474 HW 09 problems (highlighted problems are the ones to turn in)

8.8(#1)

8.8 d, e
(#2)(3, 6)

8.9
(#3) (9)

8.10a
(#4) (9)

8.16a
(#5) (6)

8.16b
(#6)

8.21
(#7)

8.21n
(#8) (12)

8.21o
(#9) (3)

7. Prove that the regular languages are closed under each of the following operations:

a. $pref(L) = \{w : \exists x \in \Sigma^*(wx \in L)\}$.

b. $suff(L) = \{w : \exists x \in \Sigma^*(xw \in L)\}$.

c. $reverse(L) = \{x \in \Sigma^* : x = w^R \text{ for some } w \in L\}$.

d. letter substitution (as defined in Section 8.3).

8. Using the defintions of *maxstring* and *mix* given in Section 8.6, give a precise definition of each of the following languages:

a. $maxstring(A^nB^n)$.

b. $maxstring(a^i b^j c^k, 1 \leq k \leq j \leq i)$.

c. $maxstring(L_1L_2)$, where $L_1 = \{w \in \{a, b\}^* : w \text{ contains exactly one a}\}$ and $L_2 = \{a\}$.

d. $mix((aba)^*)$.

e. $mix(a^*b^*)$.

9. Prove that the regular languages are not closed under *mix*.

Definitions of *maxstring* and *mix:* Examples 8.22 and 8.23.

10. Recall that $maxstring(L) = \{w : w \in L \text{ and } \forall z \in \Sigma^*(z \neq \varepsilon \rightarrow wz \notin L)\}$.

a. Prove that the regular languages are closed under *maxstring*.

b. If *maxstring*(L) is regular, must L also be regular? Prove your answer.

8.10-a I.e., given a DFSM M = (K, Σ, δ, s, A) such that L(M)=L, construct a DFSM M*=(*K**, Σ, Δ*, *s**, *A**) such that L(M*)=maxstring(L).

16. Define two integers i and j to be ***twin primes*** iff both i and j are prime and $|j - i| = 2$.

a. Let $L = \{w \in \{1\}^* : w$ is the unary notation for a natural number n such that there exists a pair p and q of twin primes, both $> n.\}$ Is L regular?

b. Let $L = \{x, y : x$ is the decimal encoding of a positive integer i, y is the decimal encoding of a positive integer j, and i and j are twin primes$\}$. Is L regular?

21. For each of the following claims, state whether it is *True* or *False*. Prove your answer.

a. There are uncountably many non-regular languages over $\Sigma = \{a, b\}$.

b. The union of an infinite number of regular languages must be regular.

c. The union of an infinite number of regular languages is never regular.

d. If L_1 and L_2 are not regular languages, then $L_1 \cup L_2$ is not regular.

e. If L_1 and L_2 are regular languages, then $L_1 \otimes L_2 = \{w : w \in (L_1 - L_2) \text{ or } w \in (L_2 - L_1)\}$ is regular.

f. If L_1 and L_2 are regular languages and $L_1 \subseteq L \subseteq L_2$, then L must be regular.

g. The intersection of a regular language and a nonregular language must be regular.

h. The intersection of a regular language and a nonregular language must not be regular.

i. The intersection of two nonregular languages must not be regular.

j. The intersection of a finite number of nonregular languages must not be regular.

k. The intersection of an infinite number of regular languages must be regular.

l. It is possible that the concatenation of two nonregular languages is regular.

m. It is possible that the union of a regular language and a nonregular language is regular.

n. Every nonregular language can be described as the intersection of an infinite number of regular languages.

o. If L is a language that is not regular, then L^* is not regular.

9.1 (#10)

9.1b
(#11) (6)

9.1d
(#12) (6)

9.1g
See note
below
(#13) (6)

9.1i
(#14) (6)

1. Define a decision procedure for each of the following questions. Argue that each of your decision procedures gives the correct answer and terminates.

a. Given two DFSMs M_1 and M_2, is $L(M_1) = L(M_2)^R$?

b. Given two DFSMs M_1 and M_2 is $|L(M_1)| < |L(M_2)|$?

c. Given a regular grammar G and a regular expression α, is $L(G) = L(\alpha)$?

d. Given two regular expressions, α and β, do there exist any even length strings that are in $L(\alpha)$ but not $L(\beta)$?

e. Let $\Sigma = \{\texttt{a},\texttt{b}\}$ and let α be a regular expression. Does the language generated by α contain all the even length strings in Σ^*.

f. Given an FSM M and a regular expression α, is it true that both $L(M)$ and $L(\alpha)$ are finite and M accepts exactly two more strings than α generates?

g. Let $\Sigma = \{\texttt{a},\texttt{b}\}$ and let α and β be regular expressions. Is the following sentence true:

$$(L(\beta) = \texttt{a}^*) \vee (\forall w\, (w \in \{\texttt{a},\texttt{b}\}^* \wedge |w| \text{ even}) \rightarrow w \in L(\alpha)).$$

h. Given a regular grammar G, is $L(G)$ regular?

i. Given a regular grammar G, does G generate any odd length strings?

9.1g:

There is a small error in the statement of the problem. a* should be {a}*

9.1b:

Note that |L(M)| means “the number of elements in the language accepted by the machine M. Note that for some machines M, the language is countably infinite.