

Name: \_\_\_\_\_ Solution \_\_\_\_\_

Grade: \_\_\_\_\_ &lt;-- (13 possible)

1. State Kleene's Theorem: A language is regular iff it is defined by some regular expression.

2. (4) On the back, draw the machines for the various cases of regExpToFSM

**Union:** New start state with  $\epsilon$ -moves to  $S_1$  and  $S_2$

**Concatenation:**  $S_1$  is the start state of new machine. Add  $\epsilon$ -move from each accepting state of  $M_1$  to  $S_2$ . Accepting states of  $M_1$  are no longer accepting in new machine

**Kleene \*:** Add a new start state, which is an accepting state.  $\epsilon$ -move from new start state to  $S_2$ .  $\epsilon$ -move from each accepting state of  $M_1$  back to the new start state. New start state can be the only accepting state, or the original accepting states can still accept.

3. (5) In the DFSMtoRegExp example machine  $M$ , show how to get

$$r_{221} = r_{220} \cup r_{210}(r_{110})^*r_{120} = \epsilon \cup 0(\epsilon)^*0 = \epsilon \cup 00$$

$$r_{132} = r_{131} \cup r_{121}(r_{221})^*r_{231} = 1 \cup 0(\epsilon \cup 00)^*(1 \cup 01) = 1 \cup 0(00)^*(\epsilon \cup 0)1$$

Note that  $0(00)^*(\epsilon \cup 0)$  is equivalent to  $0^*$ , so we get  $1 \cup 0^*1$  which is equivalent to  $0^*1$ .

$$r_{123} = r_{122} \cup r_{132}(r_{332})^*r_{322} = 0(00)^* \cup 0^*1(\epsilon \cup (0 \cup 1)0^*1)^*(0 \cup 1)(00)^* = 0(00)^* \cup 0^*1((0 \cup 1)0^*1)^*(0 \cup 1)(00)^*$$

$$r_{133} = r_{132} \cup r_{132}(r_{332})^*r_{332} = 0^*1 \cup 0^*1(\epsilon \cup (0 \cup 1)0^*1)^*(\epsilon \cup (0 \cup 1)0^*1) = 0^*1((0 \cup 1)0^*1)^*$$

A regular expression  $r$  such that  $L(R) = L(M)$

$$r_{123} \cup r_{133} = 0(00)^* \cup 0^*1((0 \cup 1)0^*1)^*(\epsilon \cup (0 \cup 1)(00)^*) \text{ [the } \epsilon \text{ is from } r_{133}]$$

4. (2) Given a DFSM for a language  $L$ , how do we construct a Machine  $M'$  for  $L^R$ ?

Let  $M = (K, \Sigma, \delta, s, A)$  be any DFSM that accepts  $L$ .  $M$  must be written out completely, without an implied dead state.

Then construct  $M' = (K', \Sigma', \delta', s', A')$  to accept  $reverse(L)$  from  $M$ :

Initially, let  $M'$  be  $M$ .

Reverse the direction of every transition in  $M'$ .

Construct a new state  $q$ . Make it the start state of  $M'$ . Create an  $\epsilon$ -transition from  $q$  to every state that was an accepting state in  $M$ .

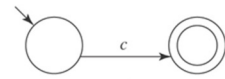
$M'$  has a single accepting state, the start state of  $M$ .

5. Tell your instructor about anything from today's session (or from the course so far) that you found confusing or still have a question about. If none, please write "None". Students must have some answer to earn this point.

$\emptyset$ :



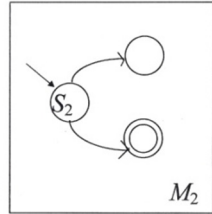
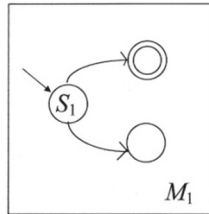
A single element of  $\Sigma$ :



$\varepsilon$  ( $\emptyset^*$ ):



Union:



Concatenation:

Kleene star: