474 HW 13 problems (highlighted problems are the ones to turn in)

13.1f
(#1) 9
13.1g
(#2)
13.1h
(#3)
13.1i
(#4) 9
13.1k
(#5)
13.1l
(#6) 9
13.1p
(#7)
13.1q
(#8)
13.1w
(#9) 9

1. For each of the following languages L, state whether L is regular, context-free but not regular, or not context-free and prove your answer.

a. $\{xy : x, y \in \{a, b\}^* \text{ and } |x| = |y|\}$.

b. $\{(ab)^n a^n b^n : n > 0\}$.

c. $\{x\#y : x, y \in \{0, 1\}^* \text{ and } x \neq y\}$.

d. $\{a^i b^n : i, n > 0 \text{ and } i = n \text{ or } i = 2n\}$.

e. $\{wx : |w| = 2 \cdot |x| \text{ and } w \in a^+b^+ \text{ and } x \in a^+b^+\}$.

f. $\{a^n b^m c^k : n, m, k \geq 0 \text{ and } m \leq min(n, k)\}$.

g. $\{xyx^R : x \in \{0, 1\}^+ \text{ and } y \in \{0, 1\}^*\}$.

h. $\{xwx^R : x, w \in \{a, b\}^+ \text{ and } |x| = |w|\}$.

i. $\{ww^Rw : w \in \{a, b\}^*\}$.

j. $\{wxw : |w| = 2 \cdot |x| \text{ and } w \in \{a, b\}^* \text{ and } x \in \{c\}^*\}$.

k. $\{a^i : i \geq 0\}\{b^i : i \geq 0\}\{a^i : i \geq 0\}$.

l. $\{x \in \{a, b\}^* : |x|$ is even and the first half of x has one more a than does the second half$\}$.

m. $\{w \in \{a, b\}^* : \#_a(w) = \#_b(w)$ and w does not contain either the substring aaa or abab$\}$.

n. $\{a^n b^{2n} c^m : n, m \geq 0\} \cap \{a^n b^m c^{2m} : n, m \geq 0\}$.

o. $\{x \text{ c } y : x, y \in \{0, 1\}^* \text{ and } y \text{ is a prefix of } x\}$.

p. $\{w : w = uu^R \text{ or } w = ua^n : n = |u|, u \in \{a, b\}^*\}$.

q. $L(G)$, where $G = S \rightarrow aSa$

$$S \rightarrow SS$$
$$S \rightarrow \varepsilon$$

r. $\{w \in$ (A-Z, a-z, ., blank)$^+$: there exists at least one duplicated, capitalized word in w). For example, the string, The history of China can be viewed from the perspective of an outsider or of someone living in China, $\in L$.

s. $\neg L_0$, where $L_0 = \{ww : w \in \{a, b\}^*\}$.

t. L^*, where $L = \{0^*1^i0^*1^i0^* : i \geq 0\}$.

u. $\neg A^nB^n$.

v. $\{ba^jb : j = n^2 \text{ for some } n \geq 0\}$. For example, baaaab $\in L$.

w. $\{w \in \{a, b, c, d\}^* : \#_b(w) \geq \#_c(w) \geq \#_d(w) \geq 0\}$.

Recall: When we use the pumping theorem to show a language is not context-free, we do not get to choose the k, we choose the w whose length is at least k. We do not get to choose how w is broken up into uvxyz (although the breakup has to meet the length constraints of the theorem), but we do get to choose how to pump the v and y (I.e. we can choose the q in uv^qxy^qz).

13.3 (#10) 9

3. Let $L = \{a^n b^m c^n d^m : n, m \geq 1\}$. L is interesting because of its similarity to a useful fragment of a typical programming language in which one must declare procedures before they can be invoked. The procedure declarations include a list of the formal parameters. So now imagine that the characters in a^n correspond to the formal parameter list in the declaration of procedure 1. The characters in b^m correspond to the formal parameter list in the declaration of procedure 2. Then the characters in c^n and d^m correspond to the parameter lists in an invocation of procedure 1 and procedure 2 respectively, with the requirement that the number of parameters in the invocations match the number of parameters in the declarations. Show that L is not context-free.

4. Without using the Pumping Theorem, prove that $L = \{w \in \{a, b, c\}^* : \#_a(w) = \#_b(w) = \#_c(w)$ and $\#_a(w) > 50\}$ is not context-free.

13.4 (#11)

5. Give an example of a context-free language L ($\neq \Sigma^*$) that contains a subset L_1 that is not context-free. Prove that L is context free. Describe L_1 and prove that it is not context-free.

6. Let $L_1 = L_2 \cap L_3$.
   a. Show values for L_1, L_2, and L_3, such that L_1 is context-free but neither L_2 nor L_3 is.
   b. Show values for L_1, L_2, and L_3, such that L_2 is context-free but neither L_1 nor L_3 is.

7. Give an example of a context-free language L, other than one of the ones in the book, where $\neg L$ is not context-free.

8. Theorem 13.7 tells us that the context-free languages are closed under intersection with the regular languages. Prove that the context-free languages are also closed under union with the regular languages.

13.8 (#12)

9. Complete the proof that the context-free languages are not closed under *maxstring* by showing that $L = \{a^i b^j c^k : k \leq i \text{ or } k \leq j\}$ is context-free but *maxstring*(L) is not context-free.

13.9 (#13) 6

12. Define the leftmost maximal P subsequence m of a string w as follows:
   - P must be a nonempty set of characters.
   - A string S is a P subsequence of w iff S is a substring of w and S is composed entirely of characters in P. For example 1, 0, 10, 01, 11, 011, 101, 111, 1111, and 1011 are $\{0, 1\}$ subsequences of 2312101121111.

13.12 (#14)

   - Let S be the set of all P subsequences of w such that, for each element t of S, there is no P subsequence of w longer than t. In the example above, $S = \{1111, 1011\}$.
   - Then m is the leftmost (within w) element of S. In the example above, m = 1011.

   a. Let $L = \{w \in \{0\text{-}9\}^* :$ if y is the leftmost maximal $\{0, 1\}$ subsequence of w then $|y|$ is even$\}$. Is L regular (but not context free), context free or neither? Prove your answer.
   b. Let $L = \{w \in \{a, b, c\}^* :$ the leftmost maximal $\{a, b\}$ subsequence of w starts with a$\}$. Is L regular (but not context free), context free or neither? Prove your answer.

Note on 13.12. What the author meant to ask and what she actually asked are quite different. **Both parts should have said**: "Is L context-Free (but not regular), regular, or neither? Prove your answer.

13.13d

(#15) 6

13.14

(#16) 6,9

14.1a

(#17) 6

14.1c

(#18) 9

14.1d

(#19)

14.1e

(#20)

13. Are the context-free languages closed under each of the following functions? Prove your answer.
   a. $chop(L) = \{w : \exists x \in L\ (x = x_1cx_2 \wedge x_1 \in \Sigma_L{}^* \wedge x_2 \in \Sigma_L{}^* \wedge c \in \Sigma_L \wedge |x_1| = |x_2| \wedge w = x_1x_2)\}$

   b. $mix(L) = \{w : \exists x, y, z: (x \in L, x = yz, |y| = |z|, w = yz^R)\}$
   c. $pref(L) = \{w : \exists x \in \Sigma^*(wx \in L)\}$
   d. $middle(L) = \{x : \exists y, z \in \Sigma^*(yxz \in L)\}$
   e. Letter substitution
   f. $shuffle(L) = \{w : \exists x \in L$ (w is some permutation of x)$\}$
   g. $copyreverse(L) = \{w : \exists x \in L\ (w = xx^R)\}$
14. Let $alt(L) = \{x : \exists y, n\ (y \in L, |y| = n, n > 0, y = a_1 \cdots a_n, \forall i \leq n\ (a_i \in \Sigma)$, and $x = a_1a_3a_5 \cdots a_k$, where $k =$ (if n is even then $n - 1$ else n))$\}$.
   a. Consider $L = \mathrm{a}^n\mathrm{b}^n$. Clearly describe $L_1 = alt(L)$.
   b. Are the context free languages closed under the function *alt*? Prove your answer.

1. Give a decision procedure to answer each of the following questions:
   a. Given a regular expression α and a PDA M, is the language accepted by M a subset of the language generated by α?
   b. Given a context-free grammar G and two strings s_1 and s_2, does G generate s_1s_2?

   c. Given a context-free grammar G, does G generate at least three strings?
   d. Given a context-free grammar G, does G generate any even length strings?
   e. Given a regular grammar G, is $L(G)$ context-free?

13.14 Examples:

If L is the language denoted by r.e. (ab)*, then alt(L) is denoted by r.e. a*.

If L is the language denoted by r.e. (abcdefg)*, then alt(L) is denoted by r.e. (aceg)* .

If L is the language denoted by r.e. a or the language denoted by r.e. a*, then alt(L) is L.