474 HW 2 problems (highlighted problems are the ones to turn in)

3.1

1. Consider the following problem: Given a digital circuit C, does C output 1 on all inputs? Describe this problem as a language to be decided.

3.2

2. Using the technique we used in Example 3.8 to describe addition, describe square root as a language recognition problem.
3. Consider the problem of encrypting a password, given an encryption key. Formulate this problem as a language recognition problem.

3.4

4. Consider the optical character recognition (OCR) problem: Given an array of black and white pixels and a set of characters, determine which character best matches the pixel array. Formulate this problem as a language recognition problem.

3.5

5. Consider the language $\text{A}^n\text{B}^n\text{C}^n = \{a^n b^n c^n : n \geq 0\}$, discussed in Section 3.3.3. We might consider the following design for a PDA to accept $\text{A}^n\text{B}^n\text{C}^n$: As each a is read, push two a's onto the stack. Then pop one a for each b and one a for each c. If the input and the stack come out even, accept. Otherwise reject. Why doesn't this work?

3.6

6. Define a PDA-2 to be a PDA with two stacks (instead of one). Assume that the stacks can be manipulated independently and that the machine accepts iff it is in an accepting state and both stacks are empty when it runs out of input. Describe the operation of a PDA-2 that accepts $\text{A}^n\text{B}^n\text{C}^n = \{a^n b^n c^n : n \geq 0\}$. (*Note*: We will see, in Section 17.5.2, that the PDA-2 is equivalent to the Turing machine in the sense that any language that can be accepted by one can be accepted by the other.)

4.1

1. Describe in clear English or pseudocode a decision procedure to answer the question, "Given a list of integers N and an individual integer n, is there any element of N that is a factor of n?"

4.2

2. Given a Java program p and the input 0, consider the question, "Does p ever output anything?"
   a. Describe a semidecision procedure that answers this question.
   b. Is there an obvious way to turn your answer to part a into a decision procedure?

4.3, 4.4

3. Recall the function $chop(L)$, defined in Example 4.10. Let $L = \{w \in \{a, b\}^* : w = w^R\}$. What is $chop(L)$?
4. Are the following sets closed under the following operations? Prove your answer. If a set is not closed under the operation, what is its closure under the operation?

4.4c

   a. $L = \{w \in \{a, b\}^* : w \text{ ends in } a\}$ under the function $odds$, defined on strings as follows: $odds(s)$ = the string that is formed by concatenating together all of the odd numbered characters of s. (Start numbering the characters at 1.) For example, $odds(ababbbb)$ = aabb.
   b. FIN (the set of finite languages) under the function $oddsL$, defined on languages as follows:

$$oddsL(L) = \{w : \exists x \in L\ (w = odds(x))\}.$$

   c. INF (the set of infinite languages) under the function $oddsL$.
   d. FIN under the function $maxstring$, defined in Example 8.22.
   e. INF under the function $maxstring$.

One more problem, not from the textbook:

#14 is described in detail on the assignment document, so I do not repeat that description here.

5.2

2. Show a DFSM to accept each of the following languages:

5.2a

   a. $\{w \in \{a, b\}^* : \text{every a in } w \text{ is immediately preceded and followed by b}\}$.

5.2b

   b. $\{w \in \{a, b\}^* : w \text{ does not end in ba}\}$.
   c. $\{w \in \{0, 1\}^* : w$ corresponds to the binary encoding, without leading 0's, of natural numbers that are evenly divisible by 4$\}$.
   d. $\{w \in \{0, 1\}^* : w$ corresponds to the binary encoding, without leading 0's, of natural numbers that are powers of 4$\}$.
   e. $\{w \in \{0\text{-}9\}^* : w$ corresponds to the decimal encoding, without leading 0's, of an odd natural number$\}$.
   f. $\{w \in \{0, 1\}^* : w \text{ has 001 as a substring}\}$.
   g. $\{w \in \{0, 1\}^* : w \text{ does not have 001 as a substring}\}$.
   h. $\{w \in \{a, b\}^* : w \text{ has bbab as a substring}\}$.
   i. $\{w \in \{a, b\}^* : w \text{ has neither ab nor bb as a substring}\}$.
   j. $\{w \in \{a, b\}^* : w \text{ has both aa and bb as a substrings}\}$.
   k. $\{w \in \{a, b\}^* : w$ contains at least two b's that are not immediately followed by an a$\}$.
   l. $\{w \in \{0, 1\}^* : w$ has no more than one pair of consecutive 0's and no more than one pair of consecutive 1's$\}$.
   m. $\{w \in \{0, 1\}^* : \text{none of the prefixes of } w \text{ ends in 0}\}$.
   n. $\{w \in \{a, b\}^* : (\#_a(w) + 2 \cdot \#_b(w)) \equiv_5 0\}$. ($\#_a(w)$ is the number of a's in w).