MA/CSSE 474 **Homework #13** 87 points

Recall: When we use the pumping theorem to show that a language is not context-free, we do not get to choose the k, we choose the w whose length is at least k. We do not get to choose how w is broken up into uvxyz (although the breakup has to meet the length constraints of the theorem), but we do get to choose how many times to pump the v and y (i.e. we can choose the q in uv^qxy^qz), and 0 is a legitimate choice for that q.

Proving that a language is context-free but not regular: You must (separately) show both of those things.

A general question from a past course's Piazza: **Q: Making PDAs to prove a language is context free.** Does adding an end of string marker invalidate using a PDA to show a language is context free? **A:** Adding an end-of-string marker is legitimate. If L$ can be accepted by a PDA M, there is also a (possibly non deterministic) PDA M' that accepts L.

1. (t-9) 13.1f
2. 13.1g
3. 13.1h
4. (t-9) 13.1i
5. 13.1k
6. (t-9) 13.1l (thirteen point one el)
7. 13.1p
8. 13.1q
9. (t-9) 13.1w
10. (t-9) 13.3
11. 13.4
12. 13.8
13. (t-6) 13.9

13.12 **Note on 13.12.** What the author meant to ask and what she actually asked are quite different. Both parts should have said: "Is L context-free (but not regular), regular, or neither? Prove your answer."

14. 13.12
15. (t-6) 13.13d
16. (t-6-9) 13.14 Examples: If L is the language denoted by r.e. (ab)*, then alt(L) is denoted by r.e. a^+.
    If L is the language denoted by r.e. (abcdefg)*,
    then alt(L) is denoted by r.e. (aceg)(bdfaceg)*(bdf ∪ ε).
    If L is the language denoted by r.e. a or the language denoted by r.e. a*,
    then alt(L) is L.
17. (t-6) 14.1a
18. (t-9) 14.1c
19. 14.1d
20. 14.1e

Previous info from Piazza

Problem 13.13d hint

A student came to my office to ask about this problem, and I could not immediately see how to do it. I knew it would involve adding lots of epsilon transitions for the "skipped-over" parts y and z, but what happens to the stack when you skip transitions that would have put things on the stack?

Here is the trick. Don't just work with the original machine. Make two copies of the original PDA M, In each transition of each copy, replace the input symbol of that transition (if there is one) by epsilon. Leave the stack parts as they are. The first copy machine deals with the y part, simulating what M would have done to the stack while reading y, without actually reading any input. the second copy does the same thing for z.

So now I have told you almost everything you need for this problem. The remaining question is how to connect the three machines into one machine that accepts exactly middle(L(M)).

Two more hints for Problem 13.13d (the "closed under middle" problem

1. What you need to do: Come up with a construction which, given any PDA M, produces a PDA M' such that L(M') = middle(L(M).

2. **Suggestion:** As a warm-up, show that the regular languages are closed under *middle* by doing the same thing as above for DFSMs. This will show you how to handle states in your PDA construction. Then use my hint from yesterday as a starting point for how to handle the stack.

Problem 13.14: Proving that CFLs are closed under alt

Q: I'm pretty sure that CFGs are closed under alt, but I'm having some trouble proving that it is. At first I was thinking to combine pairs of transitions into one transitions, but logic for that gets incredibly messy for complicated machines...

Am I on the right track of "combining" transitions to produce a machine that accepts alt(L)? Or perhaps I'm completely wrong and it's actually not context free (I don't think this is the case though...) Any hints would be appreciated.

A: I don't think you can "combine" states, but you can have a similar effect by adding epsilon transitions that "skip" states.

14.1c:

Create decision procedure that, given a CFG, generates at least 3 strings

Q: Are there any "free" functions we may use? (kinda like the minDFSM() and ndfsmtodfsm() algorithms for state machines)

A: You can assume and use any algorithm or decision procedure that we have done in class or homework or that is in the book.