

Name: _____ Key _____

Grade: _____ <-- instructor use

1. Why can't a PDA recognize the language $A^nB^nC^n$?

A stack can be used to match the counts of two things, but not three.

2. Describe (in English) the actions of a TM that recognizes $A^nB^nC^n$.

mark left and right ends.
 if a, erase, then erase a b, then a c
 move back to mark.
 keep repeating until there is no a, or
 b or c before a, or c before b.

Students may have their own variations

3. What does it mean for a language to be *semidecidable*?

\exists a TM that accepts all strings in the language

4. What is a decision problem?

A decision problem is one that has a yes/no answer for each instance.

5. Is the problem: "Are there any prime Fermat numbers greater than 1,000,000?" decidable? Explain.

Yes. One of the following algorithms is correct for this problem.

- def t(): return TRUE
- def f(): return FALSE

So there is a decision procedure, we just don't know which one it is.

6. If L is $\{b^n a : n \geq 0\}$ What is $\text{maxstring}(L)$? It is the same as L .

7. What does $(q, w) \vdash_M (q', w')$ mean? The machine M , in state q with w as the remaining input, transitions to state q' with w' as the remaining input. This happens iff $w = aw'$ for some $a \in \Sigma$, and $\delta(q, a) = q'$.

8. Prove: Every DFSA M , in configuration (q, w) , halts after $|w|$ steps .

Base case: If w is ϵ , it halts in 0 steps.

Induction step: Assume true for strings of length n and show for strings of length $n+1$.

Let $w \in \Sigma^*$, $|w| = n+1$ for some $n \in \mathbb{N}$.

Then w is ax for some $a \in \Sigma$, $x \in \Sigma^*$, $|x| = n$.

Let q' be $\delta(q, a)$. Then $(q, w) \vdash_M (q', x)$

By induction, from configuration (q', x) , M halts in n steps.

So, starting from the original configuration, M halts in $n+1$ steps.

9. Draw the transition diagram (or transition table) for a DFSA that accepts
 OddParity = $\{w \in \{0, 1\}^* : w \text{ contains an odd number of 1s}\}$

