Name:_____**Key**_____ Grade:_____/10 <-- instructor use

1. What are two main questions that Theory of Computation attempts to answer?

   * **What can be computed, and what cannot?**
   * **What are reasonable mathematical models of computation?**

2. What is a (formal) language? **A (formal) language is a (possibly infinite) set of finite-length strings over a finite alphabet.**

3. In String definitions, what is represented by Σ? **The finite alphabet of the language's symbols** by ε ? **empty string**

4. **Prove by mathematical induction:** If w and x are strings, then $(w\ x)^R = x^R\ w^R$. (continue on the back if needed)

   Proof: By induction on $|x|$: **This slide is hidden. Do it on the board.**

   $|x| = 0$: Then $x = \varepsilon$, and $(wx)^R = (w\ \varepsilon)^R = (w)^R = \varepsilon\ w^R = \varepsilon^R\ w^R = x^R\ w^R$.

   $\forall n \geq 0\ (((|x| = n) \rightarrow ((w\ x)^R = x^R\ w^R)) \rightarrow$
   $((|x| = n + 1) \rightarrow ((w\ x)^R = x^R\ w^R)))$:

   3 points for this problem

   Consider any string x, where $|x| = n + 1$. Then $x = u\ a$ for some character a and $|u| = n$. So:

$(w\ x)^R$	$= (w\ (u\ a))^R$	rewrite x as ua
	$= ((w\ u)\ a)^R$	associativity of concatenation
	$= a\ (w\ u)^R$	definition of reversal
	$= a\ (u^R\ w^R)$	induction hypothesis
	$= (a\ u^R)\ w^R$	associativity of concatenation
	$= (ua)^R\ w^R$	definition of reversal
	$= x^R\ w^R$	rewrite ua as x

5. Give purely symbolic definitions of the three languages on the "Languages and Prefixes" slide

   $L = \{w \in \{a, b\}^*$: no prefix of w contains b$\}$:
   $L = \{w \in \{a, b\}^*: \neg(\exists x,y \in \Sigma^* (w = xby)\}$ BETTER: $L = \{a\}^*$
   $L = \{w \in \{a, b\}^*$: no prefix of w starts with a$\}$
   $L = \{w \in \{a, b\}^*: \neg(\exists x \in \Sigma^* (w = ax)\}$ OR: $L = \{\varepsilon\} \cup \{w \in \{a, b\}^*: (\exists x \in \Sigma^*) (w = bx)\}$
   $L = \{w \in \{a, b\}^*$: every prefix of w starts with a$\}$
   $L = \emptyset$ because the empty string is a prefix of every string.

6. What is your biggest concern (if any) going into this course? If none, please write "None".

   If they leave it blank, take off 1 point

7. Tell your instructor about anything from today's session (or from the course so far) that you found confusing or still have a question about. If none, please write "None". Continue on the back if needed.
   If they leave it blank, take off 1 point