474 HW 13 problems (highlighted problems are the ones to turn in)

13.1a
(#1) 6
13.1b
(#2)
13.1c
(#3) 12
13.1d
(#4) 6
13.1f
(#5) 9
13.1g
(#6)
13.1h
(#7)
13.1i
(#8) 9
13.1k
(#9)
13.1l
(#10) 9
13.1p
(#11)
13.1q
(#12)
13.1w
(#13) 9

1. For each of the following languages L, state whether L is regular, context-free but not regular, or not context-free and prove your answer.
   - **a.** $\{xy : x, y \in \{a, b\}^* \text{ and } |x| = |y|\}$.
   - **b.** $\{(ab)^n a^n b^n : n > 0\}$.
   - **c.** $\{x\#y : x, y \in \{0, 1\}^* \text{ and } x \neq y\}$.
   - **d.** $\{a^i b^n : i, n > 0 \text{ and } i = n \text{ or } i = 2n\}$.
   - **e.** $\{wx : |w| = 2 \cdot |x| \text{ and } w \in a^+b^+ \text{ and } x \in a^+b^+\}$.
   - **f.** $\{a^n b^m c^k : n, m, k \geq 0 \text{ and } m \leq min(n, k)\}$.
   - **g.** $\{xyx^R : x \in \{0, 1\}^+ \text{ and } y \in \{0, 1\}^*\}$.
   - **h.** $\{xwx^R : x, w \in \{a, b\}^+ \text{ and } |x| = |w|\}$.
   - **i.** $\{ww^Rw : w \in \{a, b\}^*\}$.
   - **j.** $\{wxw: |w| = 2 \cdot |x| \text{ and } w \in \{a, b\}^* \text{ and } x \in \{c\}^*\}$.
   - **k.** $\{a^i : i \geq 0\}\{b^i : i \geq 0\}\{a^i : i \geq 0\}$.
   - **l.** $\{x \in \{a, b\}^* : |x|$ is even and the first half of x has one more a than does the second half$\}$.
   - **m.** $\{w \in \{a, b\}^* : \#_a(w) = \#_b(w)$ and w does not contain either the substring aaa or abab$\}$.
   - **n.** $\{a^n b^{2n} c^m : n, m \geq 0\} \cap \{a^n b^m c^{2m} : n, m \geq 0\}$.
   - **o.** $\{x \text{ c } y : x, y \in \{0, 1\}^* \text{ and } y \text{ is a prefix of } x\}$.
   - **p.** $\{w : w = uu^R \text{ or } w = ua^n : n = |u|, u \in \{a, b\}^*\}$.
   - **q.** $L(G)$, where $G = S \rightarrow aSa$

     $S \rightarrow SS$

     $S \rightarrow \varepsilon$
   - **r.** $\{w \in (\text{A-Z, a-z, ., blank})^+ :$ there exists at least one duplicated, capitalized word in $w)$. For example, the string, `The history of China can be viewed from the perspective of an outsider or of someone living in China`, $\in L$.
   - **s.** $\neg L_0$, where $L_0 = \{ww : w \in \{a, b\}^*\}$.
   - **t.** L^*, where $L = \{0^*1^i0^*1^i0^* : i \geq 0\}$.
   - **u.** $\neg A^nB^n$.
   - **v.** $\{ba^jb : j = n^2 \text{ for some } n \geq 0\}$. For example, baaaab $\in L$.
   - **w.** $\{w \in \{a, b, c, d\}^* : \#_b(w) \geq \#_c(w) \geq \#_d(w) \geq 0\}$.

Recall: When we use the pumping theorem to show a language is not context-free, we do not get to choose the k, we choose the w whose length is at least k. We do not get to choose how w is broken up into uvxyz (although the breakup has to meet the length constraints of the theorem), but we do get to choose how to pump the v and y (I.e. we can choose the q in uv^qxy^qz).

13.1c: When I assigned this before, I thought there was value in starting with the intuitive assumption that this language is not context-free, and only after trying many approaches to showing this, begin to think that maybe it is context-free after all. But in a survey from a previous term, some students told me that they spent more than 150 minutes on this problem, mainly because of spinning their wheels trying to use the Pumping Theorem. Thus I am telling you that this language is indeed context-free.

13.3 (#14) 9

3. Let $L = \{a^n b^m c^n d^m : n, m \geq 1\}$. L is interesting because of its similarity to a useful fragment of a typical programming language in which one must declare procedures before they can be invoked. The procedure declarations include a list of the formal parameters. So now imagine that the characters in a^n correspond to the formal parameter list in the declaration of procedure 1. The characters in b^m correspond to the formal parameter list in the declaration of procedure 2. Then the characters in c^n and d^m correspond to the parameter lists in an invocation of procedure 1 and procedure 2 respectively, with the requirement that the number of parameters in the invocations match the number of parameters in the declarations. Show that L is not context-free.

4. Without using the Pumping Theorem, prove that $L = \{w \in \{a, b, c\}^* : \#_a(w) = \#_b(w) = \#_c(w)$ and $\#_a(w) > 50\}$ is not context-free.

13.4 (#15)

5. Give an example of a context-free language $L\ (\neq \Sigma^*)$ that contains a subset L_1 that is not context-free. Prove that L is context free. Describe L_1 and prove that it is not context-free.

6. Let $L_1 = L_2 \cap L_3$.

   a. Show values for L_1, L_2, and L_3, such that L_1 is context-free but neither L_2 nor L_3 is.

   b. Show values for L_1, L_2, and L_3, such that L_2 is context-free but neither L_1 nor L_3 is.

7. Give an example of a context-free language L, other than one of the ones in the book, where $\neg L$ is not context-free.

8. Theorem 13.7 tells us that the context-free languages are closed under intersection with the regular languages. Prove that the context-free languages are also closed under union with the regular languages.

13.8 (#16)

9. Complete the proof that the context-free languages are not closed under *maxstring* by showing that $L = \{a^i b^j c^k : k \leq i$ or $k \leq j\}$ is context-free but *maxstring*(L) is not context-free.

13.9 (#17) 6

12. Define the leftmost maximal P subsequence m of a string w as follows:

   - P must be a nonempty set of characters.
   - A string S is a P subsequence of w iff S is a substring of w and S is composed entirely of characters in P. For example 1, 0, 10, 01, 11, 011, 101, 111, 1111, and 1011 are $\{0, 1\}$ subsequences of 2312101121111.

13.12 (#18)

   - Let S be the set of all P subsequences of w such that, for each element t of S, there is no P subsequence of w longer than t. In the example above, $S = \{1111, 1011\}$.
   - Then m is the leftmost (within w) element of S. In the example above, $m = 1011$.

   a. Let $L = \{w \in \{0\text{-}9\}^* :$ if y is the leftmost maximal $\{0, 1\}$ subsequence of w then $|y|$ is even$\}$. Is L regular (but not context free), context free or neither? Prove your answer.

   b. Let $L = \{w \in \{a, b, c\}^* :$ the leftmost maximal $\{a, b\}$ subsequence of w starts with a$\}$. Is L regular (but not context free), context free or neither? Prove your answer.

Note on 13.12. What the author meant to ask and what she actually asked are quite different. **Both parts should have said**: "Is L context-Free (but not regular), regular, or neither? Prove your answer.