

Name: \_\_\_\_\_ Solution \_\_\_\_\_

Grade: \_\_\_\_\_ &lt;-- (13 possible)

1. State Kleene's Theorem: A language is regular iff it is defined by some regular expression.

2. (4) On the back, draw the machines for the various cases of regExpToFSM

**Union:** New start state with  $\epsilon$ -moves to S1 and S2

**Concatenation:** S1 is the start state of new machine. Add  $\epsilon$ -move from each accepting state of M1 to S2. Accepting states of M1 are no longer accepting in new machine

**Kleene \*:** Add a new start state, which is an accepting state.  $\epsilon$ -move from new start state to S2.  $\epsilon$ -move from each accepting state of M1 back to the new start state. New start state can be the only accepting state, or the original accepting states can still accept.

3. (5) In the DFSMtoRegExp example machine M, show how to get

$$r_{221} = r_{220} \cup r_{210}(r_{110})^*r_{120} = \epsilon \cup 0(\epsilon)^*0 = \epsilon \cup 00$$

$$r_{132} = r_{131} \cup r_{121}(r_{221})^*r_{231} = 1 \cup 0(\epsilon \cup 00)^*(1 \cup 01) = 1 \cup 0(00)^*(\epsilon \cup 0)1$$

Note that  $0(00)^*(\epsilon \cup 0)$  is equivalent to  $0^*$ , so we get  $1 \cup 0^*1$  which is equivalent to  $0^*1$ .

$$r_{123} = r_{122} \cup r_{132}(r_{332})^*r_{322} = 0(00)^* \cup 0^*1(\epsilon \cup (0 \cup 1)0^*1)^*(0 \cup 1)(00)^* = 0(00)^* \cup 0^*1((0 \cup 1)0^*1)^*(0 \cup 1)(00)^*$$

$$r_{133} = r_{132} \cup r_{132}(r_{332})^*r_{332} = 0^*1 \cup 0^*1(\epsilon \cup (0 \cup 1)0^*1)^*(\epsilon \cup (0 \cup 1)0^*1) = 0^*1((0 \cup 1)0^*1)^*$$

A regular expression  $r$  such that  $L(R) = L(M)$

$$r_{123} \cup r_{133} = 0(00)^* \cup 0^*1((0 \cup 1)0^*1)^*(\epsilon \cup (0 \cup 1)(00)^*) \text{ [the } \epsilon \text{ is from } r_{133}]$$

4. (2) Given a DFSM for a language  $L$ , how do we construct a Machine  $M'$  for  $L^R$ ?

Let  $M = (K, \Sigma, \delta, s, A)$  be any DFSM that accepts  $L$ .  $M$  must be written out completely, without an implied dead state. Then construct  $M' = (K', \Sigma', \delta', s', A')$  to accept  $reverse(L)$  from  $M$ :

Initially, let  $M'$  be  $M$ .

Reverse the direction of every transition in  $M'$ .

Construct a new state  $q$ . Make it the start state of  $M'$ . Create an  $\epsilon$ -transition from  $q$  to every state that was an accepting state in  $M$ .

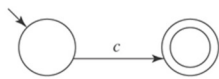
$M'$  has a single accepting state, the start state of  $M$ .

5. Tell your instructor about anything from today's session (or from the course so far) that you found confusing or still have a question about. If none, please write "None". Students must have some answer to earn this point.

$\emptyset$ :



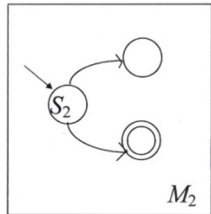
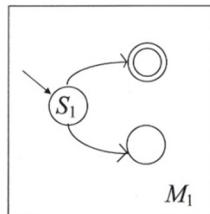
A single element of  $\Sigma$ :



$\varepsilon$  ( $\emptyset^*$ ):



Union:



Concatenation:

Kleene star: