Name:____ Grade:_____<-- instructor use

1. Give purely symbolic definitions of the three languages on the "Languages and Prefixes" slide

   * $\{a\}^*$

   * $\{\varepsilon\} \cup \{ bx : x \in \{a, b\}^* \}$

   * $\varnothing$

2. What are the two standard ways of defining a set?
   * A program that enumerates the members

   * A characteristic function, which given a value tells us whether that value is in the set.

3. What are the 3 properties that an eqiuivalence relation must satisfy?
   Reflexive, symmetric, transitive

4. For a given prime integer p, is $\{(a, b): a, b \in \mathbb{N} \wedge \exists k \in \mathbb{N}(a - b = kp)\}$ an equivalence relation? Explain.
   Yes. Reflexive: $a - a = 0p$. Symmetric. If $a-b = kp$, then $b-a = -kp$.
   Transitive: if $a - b = kp$ and $b - c = mp$, then $a - c = (k+m)p$.

5. If $L_1 = \{a^n: n \geq 0\}$ and $L_2 = \{b^n : n \geq 0\}$, what is L_1L_2? $\{a^m b^n : m,n >= 0\}$

   What is L_1^*? Same as L_1

6. When is a (propositional) wff a *tautology*? When it is true for all values of its variables

7. When we say a set of inference rules is *sound*, what do we mean? If we apply the rules to a set of axioms, we only end up with things entailed by those axioms

8. What is a *predicate*? A function whose value is Boolean

   Give an example of a predicate application with no free variables Example: contains(3, {4, 5, 6})

   with one or more free variables Example: contains(n, {4, 5, 6})

9. When is a first-order wff a *sentence* (statement)? When it has no free variables

10. Give an example of a model for $\exists x (\forall y (xy = 0))$ Integers, with standard definitions of 0 and <

11. From { ∀t(p(t)→q(t)), ∀t(q(t)→r(t)), ¬r(C)}, prove ¬p(C). Give reasons for your steps. (Continue on back)

1. ∀t(p(t)→q(t))	given
2. p(C)→q(C))	1, universal instantiation
3. ∀t(q(t)→r(t))	given
4. q(C)→r(C))	1, universal instantiation
5. p(C)→r(C))	2, 4, syllogism
6. ¬r(C)}	premise
7. ¬p(C)}	modus tollens

Tell your instructor about anything from today's session (or from the course so far) that you found confusing or still have a question about. If none, please write "None". Continue on the back if needed.