474 HW 10 problems (highlighted problems are the ones to turn in)

11.1, 2, 3 (#1, 2, 3)

11.4 (#4) (3)

11.6b (#5) (6)

11.6c (#6) (6)

11.6d (#7) (6)

11.6e (#8)

(#9) not from textbook (6)

11.6h (#10) (6)

11.6i (#11) (6)

11.6k (#12) (6)

11.8b (#13) (9)

11.9 (#14)

1. Let $\Sigma = \{a, b\}$. For the languages that are defined by each of the following grammars, do each of the following:

- **i.** List five strings that are in L.
- **ii.** List five strings that are not in L (or as many as there are, whichever is greater).
- **iii.** Describe L concisely. You can use regular expressions, expressions using variables (e.g., $a^n b^n$, or set theoretic expressions (e.g., $\{x: \ldots\}$).
- **iv.** Indicate whether or not L is regular. Prove your answer.

a. $S \rightarrow aS \mid Sb \mid \varepsilon$

b. $S \rightarrow aSa \mid bSb \mid a \mid b$

c. $S \rightarrow aS \mid bS \mid \varepsilon$

d. $S \rightarrow aS \mid aSbS \mid \varepsilon$

2. Let G be the grammar of Example 11.12. Show a third parse tree that G can produce for the string (())().

3. Consider the following grammar G:

$$S \rightarrow 0S1 \mid SS \mid 10$$

Show a parse tree produced by G for each of the following strings:

a. 010110.

b. 00101101.

4. Consider the following context free grammar G:

$$S \rightarrow aSa$$

$$S \rightarrow T$$
$$S \rightarrow \varepsilon$$
$$T \rightarrow bT$$
$$T \rightarrow cT$$
$$T \rightarrow \varepsilon$$

One of these rules is redundant and could be removed without altering $L(G)$. Which one?

6. Show a context-free grammar for each of the following languages L:

a. BalDelim = $\{w$: where w is a string of delimiters: (,), [,], {, }, that are properly balanced$\}$.

b. $\{a^i b^j : 2i = 3j + 1\}$.

c. $\{a^i b^j : 2i \neq 3j + 1\}$.

d. $\{w \in \{a, b\}^* : \#_a(w) = 2 \cdot \#_b(w)\}$.}.

e. $L = \{w \in \{a, b\}^* : w = w^R\}$.

f. $\{a^i b^j c^k : i, j, k \geq 0$ and $(i \neq j$ or $j \neq k)\}$.

g. $\{a^i b^j c^k : i, j, k \geq 0$ and $(k \leq i$ or $k \leq j)\}$.

h. $\{w \in \{a, b\}^*$: every prefix of w has at least as many a's as b's$\}$.

i. $\{a^n b^m : m \geq n, m\text{-}n$ is even$\}$.

j. $\{a^m b^n c^p d^q : m, n, p, q \geq 0$ and $m + n = p + q\}$.

k. $\{xc^n : x \in \{a, b\}^*$ and $(\#_a(x) = n$ or $\#_b(x) = n)\}$.

l. $\{b_i \# b_{i+1}{}^R : b_i$ is the binary representation of some integer i, $i \geq 0$, without leading zeros$\}$. (For example 101#011 $\in L$.)

m. $\{x^R \# y : x, y \in \{0, 1\}^*$ and x is a substring of $y\}$.

9. (t-6) Show the details of the definition of a CFG that generates $\{a^i b^j c^k : i, j, k \geq 0$ and $(i + j = k)\}$

8. Consider the unambiguous expression grammar G' of Example 11.19.

a. Trace a derivation of the string id+id*id*id in G'.

b. Add exponentiation (**) and unary minus (−) to G', assigning the highest precedence to unary minus, followed by exponentiation, multiplication, and addition, in that order.

9. Let $L = \{w \in \{a, b, \cup, \varepsilon, (,), {}^*, {}^+\}^* : w$ is a syntactically legal regular expression$\}$.

a. Write an unambiguous context-free grammar that generates L. Your grammar should have a structure similar to the arithmetic expression grammar G' that we presented in Example 11.19. It should create parse trees that:

- Associate left given operators of equal precedence, and
- Correspond to assigning the following precedence levels to the operators (from highest to lowest):
  - * and $^+$
  - concatenation
  - $\cup$

b. Show the parse tree that your grammar will produce for the string $(a \cup b)\, ba^*$.