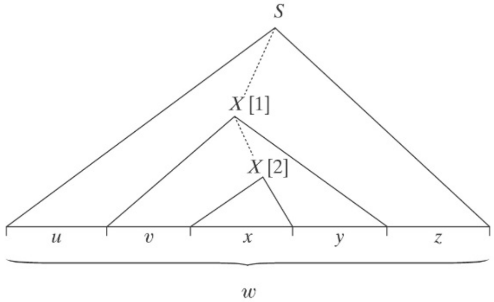
**MA/CSSE 474 Day 25 Summary** (and probably Day 26 also)

1) **Theorem from last class**: The class of languages accepted by PDAs is exactly the class of context-free languages.
   Recall: *context-free languages* are languages that can be defined by context-free grammars.
   a) CFG → PDA is the easy direction, and the one with the most practical use
      i) Two approaches: Top-down parser and Bottom-up parser
2) Top-down parser PDA M from CFG G. Discovers a rightmost derivation from beginning to end.
   a) Mirror the productions: Production A → XYZ becomes (q, ε, A) → (q, XYZ)
   b) Match terminal symbols: (q, a, a) → (q, ε)
   c) Get the process started: (s, ε, ε) → (q, S) [s is the start state of M, different from q] [S is start symbol of G]
   d) The stack holds unmatched terminals and unexpanded nonterminals.
3) Bottom-up parser PDA M from CFG G. Discovers a leftmost derivation from end to beginning.
   a) Mirror the productions: Production A → XYZ becomes (p, ε, XYZ) → (p, A) [p is the start state of M]
   b) Shift terminal symbols from input to stack: (p, a, ε) → (p, a)
   c) Get the process started: (p, ε, S) → (q, ε) [q is accepting state of M, different from q] [S is start symbol of G]
   d) The stack holds prefixes of right sides of rules.

4) Show the transitions as the parser from the "bottom-up" slide parses "id + id * id" (write small or use two colmns)
   ***state*** ***stack*** ***unread input*** ***transition to use next***

5) The number of languages over alphabet Σ is uncountable; number of context-free languages is countably infinite.
6) How to show that a language is context-free:
   a)
   b)
   c)
7) Context-free pumping theorem:




If $L$ is a context-free language, then
$\exists k \geq 1$ ($\forall$ strings $w \in L$, where $|w| \geq k$
($\exists u, v, x, y, z$ ($w = uvxyz$, $vy \neq \varepsilon$, $|vxy| \leq k$,
and
$\forall q \geq 0$ ($uv^qxy^qz$ is in $L$)))).

8) As with the reg.-lang. pumping theorem, to show a language is *not* CF, we use the contrapositive. We do not get to choose the k or the breakdown into uvxyz. We choose the w$\varepsilon$L, and for each breakdown, *q* a such that $uv^qxy^qz \notin L$.

9) Make note of the slide on similarities and differences between the two pumping theorems.

10) $A^nB^nC^n = \{a^nb^nc^n, n \geq 0\}$ Three regions, two cases, details on slides.

11) $\{ a^{n^2} : n \geq 0\}$

12) $L = \{a^nb^ma^n, n, m \geq 0 \text{ and } n \geq m\}$.

Let $w = a^kb^ka^k$

aaa … aaabbb … bbbaaa … aaa
| 1 | 2 | 3 |

13) WcW = $\{wcw : w \in \{a, b\}^*\}$ (details on slide)

14) $\{(ab)^na^nb^n : n > 0\}$

15) $\{x\#y : x, y \in \{0, 1\}^* \text{ and } x \neq y\}$