MA/CSSE 474 **Homework #14** 63 points

There are lots of Q&A from previous term's Piazza at the end of this document.

1. (t-6) 17.1a Don't just describe in English the *individual steps* the machine makes. Your answer should be a global one: in particular, describe how the final tape content is different from the original tape content.
2. 17.1b
3. (t-9) 17.3a It is worth the time it will take to learn the author's macro language for TMs. Use that language to express your answer.
4. Other parts of 17.3
5. (t-9) 17.4
6. (t-3) 17.6
7. (t-6-3) 17.11 First show that it is a subset of D by showing a construction that creates a TM from a DFSM. How do the states and transitions of the TM relate to the states and transitions of the DFSM.? Then show that it is a *proper* subset by showing that there is at least one decidable language that is not regular.
8. (t-24) 17.12 (6 points for each of the four parts) Note that c) constitutes two of those parts.

   Note on 17.12a: { [<x>, <f(x)>], x∈ℕ}, where <x> means "the binary encoding of integer x" and <f(x)> means "the binary encoding of integer f(x)"

   17.12b,c: Do these constructions for a general function-computing TM, not specifically for the successor function.

   Hint for part c: You might find the concept of "dovetailing" helpful for this problem. If you have not seen that technique before, this reference will probably help:

   http://lambda-the-ultimate.org/node/322

   And there is a dovetailing video.

9. (t-3) 17.13

Some questions and answers from previous term's Piazza discussions:

#10 (17.3a) : can we assume there will be no 0 entry in the most significant bit?

A: If you wish, you may assume that the representation of a positive integer does not begin with 0. But the representation of the number zero has to begin with 0.

General question about Turing machines: Is the input to a Turing machine a finite tape? That is, if we reach the end of the input, does it halt, or do we assume that the input is nested between infinite blank squares?

A: The tape is infinite in both directions. When the computation starts, there are finitely many non-blank symbols on the tape (which implies that there are always finitely many non-blank symbols on the tape).

Unless specified otherwise for a particular TM, the read/write head is initially positioned at the blank symbol before the leftmost non-blank symbol in the input.

For the special case where the input is the empty string, the entire tape is blank, so all tape squares are initially equivalent and indistinguishable, so it does not matter where the head begins.

Follow-up to the previous question: so in a loop that starts with a >, does that mean we go back to the blank before the input string? #12 (17.4) : Constructing TM for unary encoding of max(#a(x), #b(x))

A: The > in the macro diagrams is simply a "this is where we start" marker, so I don't think there ever is a "loop containing >". For example, in the machine in example 17.7 (page 375), the > tells where to start, but it is not part of the loop. R is the first thing that gets done each time through the loop.

If you want your TM to "go back to the last blank before the input", you must place an explicit L_{blank} (or in some cases more than one of them, if there are blanks in the middle of the current string on the tape) in your diagram.