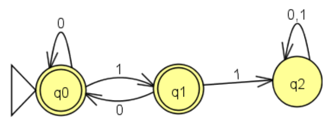
MA/CSSE 474 Day 02 Summary

Main ideas from today:

1. You should be able to quickly locate the course schedule page, the Moodle page, and the Piazza page. You should set the Piazza email option that best fits how you want to receive announcements and discussion posts. I recommend "real time" and "follow all".
2. For most students, this course will be very time-consuming. This is partly because you may need to read some sections of the textbook a few times before you "get it", and partly because some of the problems are very challenging.
3. This is a place to make notes about "policies and procedures" things that come out during our discussion.



4. Review of yesterday's "proof start". yesterday's proof.
   a. S = L(M) for this machine.
   b. T = {w∈{0, 1}* : w does not contain 11 as a substring.
   c. We want to show that S = T. First we show that S ⊆T. I.e., for all strings w: if w∈S, then w∈T
   d. We prove by induction on |w| that a slightly stronger statement is true. For all strings w,
      i. If δ(q0, w) = q0, then w does not end in 1 and w does not contain 11 as a substring.
      ii. If δ(q0, w) = q1, then w ends in 1 and w does not contain 11 as a substring.
   e. If this is true for all w, then S⊆T.
   f. We showed it to be true for the base case |w| = 0.
   g. Induction step. |w| >= 1, so w = ua for some string u and some symbol a (a is 0 or 1).
   h. We assume that (1) and (2) are true for every string that is shorter than w (for example, u is shorter than w), and use that induction hypothesis to show that (1) and(2) are true for w.

5. Continue the proof of properties (1) and (2) in d above.

   Case 1 : δ(q0, w) = q0

   Case 2: δ(q0, w) = q1

6. The part of the proof that we did not do yesterday:
   a. If *w* does not have two consecutive 1's. then *w* is accepted by M. We prove the contrapositive:
   b. If *w* is not accepted by M, then *w* has two consecutive 1's.
   c. No need for you to rewrite the proof from the slides, but here is a place to make notes about the proof.

7. Notes on reading quiz 1: In case you want to write down anything about these:

8. Operations on strings: (we probably will not get to these on Day 2, but they are here just in case.).
   a. $|w|$
   b. $\#_a(w)$
   c. Concatenation wx (it's associative, and ε is the identity for this operation)
   d. w^i
   e. w^R (recursive definition): $\varepsilon^R = \varepsilon$, $(ua)^R = au^R$

Theorem: If *w* and *x* are strings, then $(wx)^R = x^R w^R$. Prove it by induction on $|x|$

Base case: $|x| = 0$: Then $x = \varepsilon$, and $(wx)^R = (w\,\varepsilon)^R = (w)^R = \varepsilon\, w^R = \varepsilon^R\, w^R = x^R\, w^R$.

Induction step: $\forall n \geq 0\ (((|u| = n) \rightarrow ((w\,u)^R = u^R\, w^R)) \rightarrow ((|x| = n + 1) \rightarrow ((w\,x)^R = x^R\, w^R)))$:

Consider any string *x*, where $|x| = n + 1$. Then $x = u\,a$ for some symbol *a* and $|u| = n$. So:

$(w\,x)^R$	$= (w\,(u\,a))^R$	rewrite *x* as *ua*
	$= ((w\,u)\,a)^R$	associativity of concatenation
	$= a\,(w\,u)^R$	definition of reversal
	$= a\,(u^R\, w^R)$	induction hypothesis
	$= (a\,u^R)\, w^R$	associativity of concatenation
	$= (ua)^R\, w^R$	definition of reversal
	$= x^R\, w^R$	rewrite *ua* as *x*

9. ***If there is time:*** *prefixes and suffixes of strings, concatenation and powers of languages.*