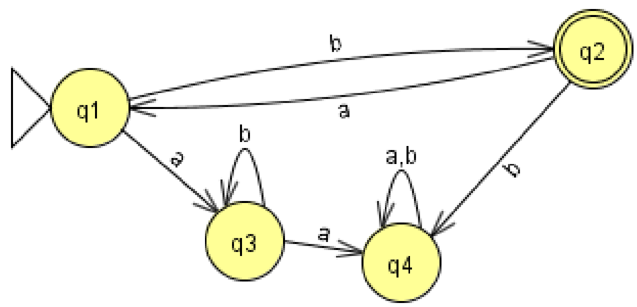
MA/CSSE 474 Day 12 Summary

1. **For Every DFSM, there is an equivalent regular expression:**
   a. Number the states $q_1, \ldots, q_n$.
   b. Define $\mathbf{R_{ijk}}$ to be the set of all strings $x \in \Sigma^*$ such that
      $(q_i,x) \mid\!-_M^* (q_j, \varepsilon)$, and
      if $(q_i,y) \mid\!-_M^* (q_\ell, \varepsilon)$, for any prefix y of x (except $y= \varepsilon$ and $y=x$), then $\ell \le k$
   c. That is, R_{ijk} is the set of all strings that take us from q_i to q_j without passing through any intermediate states numbered higher than k.
      i. In this case, "passing through" means both entering and leaving.
      ii. Note that either i or j (or both) may be greater than k.



2. **Formulas for $\mathbf{R_{ijk}}$:**
   a. Base cases (k = 0):
      i. If $i \neq j$, $R_{ij0} = \{a\in\Sigma : \delta(q_i, a) = q_j\}$
      ii. If $i = j$, $R_{ii0} = \{a\in\Sigma : \delta(q_i, a) = q_i\} \cup \{\varepsilon\}$
   b. Recursive case (k > 0):
      R_{ijk} is $R_{ij(k-1)} \cup R_{ik(k-1)}(R_{kk(k-1)})^*R_{kj(k-1)}$

3. In the DFSMtoRegExp example machine M from the slides, show how to get

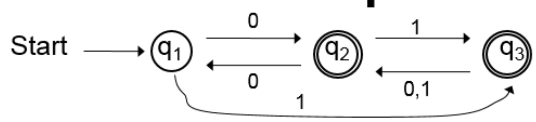
r_{221}

r_{132}

r_{123}

r_{133}

An Example



	k=0	k=1	k=2
r_{11k}	ε	ε	(00)*
r_{12k}	0	0	0(00)*
r_{13k}	1	1	0*1
r_{21k}	0	0	0(00)*
r_{22k}	ε	ε ∪ 00	(00)*
r_{23k}	1	1 ∪ 01	0*1
r_{31k}	∅	∅	(0 ∪ 1)(00)*0
r_{32k}	0 ∪ 1	0 ∪ 1	(0 ∪ 1)(00)*
r_{33k}	ε	ε	ε ∪(0 ∪ 1)0*1

A regular expression r such that L(R) = L(M)