474 HW 12 problems (highlighted problems are the ones to turn in)

12.1j (#1)	
12.3 (#2) 6+6+6	
12.4 (#3) 3 for each part	
12.5a (#4) 9	
12.6 (#5) 6	
13.1a (#6) 6	
13.1b (#7) 6	
13.1c (#8) 12	
13.1d (#9) 6	

1. Build a PDA to accept each of the following languages L:

a. BalDelim = $\{w$: where w is a string of delimiters: (,), [,], {, }, that are properly balanced$\}$.

b. $\{a^i b^j : 2i = 3j + 1\}$.

c. $\{w \in \{a, b\}^* : \#_a(w) = 2 \cdot \#_b(w)\}$.

d. $\{a^n b^m : m \leq n \leq 2m\}$.

e. $\{w \in \{a, b\}^* : w = w^R\}$.

f. $\{a^i b^j c^k : i, j, k \geq 0$ and $(i \neq j$ or $j \neq k)\}$.

g. $\{w \in \{a, b\}^*$: every prefix of w has at least as many a's as b's$\}$.

h. $\{a^n b^m a^n : n, m \geq 0$ and m is even$\}$.

i. $\{xc^n : x \in \{a, b\}^*, \#_a(x) = n$ or $\#_b(x) = n\}$.

j. $\{a^n b^m : m \geq n$, m-n is even$\}$.

k. $\{a^m b^n c^p d^q : m, n, p, q \geq 0$ and $m + n = p + q\}$.

3. Let $L = \{ba^{m_1}ba^{m_2}ba^{m_3} \ldots ba^{m_n} : n \geq 2, m_1, m_2, \ldots, m_n \geq 0$, and $m_i \neq m_j$ for some $i, j\}$.

a. Show a PDA that accepts L.

b. Show a context-free grammar that generates L.

c. Prove that L is not regular.

See hint below

4. Consider the language $L = L_1 \cap L_2$, where $L_1 = \{ww^R : w \in \{a, b\}^*\}$ and $L_2 = \{a^n b^* a^n : n \geq 0\}$.

a. List the first four strings in the lexicographic enumeration of L.

b. Write a context-free grammar to generate L.

c. Show a natural PDA for L. (In other words, don't just build it from the grammar using one of the two-state constructions presented in this chapter.)

d. Prove that L is not regular.

5. Build a deterministic PDA to accept each of the following languages:

a. $L\$$, where $L = \{w \in \{a, b\}^* : \#_a(w) = \#_b(w)\}$.

b. $L\$$ where $L = \{a^n b^+ a^m : n \geq 0$ and $\exists k \geq 0\ (m = 2k + n)\}$.

6. Complete the proof that we started in Example 12.14. Specifically, show that if M is a PDA that accepts by accepting state alone, then there exists a PDA M' that accepts by accepting state and empty stack (our definition) where $L(M') = L(M)$.

12.5a Hint: In order to be deterministic, the first thing your PDA should do, before it adds any input, is to push a special "bottom of stack" marker

12.3 Clarification: It may help you to understand the language better if you replace "for some i, j" by "at least one pair i, j".

Hints (a) Have states and transitions to handle the "don't care" sections where we aren't worried about the number of b's. (b) One nonterminal for when the $m_i > m_j$, and one for when $m_i < m_j$. (c) Pumping theorem alone probably won't do it; also use closure property.

Warning: Last time I taught the course, several students indicated that this was the most difficult problem in the assignment, and a few thought it was one of the hardest problems of the term.

1. For each of the following languages L, state whether L is regular, context-free but not regular, or not context-free and prove your answer.

a. $\{xy : x, y \in \{a, b\}^*$ and $|x| = |y|\}$.

b. $\{(ab)^n a^n b^n : n > 0\}$.

c. $\{x\#y : x, y \in \{0, 1\}^*$ and $x \neq y\}$.

d. $\{a^i b^n : i, n > 0$ and $i = n$ or $i = 2n\}$.

Recall: When we use the pumping theorem to show a language is not context-free, we do not get to choose the k, we choose the w whose length is at least k. We do not get to choose how w is broken up into uvxyz (although the breakup has to meet the length constraints of the theorem), but we do get to choose how to pump the v and y (I.e. we can choose the q in uv^qxy^qz).