MA/CSSE 474 Day 03 Summary . Main ideas from today (and probably a few that we'll get to tomorrow):

1. Operations on strings:
   a. $|w|$
   b. $\#_a(w)$
   c. Concatenation wx (it's associative, and ε is the identity for this operation)
   d. w^i
   e. w^R (recursive definition): $\varepsilon^R = \varepsilon$, $(ua)^R = au^R$

2. You should know the meaning of *substring*, *prefix*, and *suffix*, and the "proper" version of each of those.

3. Describe in simple English the language $L = \{x: \exists u \in \{a, b\}^* : x = ua\}$

4. $L = \{x\#y: x, y \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}^*$ and, when x and y are viewed as the decimal representations of natural numbers, $square(x) = y\}$.
   Examples: (in L or not?) : #9, 12#144, 3#8, 12, 12#12#12, #

5. Give a very short symbolic description of $L = \{a^n : n \geq 0\}$

6. A language that we will use a lot: $A^nB^n = \{a^kb^k : k \geq 0\}$

7. Is the Halting problem language well-specified? Can we decide which strings it contains?

   L = {w: w is a Java program that, no matter what finite input string it is given, is guaranteed to halt}.

8. Languages based on prefixes :
   $L = \{w \in \{a, b\}^*$: no prefix of w contains b}

   $L = \{w \in \{a, b\}^*$: no prefix of w starts with a}

   $L = \{w \in \{a, b\}^*$: every prefix of w starts with a}

9. Concatenation of languages, and related things
   a. L_1L_2. If L1 = {a, aa}, and L2 = {a, c, ε}, then L1 L2 = . What is $L\{\varepsilon\}$? $L\emptyset$?

   b. L^R

   c. L^3 Is this the same as $\{w3 : w \in L\}$?

   d. L^0

   e. L^k

   f. L^*

   g. L^+

10. How many different partitions of the set {1, 2, 3}?

11. Are the natural numbers closed under $f(x) = 1+x$? $g(x) = x - 1$? $h(x) = 1/x$ $k(x) = \#$ distinct factors of x?

12. What is the closure of $\mathbb{N}^+$ under division?

13. Show that $\equiv_3$ is an equivalence relation. ($a \equiv_3 b$ iff $b-a = 3k$ for some integer k).

    reflexive:

    symmetric:

    transitive:

14. Can a language be uncountable?

    Is the set of languages over a specific alphabet uncountable?

15. Questions about the *maxstring* function (defined on a slide):
    maxstring(A^nB^n) = *maxstring*({a}*) =
    Let INF be the set of infinite languages. Let FIN be the set of finite languages.
    Are the language classes FIN and INF closed under *maxstring*?

16. Questions about the *chop* function (defined on a slide):
    What is *chop*(A^nB^n)? What is *chop*($A^nB^nC^n$)?
    Are FIN and INF closed under *chop*?

17. Questions about *firstchars* function(defined on a slide)::
    What is *firstchars*(A^nB^n)? What is *firstchars*({a, b}*)?
    Are FIN and INF closed under *firstchars*?

18. A decision problem is a problem whose answer is _____.