MA/CSSE 474 Exam 2 N. Tepley Section: 01 (

No books or papers except the "notation and formula" sheet supplied by the instructor.

You may not use any devices with WI-FI/bluetooth, headphones, or earbuds.

No communication about this exam's contents with anyone besides the instructor before 11:45 AM.

If a problem is marked with a †, it means that there is information on the "notation and formulas" sheet that gives some helpful details for this problem.

Scores:

Problem	Possible	Score
1	30	
2	5	
3	4	
4	21	
5	10	
6	15	
7	10	
8	5	
Total	**100**	

1. (30 points) Circle T or F to indicate whether it is *True* or *False*. IDK means *I don't know*
If the statement can ever be False, then False is the correct answer. **You do not need to give proofs or counterexamples.**
For each part, you earn **1 point** for circling IDK, **2** for circling the correct answer, **-1** for circling the incorrect answer, and **0** if you leave it blank. Leaving it blank is silly, since you get more points for IDK.

a) T F IDK The set of all languages over alphabet {a} is countably infinite

b) T F IDK The set of all regular languages over alphabet {a, b} is countably infinite.

c) T F IDK The complement of a non-regular language must be non-regular.

d) T F IDK The complement of an infinite language must be infinite.

The next parts deal with the function *pref*(L) = $\{w \in \Sigma^* : \exists x \in \Sigma^* (wx \in L)\}$

e) T F IDK The set of regular languages is closed under *pref*.

f) T F IDK The set of non-regular languages is closed under *pref*.

g) T F IDK The set of finite languages is closed under *pref*.

h) T F IDK The set of infinite languages is closed under *pref*.

i) T F IDK If the transition graph of a DFSM M has no loops or cycles, then L(M) is finite.

j) T F IDK † If *maxstring*(L) is regular, then L must be regular.

k) T F IDK † If L is a regular language with alphabet {a, b}, then *chop*(*L*) ∩ {a, aba} = ∅ .

l) T F IDK If neither of the languages L_1, L_2 is regular, then $L_1 \cup L_2$ is not regular.

m) T F IDK Every non-regular language is the intersection of a countably infinite set of regular languages.

n) T F IDK Every non-regular language is the intersection of a countably infinite set of non-regular languages.

o) T F IDK Every regular language is the intersection of a countably infinite set of regular languages.

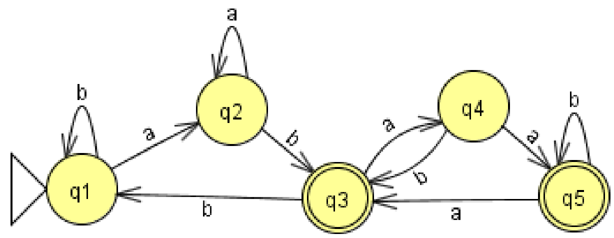
2. (5 points) † Consider the contrapositive to the Pumping Theorem (it is written out on the "Notations" handout). If we are using this to prove that a language L is non-regular, which of the values mentioned in the theorem do we get to choose? Circle them (and do not circle the ones that we are not allowed to choose).

k w x y z q

3. (4 points) Write the English name for each of these Greek letters that we frequently use in the course.

ε Σ δ κ

4. (21 points) † Consider the following transition diagram for a DFSM M, and the algorithm from class to find a regular expression α such that $L(\alpha) = L(M)$.



(6) What the value of

r_{220}?

r_{350}?

r_{321}?

(6) For each the following two r_{ijk} values, fill in the four sets of three subscripts that tell how to calculate the given r value using the recursive formula.

$r_{225} = r \quad \cup \; r \quad r^* \quad r$ $\qquad\qquad$ $r_{414} = r \quad \cup \; r \quad r^* \quad r$

(3) Write a regular expression that defines L(M), in terms of a union of some of the r_{ijk}. You do not have to evaluate that/those r_{ijk}.

(3) List all pairs (i, j) such that R_{ij1} is different than R_{ij0}.

(3) List all pairs (i, j) such that R_{ij2} is different than R_{ij1}.

5. (10 points) This problem is the same as a problem in HW5, but you do not have to prove your answer here. Let $M_1=(K_1, \Sigma, \delta_1, s_1, A_1)$ and $M_2=(K_2, \Sigma, \delta_2, s_2, A_2)$ be DFSMs that accept the regular languages $L_1 = L(M_1)$ and $L_2= L(M_2)$. Let $L = L_1 \cap L_2$. Show that L is regular by carefully constructing a DFSM $M=(K, \Sigma, \delta, s, A)$ such that $L=L(M)$. I do not expect you to have memorized the solution; you should be able to figure out the details again.

Answer:

(2) K =

(4) δ is defined as $\delta(\qquad\qquad\qquad) \;=$

(2) s =

(2) A =

6. (15 points) † Use the contrapositive of the pumping theorem directly (no closure properties or equivalence classes) to show that $L = \{w \in \{a,b\}^* : \exists x,y \in \{a,b\}(\exists n \geq 0\ (w = xa^n b^n y))\}$ is not regular.

7. (10 points) Write a decision procedure that decides the following problem:
Given regular expressions α and β over an alphabet Σ, is it true that $L(\alpha)$ is the complement of $L(\beta)$?

8. (5 points) What is the relationship between
the lexicographic ordering of all strings over alphabet Σ
and
the ordering of the states in our textbook's canonical form of a minimal DFSM over Σ?