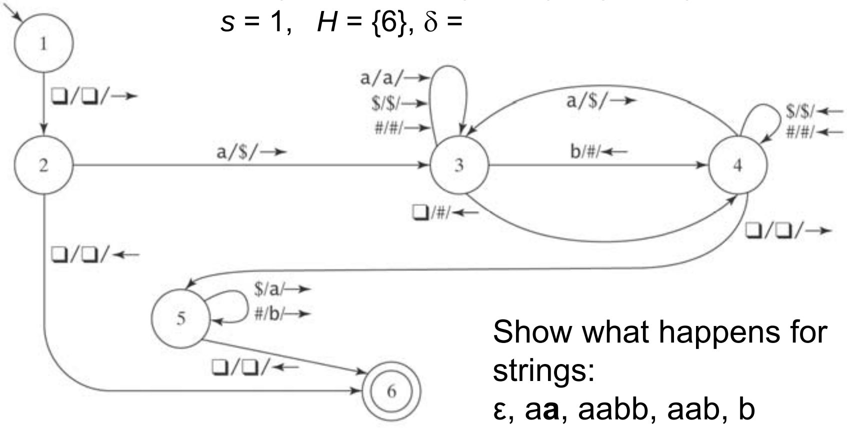
Show what happens for strings:
ε, a**a**, aabb, aab, b

❑	❑	❑	$	$	#	#	❑

↑
4

The Details(aa)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1,   *H* = {6}, δ =



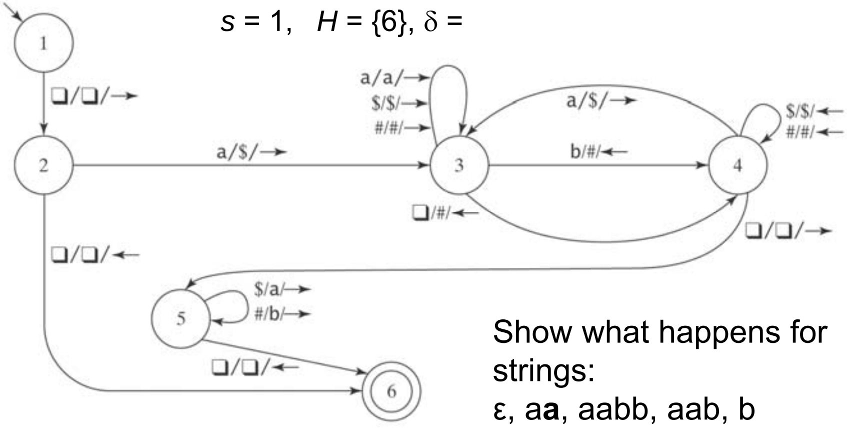
Show what happens for strings:
ε, a**a**, aabb, aab, b

❑	❑	❑	$	$	#	#	❑

↑
4

The Details(aa)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1,   *H* = {6}, δ =



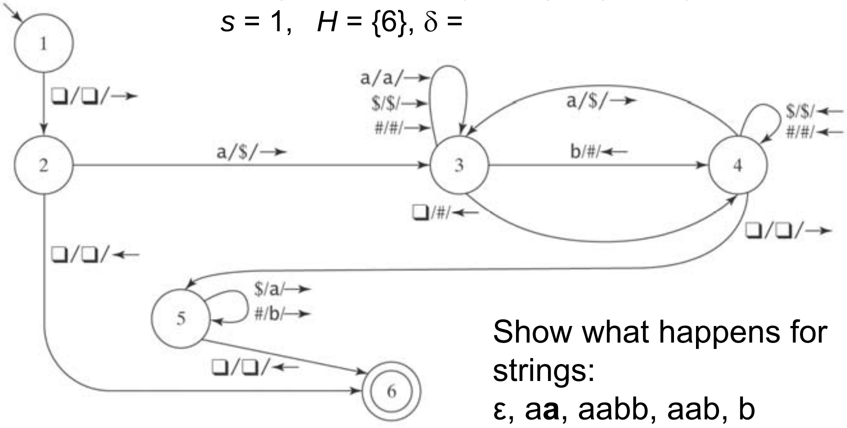
Show what happens for strings:
ε, a**a**, aabb, aab, b

❑	❑	❑	$	$	#	#	❑

↑
4

The Details(aa)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1,   *H* = {6}, δ =



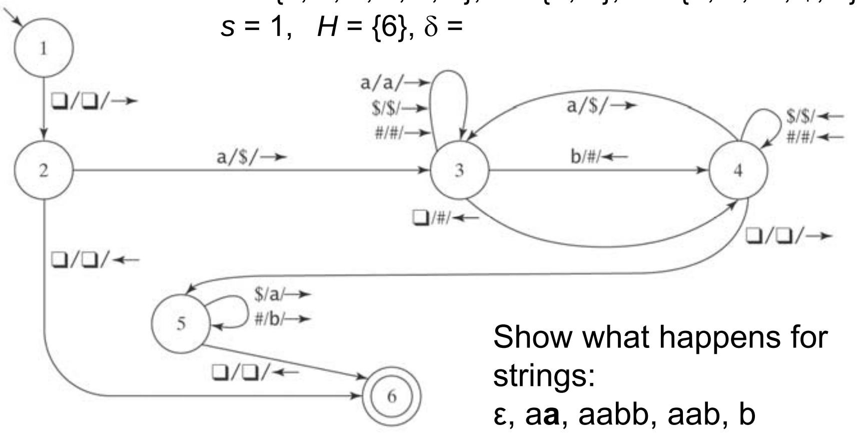
Show what happens for strings:
ε, a**a**, aabb, aab, b

❑	❑	❑	$	$	#	#	❑

↑
5

The Details(aa)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1, *H* = {6}, δ =



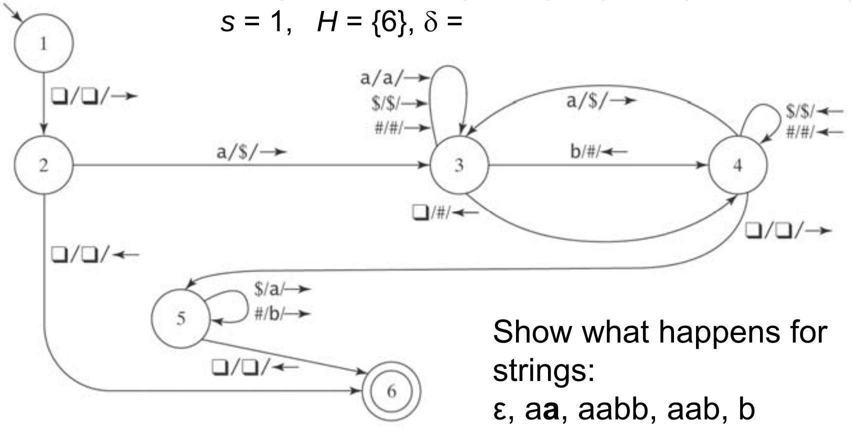
Show what happens for strings:
ε, a**a**, aabb, aab, b

❑	❑	❑	a	$	#	#	❑

5

The Details(aa)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1, *H* = {6}, δ =



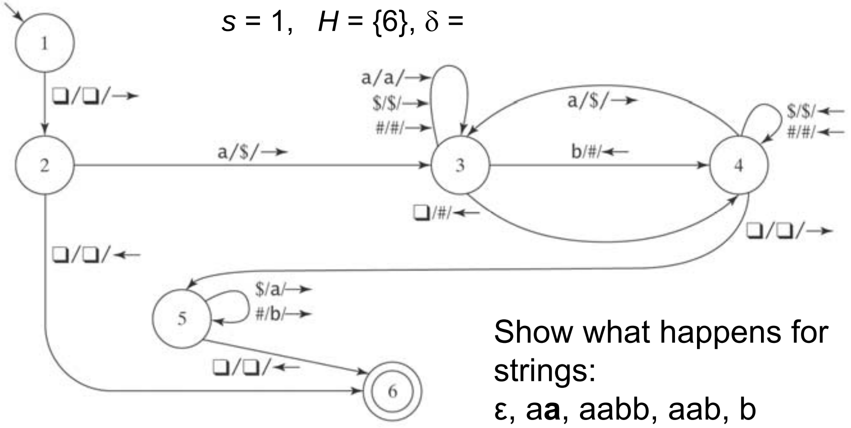
Show what happens for strings:
ε, a**a**, aabb, aab, b

❑	❑	❑	a	a	#	#	❑

5

The Details(aa)

$K = \{1, 2, 3, 4, 5, 6\}$, $\Sigma = \{a, b\}$, $\Gamma = \{a, b, \text{Æ}, \$, \#\}$,
$s = 1$, $H = \{6\}$, $\delta =$



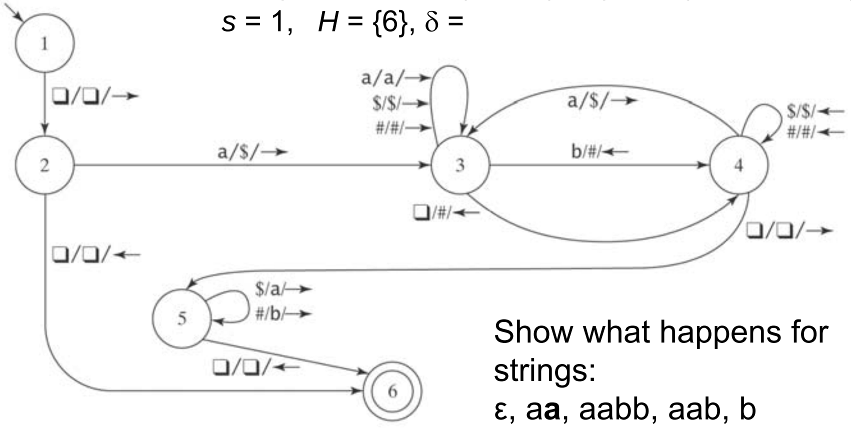
Show what happens for strings:
ε, a**a**, aabb, aab, b

□	□	□	a	a	b	#	□

5

The Details(aa)

$K = \{1, 2, 3, 4, 5, 6\}$, $\Sigma = \{a, b\}$, $\Gamma = \{a, b, \text{Æ}, \$, \#\}$,
$s = 1$, $H = \{6\}$, $\delta =$



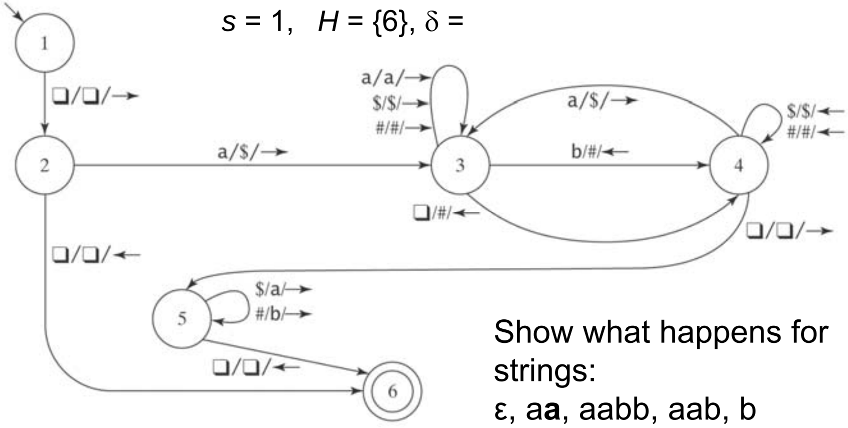
Show what happens for strings:
ε, a**a**, aabb, aab, b

□	□	□	a	a	b	b	□

5

The Details(aa)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1, *H* = {6}, δ =



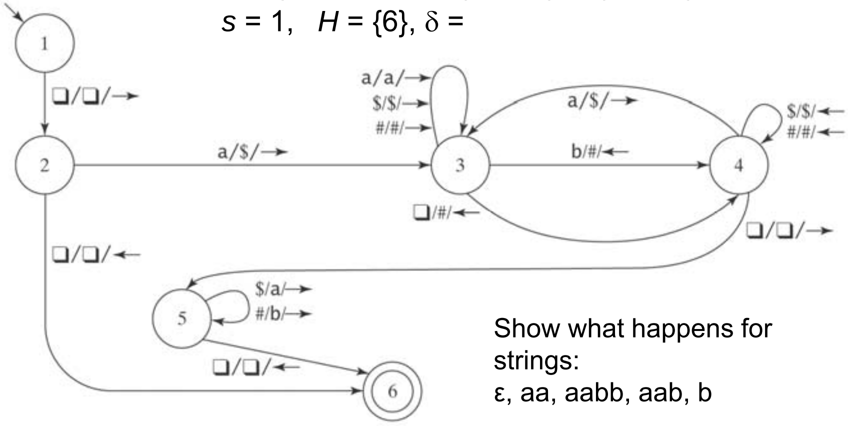
Show what happens for strings:
ε, a**a**, aabb, aab, b

❑	❑	❑	a	a	b	b	❑

6

The Details (aabb)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1, *H* = {6}, δ =



Show what happens for strings:
ε, aa, aabb, aab, b

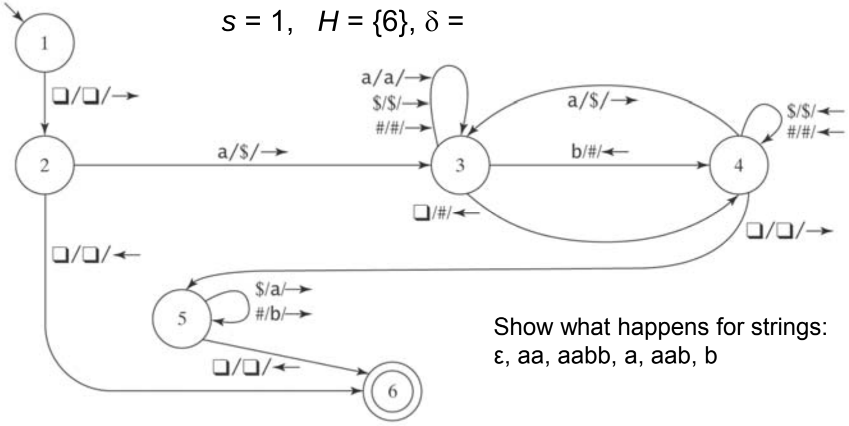
❑	❑	❑	a	a	b	b	❑

1

The steps are the same as previous example until we read the b; skip to there

The Details (aabb)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #}, *s* = 1, *H* = {6}, δ =



Show what happens for strings:
ε, aa, aabb, a, aab, b

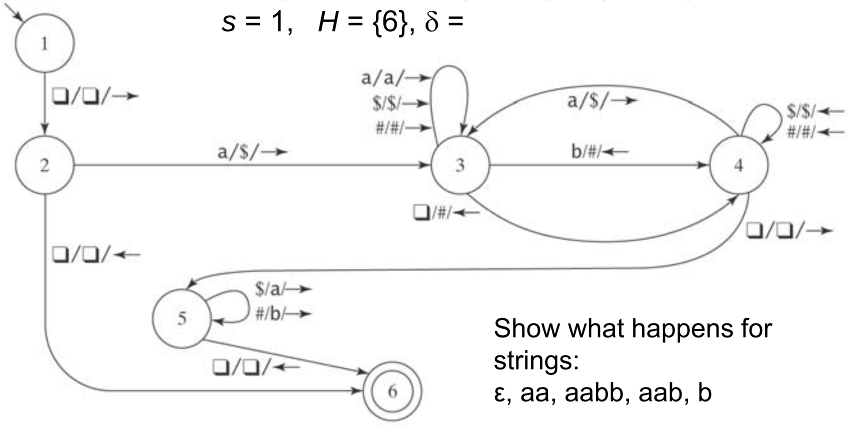
❑	❑	❑	$	a	b	b	❑

↑

3

The Details (aabb)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #}, *s* = 1, *H* = {6}, δ =



Show what happens for strings:
ε, aa, aabb, aab, b

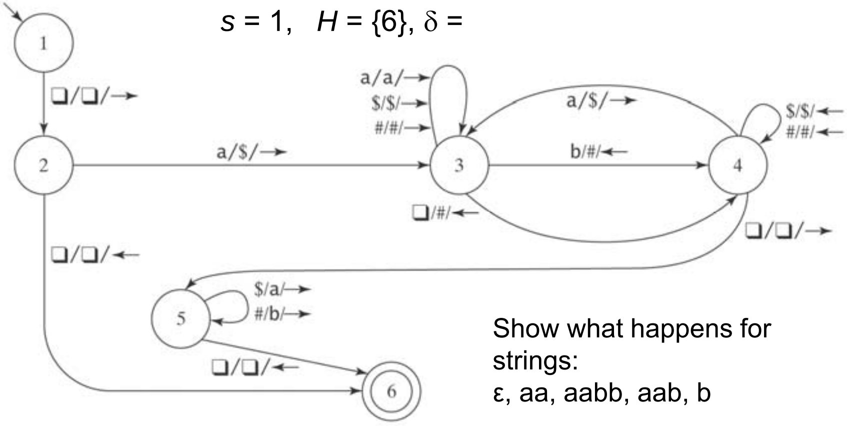
❑	❑	❑	$	a	#	b	❑

↑

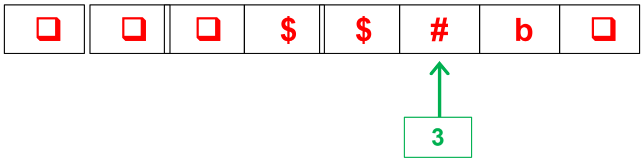
4

The Details (aabb)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1, *H* = {6}, δ =



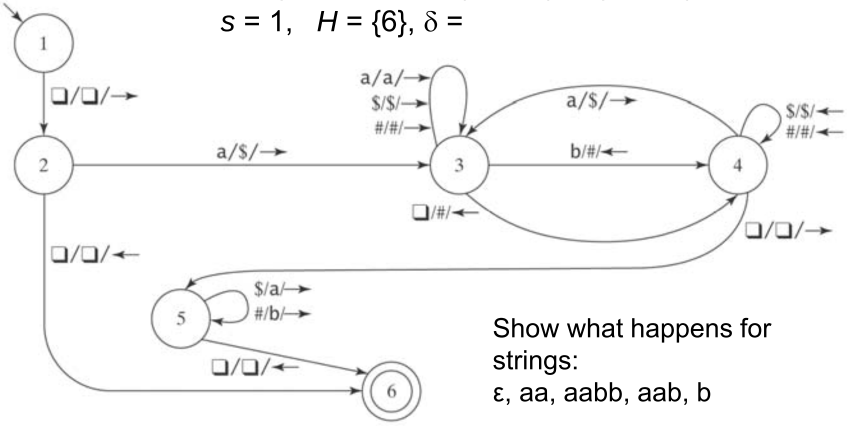
Show what happens for strings:
ε, aa, aabb, aab, b


❑ ❑ ❑ $ $ # b ❑

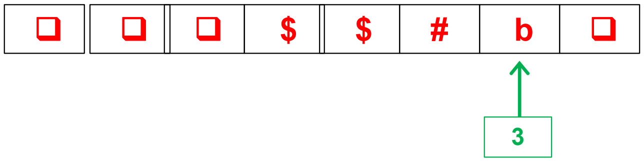
3

The Details (aabb)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1, *H* = {6}, δ =



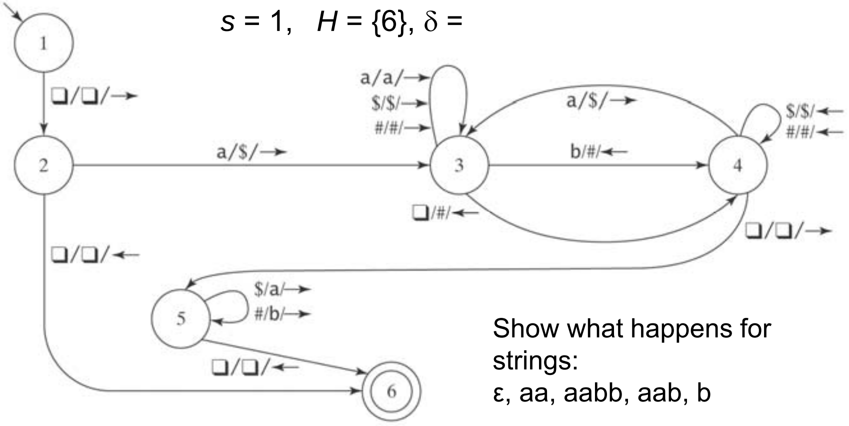
Show what happens for strings:
ε, aa, aabb, aab, b


❑ ❑ ❑ $ $ # b ❑

3

The Details (aabb)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1, *H* = {6}, δ =



Show what happens for strings:
ε, aa, aabb, aab, b

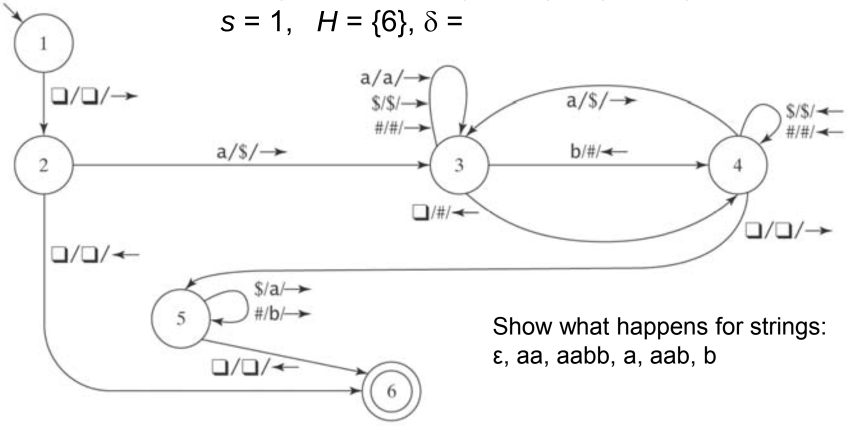
❑	❑	❑	$	$	#	#	❑

In state 4, go left until we hit a blank

4

The Details (aabb)

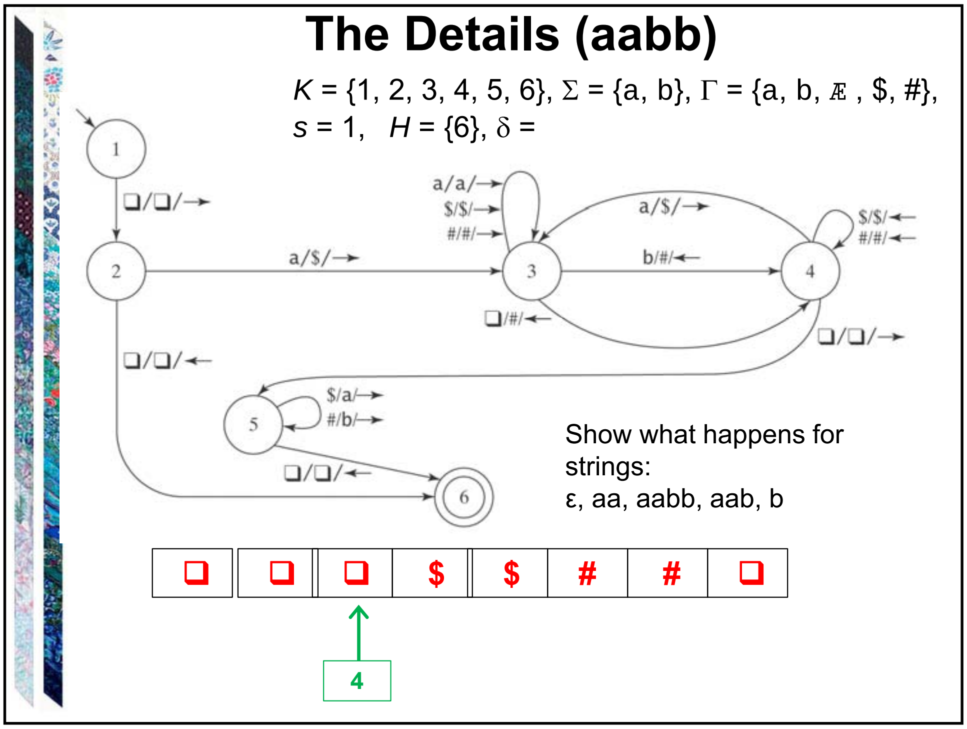
K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1, *H* = {6}, δ =



Show what happens for strings:
ε, aa, aabb, a, aab, b

❑	❑	❑	$	$	#	#	❑

4



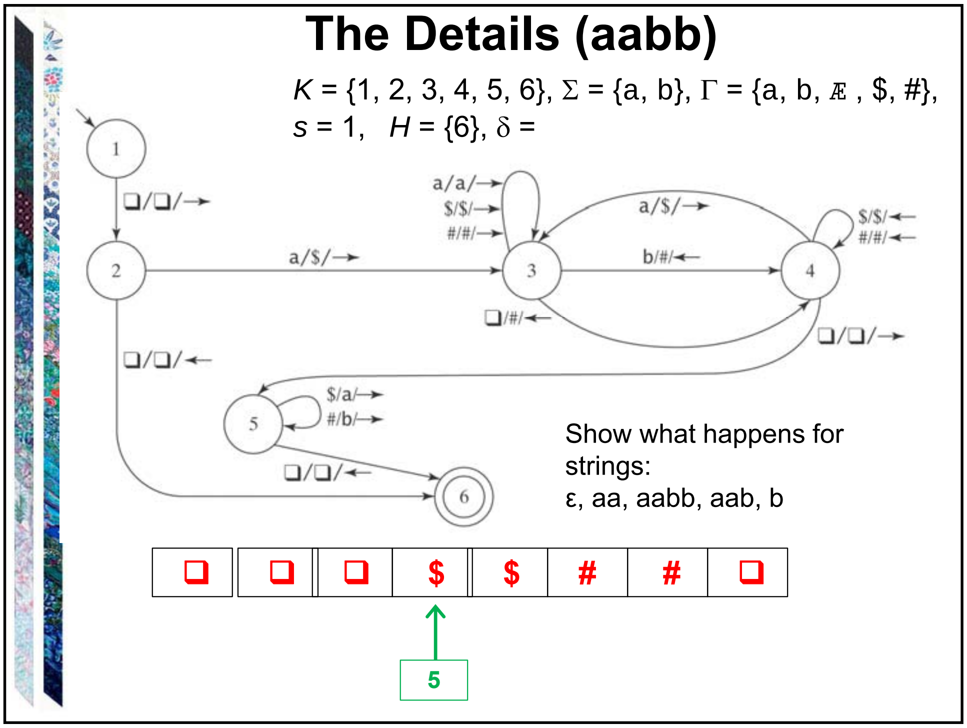
The Details (aabb)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #}, *s* = 1, *H* = {6}, δ =

Show what happens for strings:
ε, aa, aabb, aab, b

❑	❑	❑	$	$	#	#	❑

4



The Details (aabb)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #}, *s* = 1, *H* = {6}, δ =

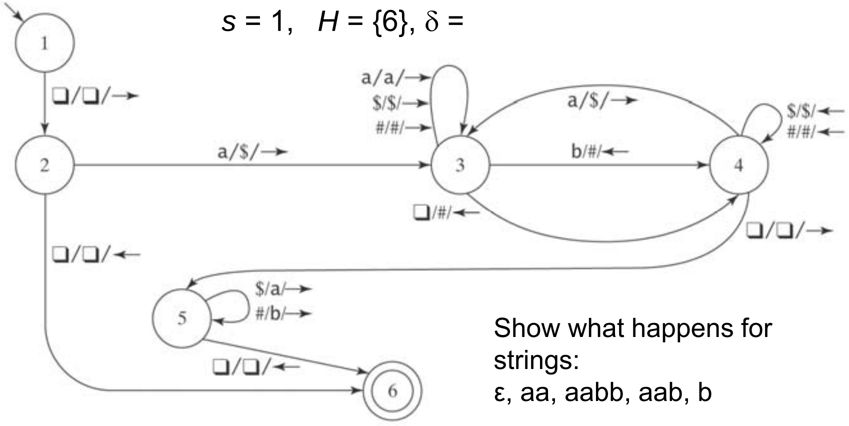
Show what happens for strings:
ε, aa, aabb, aab, b

❑	❑	❑	$	$	#	#	❑

5

The Details (aabb)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1, *H* = {6}, δ =



Show what happens for strings:
ε, aa, aabb, aab, b

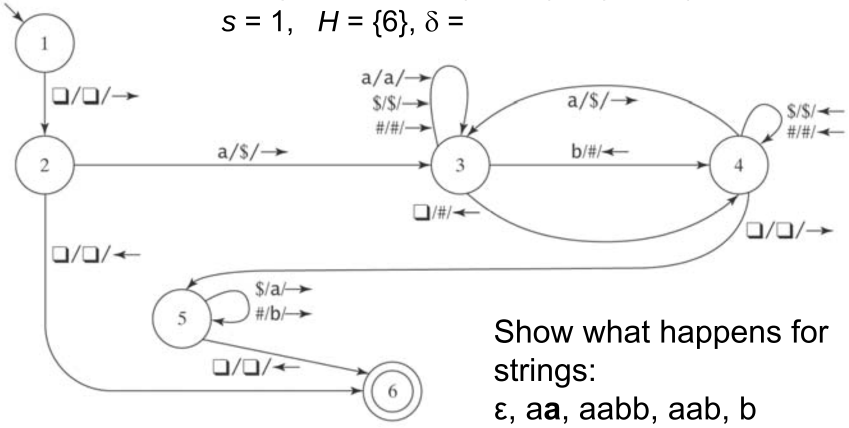
❑	❑	❑	$	$	#	#	❑

5

Go right, replacing $ with a and # with b, then move left, as in the last part of the previous example.

The Details(aabb)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1, *H* = {6}, δ =



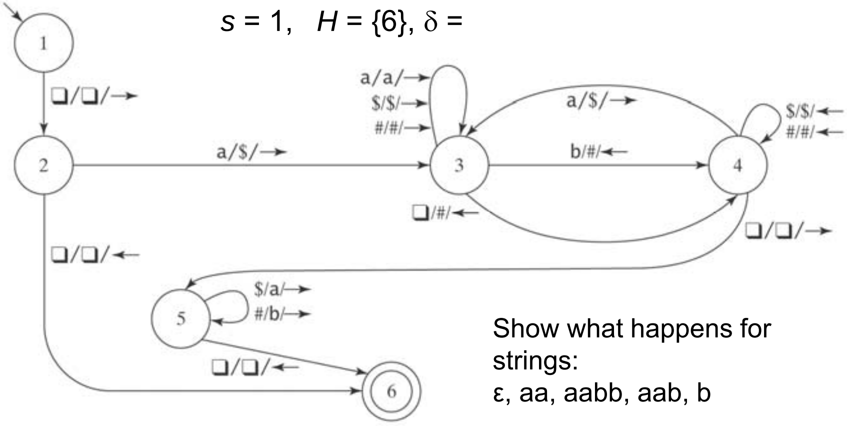
Show what happens for strings:
ε, a**a**, aabb, aab, b

❑	❑	❑	a	a	b	b	❑

6

The Details (aab)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1, *H* = {6}, δ =



Show what happens for strings:
ε, aa, aabb, aab, b

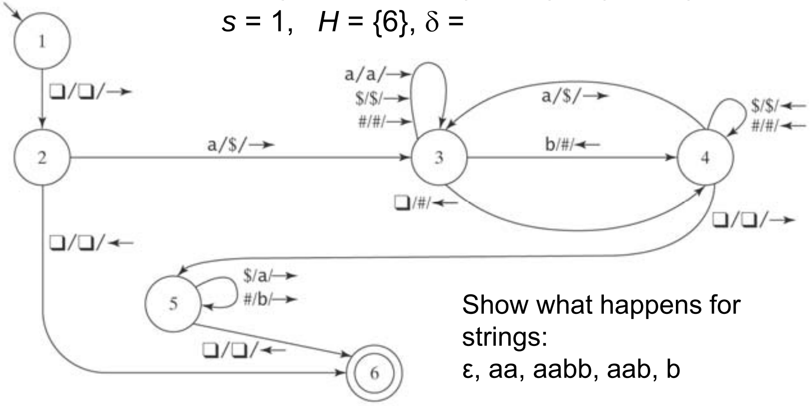
□	□	□	a	a	b	□	□

1

You should try this one.

The Details (b)

K = {1, 2, 3, 4, 5, 6}, Σ = {a, b}, Γ = {a, b, Æ , $, #},
s = 1, *H* = {6}, δ =

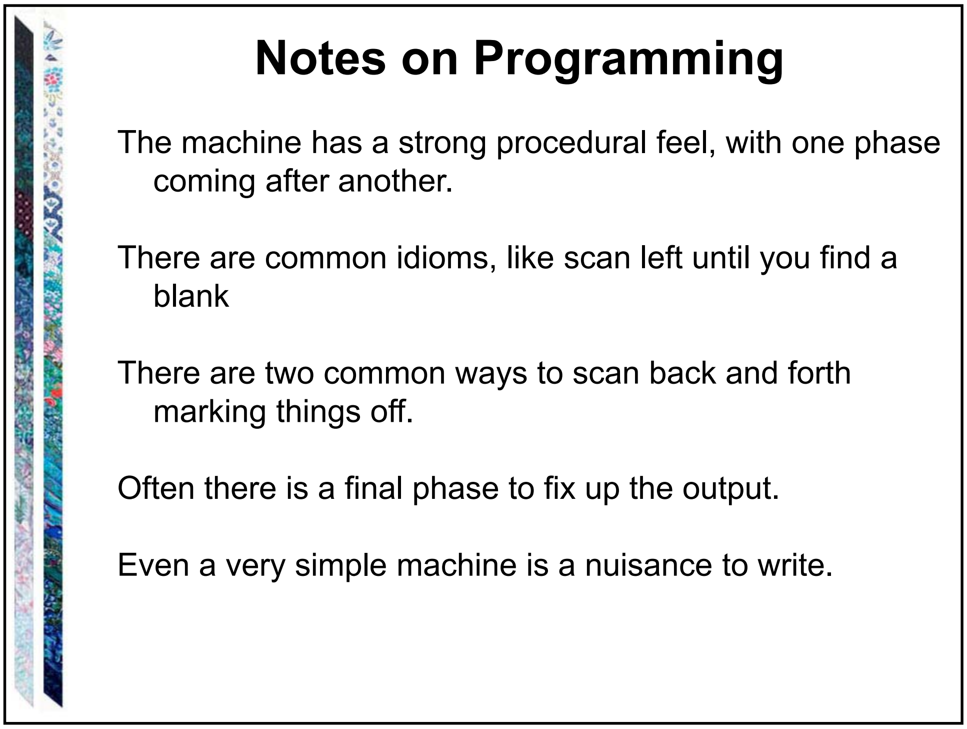


Show what happens for strings:
ε, aa, aabb, aab, b

□	□	□	b	□	□	□	□

1

In the first step, move right. Then there is no transition in the diagram. But there is an implied transition to a dead state, i.e. a new halting state that does not accept.



Notes on Programming

The machine has a strong procedural feel, with one phase coming after another.

There are common idioms, like scan left until you find a blank

There are two common ways to scan back and forth marking things off.

Often there is a final phase to fix up the output.

Even a very simple machine is a nuisance to write.