

Resources allowed:

1. Required: Working Windows or Mac laptop with webcam and microphone. You will be instructed by ProctorU to download and install some software, so be sure that you have administrative rights to the computer you are using. A mirror or other flat, shiny surface (such as a CD) so you can show the proctor your surroundings
2. Two pieces of blank paper for use as scratch paper.
3. (optional) Another 8.5" x 11" sheet of paper containing handwritten notes on one side only.

ProctorU: This service is required, and it is free if you sign up for an exam time at least 72 hours before that time, and if you don't miss your appointment. You must begin between midnight and 9 PM Indiana time on Thursday, August 14.

Material Covered:

- **HW 0 - HW 15** (including the "not to turn in" problems)
- **Assigned readings** Sections A.1-A.6, 2.1-2.2, 3.1-3.2, 4.1-4.3, 5.1-5.8, 6.1-6.4, 8.1-8.6, 9.1-9.2, 10, 11.1-11.3, 11.5-11.8, 12.1-12.4, 13.1-13.5, 14, 17.1-17.3, 17.6-17.7, 18, 19, 20, 21),
- and material from PowerPoint slides.

Question types: In order to minimize the amount of typing you must do, the exam will have several short-answer, multiple-choice and T-F-IDK questions. For questions where you have to write something, you do not have to have perfectly formatted mathematics. English and "mathematical pseudocode" is fine. For example, $\exists x > 0 (\forall y \leq x^2)(x^2 + y^2 \geq 8)$ or

"empty set $L = \text{"empty set"}$ "

- A. **T-F-IDK.** Below you will find several statements. A statement is true (T) if it is always true. It is false (F) if there is at least one counterexample (sometimes false). You may also choose IDK to indicate that you do not know the answer. *There is some value (to me and you) in knowing what you don't know.*

Point values: Correct answer: 5, incorrect answer: 0, IDK: 3, blank: 0. So if you get seven questions correct, choose IDK for three, get two wrong, and leave one blank, your score is $7*5 + 3*3 + 2*0 + 1*0 = 44$ out of 65. What if you are not sure of an answer? If you are totally guessing an answer, you are probably better off pointwise if you choose IDK. If you have a feeling that one "real" answer is correct, you are probably better off guessing that answer.

- B. **R-CF-D-SD-NSD** For each of the following language descriptions, circle

R if the language is regular,
CF if it is context-free but not regular,
D if it is decidable but not context-free
SD if it is semi-decidable but not decidable
NSD if it is not semi-decidable

Scoring: Correct answer: 3, adjacent on correct side of D|SD line: 1, wrong side of line 0. Blank 0

Example: R CF D | SD NSD $L_1 \cap L_2$, where $L_1 = \{a^n b^m : 0 \leq m \leq n\}$ and $L_2 = \{a^n b^m : 0 \leq n \leq m\}$.

- C. **Reduction proofs** (such as those in HW 14 and HW 15) will constitute a non-trivial portion of this exam.
- D. **General short-answer questions** about strings, languages, grammars, machines, computation.
- E. **Do you know how to use this algorithm/construction that we studied?** Some questions will be checking to see whether you know and understand the details of specific algorithms or constructions. **Some examples**

- NDFSM \rightarrow DFSM algorithm
- Minimal DFSM for a language or machine
- Find canonical form for a DFSM
- Construct machines to reverse, intersection, union, etc.
- Reg exp \rightarrow FSM
- FSM \rightarrow Reg exp
- Top-down and bottom-up parsing PDAs (non-deterministic) (build a PDA from a Grammar)
- Can build a grammar from a PDA (but you don't have to know how)
- Pumping Theorem (basic ideas of how it is derived, know how to use it to show a language non-CF)

Decision algorithms:

- decideFSM
- decideRegExp
- emptyFSM
- totalFSM
- finiteFSM
- infiniteFSM
- equalFSMs
- minimalFSM
- Deciding membership in a CFL (You should know algorithms for doing it using a grammar and using a PDA)
- DecideCFLempty
- DecideCFLinfinite
- CFL equivalence (decidable, but you do not have to know how to show it)

Concepts, definitions, etc.

- Propositional logic
- First-order logic (term, wff, quantifiers, free variable, sentence, interpretation, model, valid, satisfiable, unsatisfiable, theorem, proof, sound, complete)
- Sets (enumeration, characteristic function, decide, partition, Cartesian product)
- Relations(binary, n-ary, inverse of a binary relation, reflexive, symmetric, transitive, equivalence relation, equivalence classes, partial order)
- Functions (total, partial, one-to-one, onto)
- Closure (relation R closed under property P, relation R closed under function f)
- Proof techniques (construction, contradiction, counterexample, case enumeration, mathematical induction, pigeonhole principle, Diagonalization.
- String (alphabet, length, replication, concatenation, reverse, proper substring, prefix, suffix)
- Language ($\{ \}$ vs $\{ \epsilon \}$, lexicographic enumeration, uncountably infinite number of languages, functions on languages, concatenation, reverse, Kleene * and +)
- Relationship between problems and languages
- Decision procedures
- Determinism and non-determinism
- Functions on languages (for example, chop, firstchars, oddsL, maxstring)
- Finite state machine (the five parts of the definition)
- Configuration, computation, acceptance of a string, rejection, language accepted by a DFSM, dead state.
- Regular languages
- Nondeterministic FSMs, acceptance, rejection, language accepted by M, ϵ -transitions
- \approx_L equivalence classes for a language, distinguishable elements

- Regular expressions and the languages they describe
- Show a language regular or nonregular
- Closure properties
- Pumping Theorem
- Context-free grammars: terminal, nonterminal, productions (i.e. rules), derivation, language generated by a CFG
- Ambiguity
- Nullable Non-terminal
- Unit Production
- Chomsky Normal Form
- Griebach Normal form
- Pushdown Automata (deterministic and nondeterministic)
- Formal definition
- Configuration, yields, computation, accepting computation, language accepted by M
- Every regular language is context-free.
- The number of context-free languages over alphabet Σ is countably infinite.
- The set of CFLs is closed under Union, Concatenation, Kleene Star, Reverse
- Not closed under Intersection, complement, difference
- Closed under intersection with regular languages
- The list of undecidable CFL questions in section 14.3
- Turing Machine definition, configurations, computations, halting, accepting
- TM Macro language
- Definition of "TM M decides language L"
- Multiple racks and multiple tapes
- Any language accepted by a nondeterministic TM is also accepted by some deterministic TM.
- Encoding TMs as strings
- Universal TM
- Church-Turing Thesis and its implications (you do not have to know all of the equivalently powerful computation systems that the book lists)
- Understand basic ideas of the proof that H is not decidable
- Diagram on P 437
- Complement relations in D and SD (Section 20.4)
- Show that $\neg H$ is not in SD
- Relationships between enumerability and decidability; lexicographic enumeration
- Use reduction to show a language decidable, undecidable, not in SD.
- Specific undecidable problems you should know by name: The ones in the table on page 448.