MA/CSSE 474 Exam 2 Spring, 2018 Name___**SOLUTION**___ Section: 01 (2nd) ____ 02 (3rd)___ 03(4th) ___

No books or papers except the "Notation and Formulas" sheet supplied by the instructor. In the exam, it referred to as the Notes Page.

You may not use any devices with WI-FI/Bluetooth, headphones, or earbuds.

No communication about this exam's contents with anyone besides the instructor before 11:45 AM.

See my notes about problem 1 on the last page of this solution

Scores:

Problem	Possible	Score
1	40	
2	5	
3	4	
4	24	
5	9	
6	15	
7	15	
8	15	
9	6	
Total	**133**	

1. (40 points) Circle T or F to indicate whether it is *True* or *False*. IDK means *I don't know*. If the statement can ever be False, then False is the correct answer. **You do not need to give proofs or counterexamples.** For each part, you earn **2 points** for circling IDK, **4** points for circling the correct answer, **-1** for circling the incorrect answer, and **0** if you leave it blank. Leaving it blank is silly, since you get more points for IDK.

Part g had a typographical error, so I gave everyone all 4 points.

a) T (F) IDK The complement of an infinite language must be infinite.
b) (T) F IDK The complement of a non-regular language must be non-regular.
c) T (F) IDK Every subset of a regular language must be regular.
d) (T) F IDK If L is regular, then so is $\{xy : x \in L \text{ and } y \notin L\}$.
e) T (F) IDK The intersection of an infinite number of regular languages must be regular.
f) T (F) IDK If for each element w in language L, there is a DFSM that accepts w, when L must be regular.
g) (T) F IDK Let M be a DFSM such that $|K_M| = 100$, $\Sigma_M = \{a, b\}$, and $L(M)$ is finite. Then $a^{100}b^{100} \notin L(M)$.
h) (T) F IDK The finite languages are closed under intersection with the regular languages.
i) T (F) IDK $(L_1 - L_2 \text{ is regular}) \rightarrow (L_1 \text{ is regular})$.
j) T (F) IDK If *chop*(L) is regular then L is regular. (The *chop* function is defined on the Notes Page)

2. (5 points) Consider the contrapositive to the Pumping Theorem (the details are on the Notes Page). If we are using this to prove that a language L is non-regular, which of the values mentioned in the theorem do we get to choose? Circle them (and do not circle the ones that we are not allowed to choose).

k **w** x y **q**

The things we can choose correspond to the two ∃'s in the contrapositive statement.

3. (4 points) Write the English name for each of these Greek letters that we frequently use in the course. For example, if one of the letters was α, you would write *alpha*.

ε	Σ	δ	Κ
epsilon	**sigma**	**delta**	**kappa**

Why is this problem on the exam? We use these names verbally almost every day in class. If y9u don't know the names, it will be hard to follow our discussions.

4. (24 points) **In the pictured DFSM M, the start state is S1, and the accepting states are S1 and S2.** Consider the algorithm from class for finding a regular expression α such that L(α) = L(M).

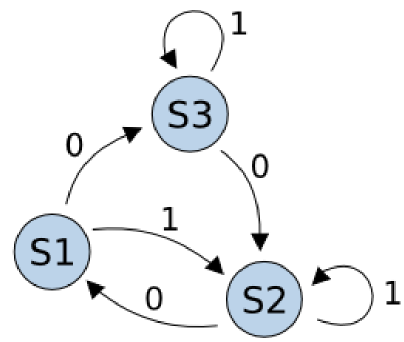
(12) Write your answers in the simplest form that you can. What is the value of

r_{220}? $1 \cup \varepsilon$

r_{221}? $(01) \cup 1 \cup \varepsilon$

r_{312}? $0((01) \cup 1)^*0$

We tried hard to determine whether your answer is equivalent to mine. If you are convinced that yours is equivalent, but we did not give you credit, please show your exam to me and I will fix your score if I agree with you.



(8) For each the following two r_{ijk} values, fill in the four sets of three subscripts that tell how to calculate the given r value using the recursive formula.

$r_{221} = r_{220} \cup r_{210} \; r_{110}^* \; r_{120}$

One point for each of the eight answers.

$r_{313} = r_{312} \cup r_{332} \; r_{332}^* \; r_{312}$

(4) Write a regular expression that defines L(M). Express your answer in terms of a union of some of the r_{ijk}. You do not have to actually evaluate the r_{ijk}.

L($r_{113} \cup r_{123}$) = L(M)

5. (9 points) This problem is the same as a problem in HW5, but you do not have to prove your answer here. Just show that you understand the construction. Let $M_1=(K_1, \Sigma, \delta_1, s_1, A_1)$ and $M_2=(K_2, \Sigma, \delta_2, s_2, A_2)$ be DFSMs that accept the regular languages $L_1 = L(M_1)$ and $L_2= L(M_2)$. Let $L = L_1 \cap L_2$. Show that L is regular by directly constructing a DFSM $M=(K , \Sigma, \delta, s, A)$ such that L=L(M). I do not expect you to have memorized the solution; you should be able to figure out the details again.

(2) K = $K_1 \times K_2$

(4) δ is defined as $\delta((q_1, q_2), a) = (\delta(q_1, a), \delta(q_2, a))$

(1) s = (s_1, s_2)

(2) A = $\{ (q_1, q_2) : q_1 \in A_1 \wedge q_2 \in A_2 \}$ Alternate answer: $A_1 \times A_2$

6. (15 points) L = { $u \in \{a, b\}^*$: $\exists s, t$ ($u=st$ $\wedge$ $|s| = |t|$ $\wedge$ $\#_a(s) \geq \#_a(t)$ }.

Regular or not? Prove your answer. 5 points for the correct answer, 10 for the informal proof.

Circle the correct answer: REGULAR **NOT REGULAR**

Proof:

Let w = $a^k bba^k$. No matter how we write w=xyz where y meets the pumping theorem constraints of being non-empty and having a length that is $\leq$ k, we must have y=a^p for some p with $1 \leq p \leq k$.

Let q=0, so we get xz = $a^{k-p}bba^k$.

If p is odd, then xz is outside L, because every string in L has even length.

If p is even, then both b's are in the first half of xz, but the second half is all a's, so the second half has more a's.

Other *w* choices are possible, and in particular, w= $b^k aab^k$ is also a simple choice.

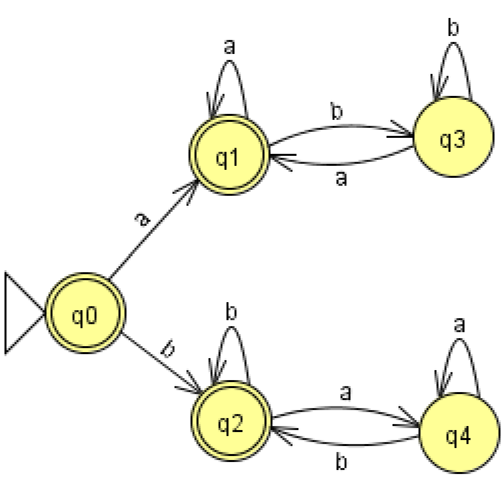
Partial credit: Based on our perception of how well you appear to understand the Pumping Theorem, plus your appropriate choice of w. **Any *w* that has a chance of working must contain both a's and b's**

7. (15 points) L= { $w \in \{a, b\}^*$: the number of occurrences of the substring *ab* in *w* equals the number of occurrences of the substring *ba* in *w*}.

Regular or not? Prove your answer. 5 points for the correct answer, 10 for the informal proof.

Circle the correct answer: **REGULAR** NOT REGULAR

Proof: **A DFSM. You could also show a NDSFM or a regexp**

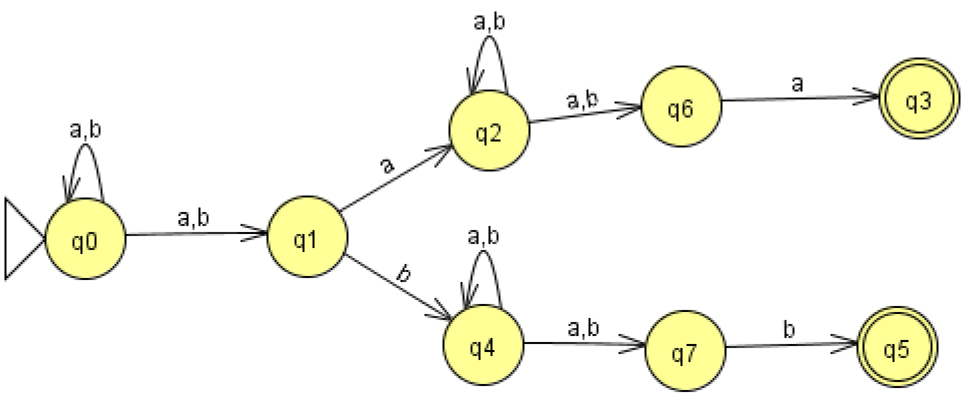


8. (15 points) L = { $w = xyzy : x, y, z \in \{a, b\}^+$ }.
Regular or not? Prove your answer. 5 points for the correct answer, 10 for the informal proof.

Circle the correct answer: **REGULAR** NOT REGULAR

Proof:

Both y's can be a single a or b. Here is one NDFSM for L:



9. (6 points)
a) Give an example of a non-regular language L such that L* is regular.
Your L can be a language that we have seen in this course. No proof is required.

A couple of examples:

L = { a^p: p is prime} **L* = {$a^n : n \geq 2$} ∪ {ε}**

L = { a^k: k is a power of 2} **L* = $\{a\}^*$**

Note that the schedule page, listing things that can be on the exam, says, "(be sure to consider the T/F problems in Problem 8.21)"

b) Give an example of a non-regular language L such that L* is not regular.
Your L can be a language that we have seen in this course. No proof is required.

A couple of examples:

L = BAL **L* = BAL, which we showed to be non-regular.**

L = A^nB^n **L* is clearly not regular.**

The *w* in the pumping theorem can be a^kb^k.

Notes on problem 1.

a. F. Let L = {a}* - {aaa}. Infinite; its complement is finite.
b. T. We know that the complement of a regular language must be regular. Suppose that A is non-regular and its complement B is regular. Note that A is the complement of B; this is a contradiction.
c. F. Every non-regular language is a subset of Σ*.
d. T. This language is LL^R. The set of regular languages is closed under concatenation.
e. F. Example: let n_1, n_2, … be all of the non-prime natural numbers. The intersection of {a}* - $\{a^{n1}\}$, {a}* - $\{a^{n2}\}$, …, is $\{a^p$: p is prime}. There is something similar in problem 8.21, but this problem is simpler.
f. F. "when" in this problem was supposed to be "then". With the wording I gave, it is hard to know what it means. My intention was to give everyone full credit for this one. Of I did not do that for you, let me know and I will fix it for you.
g. T. If a machine accepts any string whose length is as large as its number of states or larger, then that string can be pumped, and so the language is infinite.
h. T. The intersection of a finite language with *any* language is finite.
i. F. Let L_1 be any non-regular language and let L_2 be L_1. Then L_1 - L_2 is ∅, which is regular.
j. F. Let L = { a^nca^n : n ≥ 0 }. It's easy to show that L is not regular. chop(L) = { a^{2n} : n ≥ 0 }, which is regular.