474 HW 16 problems (highlighted problems are the ones to turn in)

19.1 (#1) 6

1. Consider the language $L = \{<M> : \text{Turing machine } M \text{ accepts at least two strings}\}$.
   - **a.** Describe in clear English a Turing machine M that semidecides L.
   - **b.** Now change the definition of L just a bit. Consider:

     $L' = \{<M> : \text{Turing machine } M \text{ accepts } exactly \text{ 2 strings}>$.

     Can you tweak the Turing machine you described in part a to semidecide L'?

19.2 (#2) 12

2. Consider the language $L = \{<M> : \text{Turing machine } M \text{ accepts the binary encodings of the first three prime numbers}\}$.
   - **a.** Describe in clear English a Turing machine M that semidecides L.
   - **b.** Suppose (contrary to fact, as established by Theorem 19.2) that there were a Turing machine *Oracle* that decided H. Using it, describe in clear English a Turing machine M that decides L.

20.1 (#3)

1. Show that the set D (the decidable languages) is closed under:
   - **a.** Union
   - **b.** Concatenation
   - **c.** Kleene star
   - **d.** Reverse
   - **e.** Intersection

20.2 (#4)

2. Show that the set SD (the semidecidable languages) is closed under:
   - **a.** Union
   - **b.** Concatenation
   - **c.** Kleene star
   - **d.** Reverse
   - **e.** Intersection

20.3 (#5) 9

3. Let $L_1, L_2, \ldots, L_k$ be a collection of languages over some alphabet Σ such that:
   - For all $i \neq j, L_i \cap L_j = \varnothing$.
   - $L_1 \cup L_2 \cup \ldots \cup L_k = \Sigma^*$.
   - $\forall i$ (L_i is in SD).

   Prove that each of the languages L_1 through L_k is in D.

20.4 (#6) 6

4. If L_1 and L_3 are in D and $L_1 \subseteq L_2 \subseteq L_3$, what can we say about whether L_2 is in D?

20.5 (#7)

5. Let L_1 and L_2 be any two decidable languages. State and prove your answer to each of the following questions:
   - **a.** Is it necessarily true that $L_1 - L_2$ is decidable?
   - **b.** Is it possible that $L_1 \cup L_2$ is regular?

20.6 (#8)

6. Let L_1 and L_2 be any two undecidable languages. State and prove your answer to each of the following questions:
   - **a.** Is it possible that $L_1 - L_2$ is regular?
   - **b.** Is it possible that $L_1 \cup L_2$ is in D?

20.7 (#9) 6

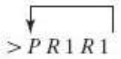
7. Let M be a Turing machine that lexicographically enumerates the language L. Prove that there exists a Turing machine M' that decides L^R.

20.8 (#10) 9

8. Construct a standard one-tape Turing machine M to enumerate the language:

   $\{w : w \text{ is the binary encoding of a positive integer that is divisible by } 3\}$.

   Assume that M starts with its tape equal to ❑. Also assume the existence of the printing subroutine P, defined in Section 20.5.1. As an example of how to use P, consider the following machine, which enumerates L', where $L' = \{w : w \text{ is the unary encoding of an even number}\}$:



20.13 (#11) 9

13. Show that every infinite semidecidable language has a subset that is not decidable.