Some 474 HW 1 problems (highlighted problems are the ones to turn in). This document is not intended to be a substitute for the real HW1 assignment document, which contains many clarifying comments. It is simply here so that you do not have to look up the problem statements in the book

Shaded problems are to be turned in.

2-2

2-4

2-5

2. Let $L_1 = \{a^n b^n : n > 0\}$. Let $L_2 = \{c^n : n > 0\}$. For each of the following strings, state whether or not it is an element of L_1L_2:

- **a.** ε.
- **b.** aabbcc.
- **c.** abbcc.
- **d.** aabbcccc.

3. Let L_1 = {peach, apple, cherry} and L_2 = {pie, cobbler, ε}. List the elements of L_1L_2 in lexicographic order.

4. Let $L = \{w \in \{a, b\}^* : |w| \equiv_3 0\}$. List the first six elements in a lexicographic enumeration of L.

5. Consider the language L of all strings drawn from the alphabet {a, b} with at least two different substrings of length 2.

- **a.** Describe L by writing a sentence of the form $L = \{w \in \Sigma^* : P(w)\}$, where Σ is a set of symbols and P is a first-order logic formula. You may use the function $|s|$ to return the length of s. You may use all the standard relational symbols (e.g., $=$, $\neq$, $<$, etc.), plus the predicate *Substr*(s, t), which is *True* iff s is a substring of t.
- **b.** List the first six elements of a lexicographic enumeration of L.

2.6

2.6c

6. For each of the following languages L, give a simple English description. Show two strings that are in L and two that are not (unless there are fewer than two strings in L or two not in L, in which case show as many as possible).

- **a.** $L = \{w \in \{a, b\}^* :$ exactly one prefix of w ends in a $\}$.
- **b.** $L = \{w \in \{a, b\}^* :$ all prefixes of w end in a $\}$.
- **c.** $L = \{w \in \{a, b\}^* : \exists x \in \{a, b\}^+ (w = axa)\}$.

2-7, 2-8

7. Are the following sets closed under the following operations? If not, what are their respective closures?

- **a.** The language {a, b} under concatenation.
- **b.** The odd length strings over the alphabet {a, b} under Kleene star.
- **c.** $L = \{w \in \{a, b\}^*\}$ under reverse.
- **d.** $L = \{w \in \{a, b\}^* : w$ starts with a $\}$ under reverse.
- **e.** $L = \{w \in \{a, b\}^* : w$ ends in a $\}$ under concatenation.

See my notes about closure in the main assignment document.

2-8 acdgl

8. For each of the following statements, state whether it is *True* or *False*. Prove your answer.

- **a.** $\forall L_1, L_2 (L_1 = L_2 \text{ iff } L_1^* = L_2^*)$.
- **b.** $(\varnothing \cup \varnothing^*) \cap (\neg\varnothing - (\varnothing\varnothing^*)) = \varnothing$ (where $\neg\varnothing$ is the complement of $\varnothing$).
- **c.** Every infinite language is the complement of a finite language.
- **d.** $\forall L ((L^R)^R = L)$.
- **e.** $\forall L_1, L_2 ((L_1L_2)^* = L_1^*L_2^*)$.
- **f.** $\forall L_1, L_2 ((L_1^*L_2^*L_1^*)^* = (L_2 \cup L_1)^*)$.
- **g.** $\forall L_1, L_2 ((L_1 \cup L_2)^* = L_1^* \cup L_2^*)$.
- **h.** $\forall L_1, L_2, L_3 ((L_1 \cup L_2)L_3 = (L_1L_3) \cup (L_2L_3))$.
- **i.** $\forall L_1, L_2, L_3 ((L_1L_2) \cup L_3 = (L_1 \cup L_3)(L_2 \cup L_3))$.
- **j.** $\forall L ((L^+)^* = L^*)$.
- **k.** $\forall L (\varnothing L^* = \{\varepsilon\})$.
- **l.** $\forall L (\varnothing \cup L^+ = L^*)$.
- **m.** $\forall L_1, L_2 ((L_1 \cup L_2)^* = (L_2 \cup L_1)^*)$.

You only have to prove your answer if the statement is false.

A 21 de

A 22

A 23

There is also a required problem (#12) described on the assignment page itself that is not from the textbook.

21. Use induction to prove each of the following claims:

a. $\forall n > 0 \left(\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \right)$.

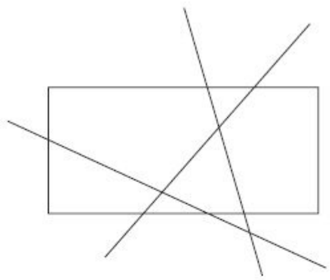
b. $\forall n > 0\ (n! \geq 2^{n-1})$. Recall that $0! = 1$ and $\forall n > 0\ (n! = n(n-1)(n-2)\cdots 1)$.

c. $\forall n > 0 \left(\sum_{k=0}^{n} 2^k = 2^{n+1} - 1 \right)$.

d. $\forall n \geq 0 \left(\sum_{k=0}^{n} r^k = \frac{r^{n+1} - 1}{r - 1} \right)$, given $r \neq 0, 1$.

e. $\forall n \geq 0 \left(\sum_{k=0}^{n} f_k^2 = f_n \cdot f_{n+1} \right)$, where f_n is the n^{th} element of the Fibonacci sequence, as defined in Example 24.4.

22. Consider a finite rectangle in the plane. We will draw some number of (infinite) lines that cut through the rectangle. So, for example, we might have:



In Section 28.7.6, we define what we mean when we say that a map can be colored using two colors. Treat the rectangle that we just drew as a map, with regions defined by the lines that cut through it. Use induction to prove that, no matter how many lines we draw, the rectangle can be colored using two colors.

23. Let $div_2(n) = \lfloor n/2 \rfloor$ (i.e., the largest integer that is less than or equal to $n/2$). Alternatively, think of it as the function that performs division by 2 on a binary number by shifting right one digit. Prove that the following program correctly multiplies two natural numbers. Clearly state the loop invariant that you are using.

$mult(n, m$: natural numbers$) =$
$\quad result = 0.$
$\quad$ While $m \neq 0$ do
$\quad\quad$ If $odd(m)$ then $result = result + n$.
$\quad\quad n = 2n.$
$\quad\quad m = div_2(m).$

A.22 (colored in such a way that no two regions that share a common edge are assigned the same color). **Hint:** Induction on the number of lines. How do you get from a correct coloring with n lines through the rectangle to a correct coloring with n+1 lines?

Note that **the last problem** (a turnin problem, induction problem about counting parentheses in expressions) is **not** from the textbook. It is stated directly on the HW 1 assignment page, **so I did not repeat it here**. The main assignment page has a lot to say about this problem.