474 HW 12 problems (highlighted problems are the ones to turn in)

12.1j (#1)

12.3 (#2) 6+6+6

12.4 (#3) 3 for each part

12.5a (#4) 9

12.6 (#5) 6

13.1a (#1) 6

13.1b (#2)

13.1c (#3) 12

13.1d (#4) 6

1. Build a PDA to accept each of the following languages L:

a. BalDelim = $\{w :$ where w is a string of delimiters: (,), [,], {, }, that are properly balanced$\}$.

b. $\{a^i b^j : 2i = 3j + 1\}$.

c. $\{w \in \{a, b\}^* : \#_a(w) = 2 \cdot \#_b(w)\}$.

d. $\{a^n b^m : m \le n \le 2m\}$.

e. $\{w \in \{a, b\}^* : w = w^R\}$.

f. $\{a^i b^j c^k : i, j, k \ge 0$ and $(i \ne j$ or $j \ne k)\}$.

g. $\{w \in \{a, b\}^* :$ every prefix of w has at least as many a's as b's$\}$.

h. $\{a^n b^m a^n : n, m \ge 0$ and m is even$\}$.

i. $\{xc^n : x \in \{a, b\}^*, \#_a(x) = n$ or $\#_b(x) = n\}$.

j. $\{a^n b^m : m \ge n, m\text{-}n$ is even$\}$.

k. $\{a^m b^n c^p d^q : m, n, p, q \ge 0$ and $m + n = p + q\}$.

3. Let $L = \{ba^{m_1}ba^{m_2}ba^{m_3} \ldots ba^{m_n} : n \ge 2, m_1, m_2, \ldots, m_n \ge 0$, and $m_i \ne m_j$ for some $i, j\}$.

a. Show a PDA that accepts L.

b. Show a context-free grammar that generates L.

c. Prove that L is not regular.

See hint below

4. Consider the language $L = L_1 \cap L_2$, where $L_1 = \{ww^R : w \in \{a, b\}^*\}$ and $L_2 = \{a^n b^* a^n : n \ge 0\}$.

a. List the first four strings in the lexicographic enumeration of L.

b. Write a context-free grammar to generate L.

c. Show a natural PDA for L. (In other words, don't just build it from the grammar using one of the two-state constructions presented in this chapter.)

d. Prove that L is not regular.

5. Build a deterministic PDA to accept each of the following languages:

a. $L\$$, where $L = \{w \in \{a, b\}^* : \#_a(w) = \#_b(w)\}$.

b. $L\$$ where $L = \{a^n b^+ a^m : n \ge 0$ and $\exists k \ge 0\ (m = 2k + n)\}$.

6. Complete the proof that we started in Example 12.14. Specifically, show that if M is a PDA that accepts by accepting state alone, then there exists a PDA M' that accepts by accepting state and empty stack (our definition) where $L(M') = L(M)$.

12.5a Hint: In order to be deterministic, the first thing your PDA should do, before it adds any input, is to push a special "bottom of stack" marker

12.3 Clarification: It may help you to understand the language better if you replace "for some i, j" by "at least one pair i, j" .

Hints (a) Have states and transitions to handle the "don't care" sections where we aren't worried about the number of b's. (b) One nonterminal for when the $m_i > m_j$, and one for when $m_i < m_j$. (c) Pumping theorem alone probably won't do it; also use closure property.

Warning: Last time I taught the course, several students indicated that this was the most difficult problem in the assignment, and a few thought it was one of the hardest problems of the term.

1. For each of the following languages L, state whether L is regular, context-free but not regular, or not context-free and prove your answer.

a. $\{xy : x, y \in \{a, b\}^*$ and $|x| = |y|\}$.

b. $\{(ab)^n a^n b^n : n > 0\}$.

c. $\{x\#y : x, y \in \{0, 1\}^*$ and $x \ne y\}$.

d. $\{a^i b^n : i, n > 0$ and $i = n$ or $i = 2n\}$.

Recall: When we use the pumping theorem to show a language is not context-free, we do not get to choose the k, we choose the w whose length is at least k. We do not get to choose how w is broken up into uvxyz (although the breakup has to meet the length constraints of the theorem), but we do get to choose how to pump the v and y (I.e. we can choose the q in uv^qxy^qz).