**474 HW 08 problems (highlighted problems are the ones to turn in)**

8.1acegklptuz (#1)

8.1 e (#2)

8.1f

8.1g

8.1h

8.1k

8.1m

(#2 - #7)
3 pts. each

8.7a
(#8) 9

**1.** For each of the following languages $L$, state whether $L$ is regular or not and prove your answer:

**a.** $\{a^i b^j : i, j \geq 0 \text{ and } i + j = 5\}$.

**b.** $\{a^i b^j : i, j \geq 0 \text{ and } i - j = 5\}$.

**c.** $\{a^i b^j : i, j \geq 0 \text{ and } |i - j| \equiv_5 0\}$.

**d.** $\{w \in \{0, 1, \#\}^* : w = x\#y, \text{ where } x, y \in \{0, 1\}^* \text{ and } |x| \cdot |y| \equiv_5 0\}$.

**e.** $\{a^i b^j : 0 \leq i < j < 2000\}$.

**f.** $\{w \in \{Y, N\}^* : w$ contains at least two Y's and at most two N's$\}$.

**g.** $\{w = xy : x, y \in \{a, b\}^* \text{ and } |x| = |y| \text{ and } \#_a(x) \geq \#_a(y)\}$.

**h.** $\{w = xyzy^R x : x, y, z \in \{a, b\}^*\}$.

**i.** $\{w = xyzy : x, y, z \in \{0, 1\}^+\}$.

**j.** $\{w \in \{0, 1\}^* : \#_0(w) \neq \#_1(w)\}$.

**k.** $\{w \in \{a, b\}^* : w = w^R\}$.

**l.** $\{w \in \{a, b\}^* : \exists x \in \{a, b\}^+ (w = x\, x^R x)\}$.

**m.** $\{w \in \{a, b\}^*$ : the number of occurrences of the substring ab equals the number of occurrences of the substring ba$\}$.

**n.** $\{w \in \{a, b\}^* : w$ contains exactly two more b's than a's$\}$.

**o.** $\{w \in \{a, b\}^* : w = xyz, |x| = |y| = |z|$, and $z = x$ with every a replaced by b and every b replaced by a$\}$. Example: abbbabbaa $\in L$, with $x =$ abb, $y =$ bab, and $z =$ baa.

**p.** $\{w : w \in \{a - z\}^*$ and the letters of $w$ appear in reverse alphabetical order$\}$. For example, spoonfeed $\in L$.

**q.** $\{w : w \in \{a - z\}^*$ every letter in $w$ appears at least twice$\}$. For example, unprosperousness $\in L$.

**r.** $\{w : w$ is the decimal encoding of a natural number in which the digits appear in a non-decreasing order without leading zeros$\}$.

**s.** $\{w$ of the form: $<integer_1> + <integer_2> = <integer_3>$, where each of the substrings $<integer_1>$, $<integer_2>$, and $<integer_3>$ is an element of $\{0 - 9\}^*$ and $integer_3$ is the sum of $integer_1$ and $integer_2\}$. For example, 124+5=129 $\in L$.

**t.** $L_0^*$, where $L_0 = \{ba^i b^j a^k, j \geq 0, 0 \leq i \leq k\}$.

**u.** $\{w : w$ is the encoding of a date that occurs in a year that is a prime number$\}$. A date will be encoded as a string of the form *mm*/*dd*/*yyyy*, where each $m$, $d$, and $y$ is drawn from $\{0–9\}$.

**v.** $\{w \in \{1\}^* : w$ is, for some $n \geq 1$, the unary encoding of $10^n\}$. (So $L = \{1111111111, 1^{100}, 1^{1000}, \ldots\}$.)

**6.** Prove *by construction* that the regular languages are closed under:

**a.** intersection.

**b.** set difference.

**7.** Prove that the regular languages are closed under each of the following operations:

**a.** $pref(L) = \{w : \exists x \in \Sigma^* (wx \in L)\}$.

**b.** $suff(L) = \{w : \exists x \in \Sigma^* (xw \in L)\}$.

**c.** $reverse(L) = \{x \in \Sigma^* : x = w^R \text{ for some } w \in L\}$.

**d.** letter substitution (as defined in Section 8.3).

**8.7a** Do this by construction, i.e., produce an algorithm that takes as input a DFSM $M = (K, \Sigma, \delta, s, A)$ that accepts L, and produces a DFSM $M' = (K', \Sigma', \delta', s', A')$ that accepts *pref*($L$). Describe how to get from M to M'

**Hint:** M' will have a lot of its elements in common with M, but it takes a somewhat complex calculation (based on M) to determine exactly what has to be changed.

On the main HW8 assignment document, I posted **the author's solutions to the other three parts of problem 8.7**, so that you will have more examples.

8.2ac
(#9)

**2.** For each of the following languages $L$, state whether $L$ is regular or not and prove your answer:

**a.** $\{w \in \{\mathrm{a,b,c}\}^* : \text{in each prefix } x \text{ of } w, \#_a(x) = \#_b(x) = \#_c(x))\}$.

**b.** $\{w \in \{\mathrm{a,b,c}\}^* : \exists \text{ some prefix } x \text{ of } w\ (\#_a(x) = \#_b(x) = \#_c(x))\}$.

**c.** $\{w \in \{\mathrm{a,b,c}\}^* : \exists \text{ some prefix } x \text{ of } w\ (x \neq \varepsilon \text{ and } \#_a(x) = \#_b(x) = \#_c(x))\}$.

8.3
(#10)

**3.** Define the following two languages:

$L_a = \{w \in \{\mathrm{a,b}\}^* : \text{in each prefix } x \text{ of } w, \#_a(x) \geq \#_b(x)\}$.

$L_b = \{w \in \{\mathrm{a,b}\}^* : \text{in each prefix } x \text{ of } w, \#_b(x) \geq \#_a(x)\}$.

**a.** Let $L_1 = L_a \cap L_b$. Is $L_1$ regular? Prove your answer.

**b.** Let $L_2 = L_a \cup L_b$. Is $L_2$ regular? Prove your answer.

8.4a
(#11) 6

8.4b
(#12)

**4.** For each of the following languages $L$, state whether $L$ is regular or not and prove your answer:

**a.** $\{uww^Rv : u, v, w \in \{\mathrm{a,b}\}^+\}$.

**b.** $\{xyzy^Rx : x, y, z \in \{\mathrm{a,b}\}^+\}$.

8.7
(#13)

**7.** Prove that the regular languages are closed under each of the following operations:

**a.** $pref(L) = \{w : \exists x \in \Sigma^*(wx \in L)\}$.

**b.** $suff(L) = \{w : \exists x \in \Sigma^*(xw \in L)\}$.

**c.** $reverse(L) = \{x \in \Sigma^* : x = w^R \text{ for some } w \in L\}$.

**d.** letter substitution (as defined in Section 8.3).

**8.** Using the defintions of *maxstring* and *mix* given in Section 8.6, give a precise definition of each of the following languages:

**a.** $maxstring(A^nB^n)$.

**b.** $maxstring(\mathrm{a}^i\mathrm{b}^j\mathrm{c}^k, 1 \leq k \leq j \leq i)$.

6.18
(#14) 9

**c.** $maxstring(L_1L_2)$, where $L_1 = \{w \in \{\mathrm{a,b}\}^* : w \text{ contains exactly one a}\}$ and $L_2 = \{\mathrm{a}\}$.

**d.** $mix((\mathrm{aba})^*)$.

**e.** $mix(\mathrm{a}^*\mathrm{b}^*)$.

**9.** Prove that the regular languages are not closed under *mix*.

6.20
(#15)

18. Let $\Sigma = \{\mathrm{a, b}\}$. Let $L = \{\varepsilon, \mathrm{a, b}\}$. Let $R$ be a relation defined on $\Sigma^*$ as follows: $\forall xy\ (xRy \text{ iff } y = x\mathrm{b})$. Let $R'$ be the reflexive, transitive closure of $R$. Let $L' = \{x : \exists y \in L\ (yR'x)\}$. Write a regular expression for $L'$.

Good practice problems for exams (no proof necessary)

Note on 6.18 Transitive and reflexive closures are introduced in Section A.5 Closures under various operations are also mentioned on pages 17, 57, and 72.

**20.** For each of the following statements, state whether it is *True* or *False*. Prove your answer.

**a.** $(\mathrm{ab})^*\mathrm{a} = \mathrm{a}(\mathrm{ba})^*$.

**b.** $(\mathrm{a} \cup \mathrm{b})^*\mathrm{b}(\mathrm{a} \cup \mathrm{b})^* = \mathrm{a}^*\mathrm{b}(\mathrm{a} \cup \mathrm{b})^*$.

**c.** $(\mathrm{a} \cup \mathrm{b})^*\mathrm{b}(\mathrm{a} \cup \mathrm{b})^* \cup (\mathrm{a} \cup \mathrm{b})^*\mathrm{a}(\mathrm{a} \cup \mathrm{b})^* = (\mathrm{a} \cup \mathrm{b})^*$.

**d.** $(\mathrm{a} \cup \mathrm{b})^*\mathrm{b}(\mathrm{a} \cup \mathrm{b})^* \cup (\mathrm{a} \cup \mathrm{b})^*\mathrm{a}(\mathrm{a} \cup \mathrm{b})^* = (\mathrm{a} \cup \mathrm{b})^+$.

**e.** $(\mathrm{a} \cup \mathrm{b})^*\mathrm{ba}(\mathrm{a} \cup \mathrm{b})^* \cup \mathrm{a}^*\mathrm{b}^* = (\mathrm{a} \cup \mathrm{b})^*$.

**f.** $\mathrm{a}^*\mathrm{b}(\mathrm{a} \cup \mathrm{b})^* = (\mathrm{a} \cup \mathrm{b})^*\mathrm{b}(\mathrm{a} \cup \mathrm{b})^*$.

**g.** If $\alpha$ and $\beta$ are any two regular expressions, then $(\alpha \cup \beta)^* = \alpha(\beta\alpha \cup \alpha)$.

**h.** If $\alpha$ and $\beta$ are any two regular expressions, then $(\alpha\beta)^*\alpha = \alpha(\beta\alpha)^*$.