474 HW 16 problems (highlighted problems are the ones to turn in)

20.1 (#3)

20.2 (#4)

1. Show that the set D (the decidable languages) is closed under:
   a. Union
   b. Concatenation
   c. Kleene star
   d. Reverse
   e. Intersection
2. Show that the set SD (the semidecidable languages) is closed under:
   a. Union
   b. Concatenation
   c. Kleene star
   d. Reverse
   e. Intersection

20.3 (#3) 9

3. Let $L_1, L_2, \ldots, L_k$ be a collection of languages over some alphabet Σ such that:
   - For all $i \neq j$, $L_i \cap L_j = \varnothing$.
   - $L_1 \cup L_2 \cup \ldots \cup L_k = \Sigma^*$.
   - $\forall i$ (L_i is in SD).

   Prove that each of the languages L_1 through L_k is in D.

20.4 (#4) 6

4. If L_1 and L_3 are in D and $L_1 \subseteq L_2 \subseteq L_3$, what can we say about whether L_2 is in D?

20.5 (#7)

5. Let L_1 and L_2 be any two decidable languages. State and prove your answer to each of the following questions:
   a. Is it necessarily true that $L_1 - L_2$ is decidable?
   b. Is it possible that $L_1 \cup L_2$ is regular?

20.6 (#8)

20.7 (#7) 6

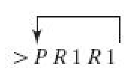
6. Let L_1 and L_2 be any two undecidable languages. State and prove your answer to each of the following questions:
   a. Is it possible that $L_1 - L_2$ is regular?
   b. Is it possible that $L_1 \cup L_2$ is in D?

20.8 (#8) 9

7. Let M be a Turing machine that lexicographically enumerates the language L. Prove that there exists a Turing machine M' that decides L^R.
8. Construct a standard one-tape Turing machine M to enumerate the language:

   $\{w : w$ is the binary encoding of a positive integer that is divisible by $3\}$.

   Assume that M starts with its tape equal to ❑. Also assume the existence of the printing subroutine P, defined in Section 20.5.1. As an example of how to use P, consider the following machine, which enumerates L', where $L' = \{w : w$ is the unary encoding of an even number$\}$:



20.11 (#9) 9

11) Recall the function *mix*, defined in Example 8.23. Neither the regular languages nor the context-free languages are closed under *mix*. Are the decidable languages closed under *mix*? Prove your answer.

10.12 (#10) 6

12) Let Σ = {a, b}. Consider the set of all languages over Σ that contain only even length strings.
   a) How many such languages are there?
   b) How many of them are semidecidable?

20.13 (#11) 9

13. Show that every infinite semidecidable language has a subset that is not decidable.