

474 HW 15 problems (highlighted problems are the ones to turn in)

18.1a

(#1) 9

1. Church's Thesis makes the claim that all reasonable formal models of computation are equivalent. And we showed in, Section 17.4, a construction that proved that a simple accumulator/register machine can be implemented as a Turing machine. By extending that construction, we can show that any computer can be implemented as a Turing machine. So the existence of a decision procedure (stated in any notation that makes the algorithm clear) to answer a question means that the question is decidable by a Turing machine.

Now suppose that we take an arbitrary question for which a decision procedure exists. If the question can be reformulated as a language, then the language will be in D iff there exists a decision procedure to answer the question. For each of the following problems, your answers should be a precise description of an algorithm. It need not be the description of a Turing Machine:

- a. Let $L = \{ \langle M \rangle : M \text{ is a DFSM that doesn't accept any string containing an odd number of 1's} \}$. Show that L is in D .

18.1b

(#2) 9

Problem #3 A TM M has tape alphabet $\{ \square, a, b \}$ (this is the order used in the encoding $\langle M \rangle$).

$\langle M \rangle = (q00, a00, q00, a00, \rightarrow), (q00, a10, q01, a10, \rightarrow), (q00, a01, y10, a10, \leftarrow), (q01, a01, q00, a10, \rightarrow), (q01, a10, n11, a01, \leftarrow)$

- (a) (6) Provide a transition diagram or a transition table for the TM M .
- (b) (3) For each of the following outcomes of running M , provide a short string of a's and b's that is accepted by M , rejected by M , neither.

1. Consider the language $L = \{ \langle M \rangle : \text{Turing machine } M \text{ accepts at least two strings} \}$.

- a. Describe in clear English a Turing machine M that semidecides L .
- b. Now change the definition of L just a bit. Consider:

$L' = \{ \langle M \rangle : \text{Turing machine } M \text{ accepts exactly 2 strings} \}$.

Can you tweak the Turing machine you described in part a to semidecide L' ?

19.1

(#4) 6

2. Consider the language $L = \{ \langle M \rangle : \text{Turing machine } M \text{ accepts the binary encodings of the first three prime numbers} \}$.

- a. Describe in clear English a Turing machine M that semidecides L .
- b. Suppose (contrary to fact, as established by Theorem 19.2) that there were a Turing machine *Oracle* that decided H . Using it, describe in clear English a Turing machine M that decides L .

19.2

(#5) 12