MA/CSSE 474 Day 23 Summary Review PDA definitions; lots of PDA examples

1. **PDA definition:** $M = (K, \Sigma, \Gamma, \Delta, s, A)$, where
   a) ***K*** is a finite set of **states**
   b) Σ is the finite **input alphabet**
   c) Γ is the finite **stack alphabet** **[**note that Σ and Γ can contain some of the same symbols]
   d) $s \in K$ is the **initial (start) state**
   e) $A \subseteq K$ is the set of **accepting states**, and
   f) Δ is the **transition relation.** It is a finite subset of $(K \times (\Sigma \cup \{\varepsilon\}) \times \Gamma^*) \times (K \times \Gamma^*)$
      i) i.e. (state, single input symbol or ε, string of stack symbols) → (state, string of stack symbols)
      ii) The first "string of stack symbols" will almost always be a single symbol or ε.
      iii) Note that this is nondeterministic; there can be one, many, or zero transitions out of a given configuration.
2. Configurations:
   a) A **configuration** of *M* is an element of $K \times \Sigma^* \times \Gamma^*$.
      i) (current state, remaining unread input, what's on the stack (left end is top of stack)
   b) The **initial configuration** of *M* is (s, w, ε), where w is the input string.
3. The stack.
   a) Left end of the string is top of stack
   b) If the stack contains *def* and we push *abc*, the new stack content is *abcdef*.
4. Machine transitions: $(q_1, cw, \gamma_1\gamma) \vdash_M (q_2, w, \gamma_2\gamma)$ iff $((q_1, c, \gamma_1), (q_2, \gamma_2)) \in \Delta$.
5. Yields, Computations, Acceptance, L(M), Rejection
   a) Let $\vdash_M{}^*$ be the reflexive, transitive closure of $\vdash_M$.
   b) *Configuration* C_1 ***yields*** configuration C_2 iff $C_1 \vdash_M{}^* C_2$
   c) A ***computation*** by *M* is a finite sequence of configurations $C_0, C_1, \ldots, C_n$ for some $n \geq 0$ such that:
      i) C_0 is an initial configuration,
      ii) C_n is of the form (q, ε, γ), for some state $q \in K_M$ and some string γ in Γ^*, and
      iii) $C_0 \vdash_M C_1 \vdash_M C_2 \vdash_M \ldots \vdash_M C_n$.
   d) In an ***accepting computation*** *of M,* $C = (s, w, \varepsilon) \vdash_M{}^* (q, \varepsilon, \varepsilon)$, *and* $q \in A$.
      i) *M* ***accepts*** a string *w* iff it has at least one accepting computation that begins with (s, w, ε).
   e) **Messy:** Note that there are many possibilities for non-acceptance:
      i) Read all the input and halt in a non-accepting state,
      ii) Read all the input and halt in an accepting state with non-empty stack,
      iii) Loop forever doing epsilon-transitions and never finish reading the input, or
      iv) Reach a dead end where there are no legal transitions.
   f) L(M), the ***language accepted by*** *M*, is $\{ w \in \Sigma : \text{M accepts w} \}$
   g) A computation *C* of *M* is a ***rejecting computation*** iff:
      i) $C = (s, w, \varepsilon) \vdash_M{}^* (q, w', \alpha)$,
      ii) *C* is not an accepting computation, and
      iii) *M* has no moves that it can make from (q, ε, α).
   h) *M* ***rejects*** a string *w* iff all of its computations reject.
      i) Note that it is possible that, on input *w*, *M* neither accepts nor rejects.
6. We look at PDA's for BAL, A^nB^n, wcw^R. Make sure that you understand how these work. Notes here:

7. A PDA for $\{a^n b^{2n}: n \geq 0\}$

8. A PDA for PalEven $=\{ww^R: w \in \{a, b\}^*\}$

9. A PDA for $\{w \in \{a, b\}^* : \#_a(w) = \#_b(w)\}$

10. $L = \{a^m b^n : m \neq n;\ m, n > 0\}$ (Details on slides)

11. To reduce non-determinism we can add markers for bottom-of-stack and end-of-input.

12. PDA for $A^nB^nC^n = \{a^n b^n c^n: n \geq 0\}$