474 Instructor Notes from Day 13 slides:

Slide 11: For Every FSM There is a Corresponding Regular Expression

I do this construction because I think every math or CS major should see this kind of dynamic programming algorithm, and there is no required course that does it.

Another algorithm that uses a similar approach is the Floyd-Warshall "all-pairs shortest paths algorithm", which solves a problem) in n3 time that at first seems like it must be O(n4).

Write "Floyd-Warshall" on the board.

Slide 12: **DFA→Reg. Exp. construction**

R_{ij0} is {a in Sigma : delta(q_i, a) = q_j. This set has one or zero elements.

R_{ij0} is set of all strings that take M from q_i to q_j, because all states are numbered $\leq$ n.

Slide 13: **DFA→Reg. Exp. continued**

Recursive case: if a string is in R_{ijjk}, it either

takes us form state i to state j without passing through state k, or

it does pass through k. It takes us to k for the first time, possibly does some loops that pass through k, then goes to j.

Slide 17: An Example

Look quickly at the k=0 cases.

Tell students that for practice they should look at some of the others that we do not do together.

Look together at these examples:

$r_{221} = r_{220} \cup r_{210}(r_{110})^* r_{120} = \varepsilon \cup 0(\varepsilon)^*0 = = \varepsilon \cup 00$

$r_{132} = r_{131} \cup r_{121}(r_{221})^* r_{231} = 1 \cup 0(\varepsilon \cup 00)^*(1 \cup 01) = 1 \cup 0(00)^*(\varepsilon \cup 0)\ 1$.

Note that $0(00)^*(\varepsilon \cup 0)$ is equivalent to 0^*, so we get $1 \cup 0^*1$ which is equivalent to 0^*1.

Have students (on the quiz) do r123 and r133 and simplify them. Compare notes with another student.

$r_{123} = r_{122} \cup r_{132}(r_{332})^* r_{322} = 0(00)^* \cup 0^*1(\varepsilon \cup (0 \cup 1)0^*1)^*(0 \cup 1)(00)^* = 0(00)^* \cup 0^*1((0 \cup 1)0^*1)^*(0 \cup 1)(00)^*$

$r_{133} = r_{132} \cup r_{132}(r_{332})^* r_{332} = 0^*1 \cup 0^*1(\varepsilon \cup (0 \cup 1)0^*1)^*(\varepsilon \cup (0 \cup 1)0^*1 = 0^*1((0 \cup 1)0^*1)^*$

For the entire machine we get $r_{123} \cup r_{133} = 0(00)^* \cup 0^*1((0 \cup 1)0^*1)^*(\varepsilon \cup (0 \cup 1)(00)^*)$