**474 HW 09 problems (highlighted problems are the ones to turn in)**

8.8(#1)

8.8 d, e
(#2)( 3, 6)

8.9
(#3) (9)

8.10a
(#4) (9)

8.16a
(#5) (9)

8.16b
(#6)

8.21
(#7)

8.21n
(#8) (12)

8.21o
(#9) (3)

**7.** Prove that the regular languages are closed under each of the following operations:

**a.** $pref(L) = \{w : \exists x \in \Sigma^*(wx \in L)\}$.

**b.** $suff(L) = \{w : \exists x \in \Sigma^*(xw \in L)\}$.

**c.** $reverse(L) = \{x \in \Sigma^* : x = w^R \text{ for some } w \in L\}$.

**d.** letter substitution (as defined in Section 8.3).

**8.** Using the defintions of *maxstring* and *mix* given in Section 8.6, give a precise definition of each of the following languages:

**a.** $maxstring(\mathrm{A}^n\mathrm{B}^n)$.

**b.** $maxstring(\mathrm{a}^i\,\mathrm{b}^j\,\mathrm{c}^k, 1 \le k \le j \le i)$.

**c.** $maxstring(L_1L_2)$, where $L_1 = \{w \in \{\mathrm{a}, \mathrm{b}\}^* : w \text{ contains exactly one } \mathrm{a}\}$ and $L_2 = \{\mathrm{a}\}$.

**d.** $mix((\mathrm{aba})^*)$.

**e.** $mix(\mathrm{a}^*\mathrm{b}^*)$.

**9.** Prove that the regular languages are not closed under *mix*.

Definitions of *maxstring* and *mix:* Examples 8.22 and 8.23.

**10.** Recall that $maxstring(L) = \{w : w \in L \text{ and } \forall z \in \Sigma^*(z \ne \varepsilon \rightarrow wz \notin L)\}$.

**a.** Prove that the regular languages are closed under *maxstring*.

**b.** If $maxstring(L)$ is regular, must $L$ also be regular? Prove your answer.

**8.10-a** I.e., given a DFSM M = (K, Σ, δ, s, A) such that L(M)=L, construct a DFSM $M^*=(K^*, \Sigma, \Delta^*, s^*, A^*)$ such that L(M*)=maxstring(L).

**16.** Define two integers $i$ and $j$ to be ***twin primes*** iff both $i$ and $j$ are prime and $|j - i| = 2$.

**a.** Let $L = \{w \in \{1\}^* : w$ is the unary notation for a natural number $n$ such that there exists a pair $p$ and $q$ of twin primes, both $> n.\}$ Is $L$ regular?

**b.** Let $L = \{x, y : x$ is the decimal encoding of a positive integer $i$, $y$ is the decimal encoding of a positive integer $j$, and $i$ and $j$ are twin primes$\}$. Is $L$ regular?

**21.** For each of the following claims, state whether it is *True* or *False*. Prove your answer.



**a.** There are uncountably many non-regular languages over $\Sigma = \{\mathrm{a}, \mathrm{b}\}$.

**b.** The union of an infinite number of regular languages must be regular.

**c.** The union of an infinite number of regular languages is never regular.

**d.** If $L_1$ and $L_2$ are not regular languages, then $L_1 \cup L_2$ is not regular.

**e.** If $L_1$ and $L_2$ are regular languages, then $L_1 \otimes L_2 = \{w : w \in (L_1 - L_2) \text{ or } w \in (L_2 - L_1)\}$ is regular.



**f.** If $L_1$ and $L_2$ are regular languages and $L_1 \subseteq L \subseteq L_2$, then $L$ must be regular.

**g.** The intersection of a regular language and a nonregular language must be regular.

**h.** The intersection of a regular language and a nonregular language must not be regular.

**i.** The intersection of two nonregular languages must not be regular.

**j.** The intersection of a finite number of nonregular languages must not be regular.

**k.** The intersection of an infinite number of regular languages must be regular.

**l.** It is possible that the concatenation of two nonregular languages is regular.



**m.** It is possible that the union of a regular language and a nonregular language is regular.

**n.** Every nonregular language can be described as the intersection of an infinite number of regular languages.

**o.** If $L$ is a language that is not regular, then $L^*$ is not regular.

9.1 (#10)

9.1b
(#11) (6)

9.1d
(#12) (6)

9.1g
See note
below
(#13) (6)

9.1i
(#14) (6)

1. Define a decision procedure for each of the following questions. Argue that each of your decision procedures gives the correct answer and terminates.
   **a.** Given two DFSMs $M_1$ and $M_2$, is $L(M_1) = L(M_2)^R$?
   **b.** Given two DFSMs $M_1$ and $M_2$ is $|L(M_1)| < |L(M_2)|$?
   **c.** Given a regular grammar $G$ and a regular expression $\alpha$, is $L(G) = L(\alpha)$?
   **d.** Given two regular expressions, $\alpha$ and $\beta$, do there exist any even length strings that are in $L(\alpha)$ but not $L(\beta)$?
   **e.** Let $\Sigma = \{\mathtt{a},\mathtt{b}\}$ and let $\alpha$ be a regular expression. Does the language generated by $\alpha$ contain all the even length strings in $\Sigma^*$.
   **f.** Given an FSM $M$ and a regular expression $\alpha$, is it true that both $L(M)$ and $L(\alpha)$ are finite and $M$ accepts exactly two more strings than $\alpha$ generates?
   **g.** Let $\Sigma = \{\mathtt{a},\mathtt{b}\}$ and let $\alpha$ and $\beta$ be regular expressions. Is the following sentence true:

   $$(L(\beta) = \mathtt{a}^*) \vee (\forall w\,(w \in \{\mathtt{a},\mathtt{b}\}^* \wedge |w| \text{ even}) \rightarrow w \in L(\alpha)).$$

   **h.** Given a regular grammar $G$, is $L(G)$ regular?
   **i.** Given a regular grammar $G$, does $G$ generate any odd length strings?

## 9.1g:

There is a small error in the statement of the problem. a* should be {a}*

## 9.1b:

Note that |L(M)| means "the number of elements in the language accepted by the machine M. Note that for some machines M, the language is countably infinite.