MA/CSSE 474
Theory of Computation

More Reduction Examples
Non-SD Reductions
Slides for Days 38 and 39, 201830

Your Questions?

- Previous class days' material
- Reading Assignments
- HW 15 problems
- Final Exam
- Anything else

When released in 1969, *Abbey Road* was considered by many critics to be a disappointment. Now it is on many "best albums of all time lists". For example, #14 on *Rolling Stone's* list, #5 on thetoptens.com.

Reducing *Language* L_1 to L_2

- Language L_1 (over alphabet Σ_1) is **mapping reducible** to language L_2 (over alphabet Σ_2) and we write $L_1 \leq L_2$ if there is a Turing-computable function $f : \Sigma_1^* \rightarrow \Sigma_2^*$ such that

$$\forall x \in \Sigma_1^*,\ x \in L_1 \text{ if and only if } f(x) \in L_2$$

Application: If L1 is a language that is known to not be in D, and we can find a reduction from L1 to L2, then L2 is also not in D.

H_{ANY} is not in D (reduction 1)

H = {<*M*, *w*> : TM M halts on input string *w*}

R ↓

(?*Oracle*) H_{ANY} = {<*M*> : there exists at least one string on which TM *M* halts}

R(<*M*, *w*>) =

1. Construct <*M#*>, where *M#*(*x*) operates as follows:
   1.1. Examine *x*.
   1.2. If *x* = *w*, run *M* on *w*, else loop.
2. Return <*M#*>.

If *Oracle* exists, then *C* = *Oracle*(*R*(<*M*, *w*>)) decides H:

- *R* can be implemented as a Turing machine.
- *C* is correct: The only string on which *M#* can halt is *w*. So:
  - <*M*, *w*> $\in$ H: *M* halts on *w*. So *M#* halts on *w*. There exists at least one string on which *M#* halts. *Oracle* accepts.
  - <*M*, *w*> $\notin$ H: *M* does not halt on *w*, so neither does *M#*. So there exists no string on which *M#* halts. *Oracle* rejects.

But no machine to decide H can exist, so neither does *Oracle*.

H_{ANY} is not in D (reduction 2)

Proof: We show that H_{ANY} is not in D by reduction from H:

H = {<*M*, *w*> : TM M halts on input string *w*}

R ↓

(?*Oracle*) H_{ANY} = {<*M*> : there exists at least one string on which TM *M* halts}

R(<*M*, *w*>) =

1. Construct the description <*M#*>, where *M#*(*x*) operates as follows:
   1.1. Erase the tape.
   1.2. Write *w* on the tape.
   1.3. Run *M* on *w*.
2. Return <*M#*>.

If *Oracle* exists, then *C* = *Oracle*(*R*(<*M*, *w*>)) decides H:

- *C* is correct: *M#* ignores its own input. It halts on everything or nothing. So:
  - <*M*, *w*> ∈ H: *M* halts on *w*, so *M#* halts on everything. So it halts on at least one string. *Oracle* accepts.
  - <*M*, *w*> ∉ H: *M* does not halt on *w*, so *M#* halts on nothing. So it does not halt on at least one string. *Oracle* rejects.

But no machine to decide H can exist, so neither does *Oracle*.

The Steps Proving L_2 undecidable

1. ✪ Choose an undecidable language L_1 to reduce from.

2. ✪ Define the reduction *R*.

3. Show that *C* (the composition of *R* with *Oracle, if Oracle exists*) is correct. I.e. it decides L_1 (this is a contradiction)

✪ indicates where we make choices.

Undecidable Problems (Languages That Aren't In D)

The Problem View	**The Language View**
Does TM *M* halt on *w*?	$H = \{\langle M, w\rangle : M \text{ halts on } w\}$
Does TM *M* not halt on *w*?	$\neg H = \{\langle M, w\rangle : M \text{ does not halt on } w\}$
Does TM *M* halt on the empty tape?	$H_\varepsilon = \{\langle M\rangle : M \text{ halts on } \varepsilon\}$
Is there any string on which TM *M* halts?	$H_{ANY} = \{\langle M\rangle :$ there exists at least one string on which TM M halts $\}$
Does TM *M* accept all strings?	$A_{ALL} = \{\langle M\rangle : L(M) = \Sigma^*\}$
Do TMs M_a and M_b accept the same languages?	$EqTMs = \{\langle M_a, M_b\rangle : L(M_a) = L(M_b)\}$
Is the language that TM *M* accepts regular?	$TMreg = \{\langle M\rangle : L(M) \text{ is regular}\}$

Next: We examine proofs of some of these (some are also done in the textbook)

$H_{ALL} = \{\langle M\rangle$: TM M halts on all inputs$\}$

We show that H_{ALL} is not in D by reduction from H_ε. **Note:** We reduce from H_ε, not H

$H_\varepsilon = \{\langle M\rangle$: TM M halts on $\varepsilon\}$

R ↓

(?*Oracle*) $H_{ALL} = \{\langle M\rangle$: TM M halts on all inputs $\}$

R(<*M*>) =

1. Construct the description <*M#*>, where *M#*(*x*) operates as follows:
   1.1. Erase the tape.
   1.2. Run *M*.
2. Return <*M#*>.

If *Oracle* exists, then *C* = *Oracle*(*R*(<*M*>)) decides H_ε:

- *R* can be implemented as a Turing machine.
- *C* is correct: *M#* halts on everything or nothing, depending on whether *M* halts on ε. So:
  - $\langle M\rangle \in H_\varepsilon$: *M* halts on ε, so *M#* halts on all inputs. *Oracle* accepts.
  - $\langle M\rangle \notin H_\varepsilon$: *M* does not halt on ε, so *M#* halts on nothing. *Oracle* rejects.

But no machine to decide H_ε can exist, so neither does *Oracle*.

The Membership Question for TMs

We next define a new language:

A = {<*M*, *w*> : *M* accepts *w*}

Note that A is different from H since it is possible that *M* halts but does not accept.

An alternative definition of A is:

A = {<*M*, *w*> : *w* ∈ *L*(*M*)}

A = {<*M*, *w*> : *w* ∈ *L*(*M*)}

We show that A is not in D by reduction from H.

H = {<*M, w*> : TM *M* halts on input string *w*}

R ↓

(?*Oracle*) A = {<*M, w* > : *w* ∈ *L*(*M*) }

R(<*M, w*>) =

1. Construct the description <*M#*>, where *M#*(*x*) operates as follows:
   1.1. Erase the tape.
   1.2. Write *w* on the tape.
   1.3. Run *M* on *w*.
   1.4. **Accept**
2. Return <*M#, w*>.

If *Oracle* exists, then *C* = *Oracle*(*R*(<*M*, *w*>)) decides H:

- *R* can be implemented as a Turing machine.
- *C* is correct: *M#* accepts everything or nothing. So:
  - <*M, w*> ∈ H: *M* halts on *w*, so *M#* accepts everything. In particular, it accepts *w*. *Oracle* accepts.
  - <*M, w* > ∉ H: *M* does not halt on *w*. *M#* gets stuck in step 1.3 and so accepts nothing. *Oracle* rejects.

But no machine to decide H can exist, so neither does *Oracle*.

A_ε, A_{ANY}, and A_{ALL}

Theorem: $A_\varepsilon = \{<M> : \text{TM } M \text{ accepts } \varepsilon\}$ is not in D.

Proof: Analogous to that for H_ε.

Theorem:

$A_{ANY} = \{<M> : \text{TM } M \text{ accepts at least one string}\}$

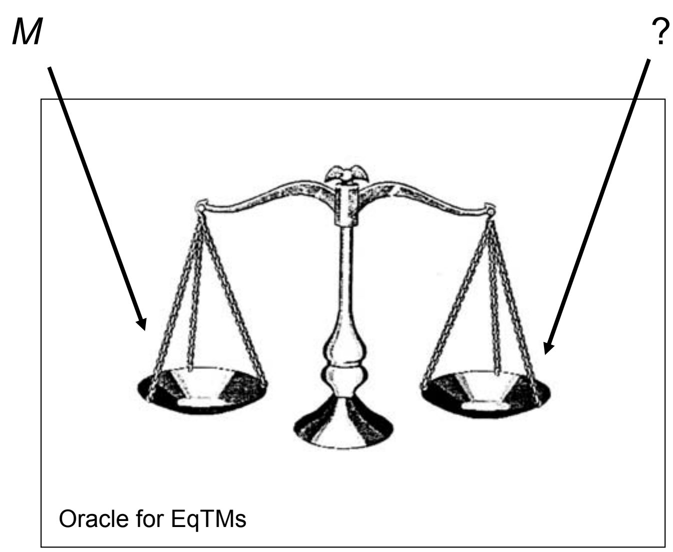
is not in D.

Proof: Analogous to that for H_{ANY}.

Theorem: $A_{ALL} = \{<M> : = L(M) = \Sigma^*\}$ is not in D.

Proof: Analogous to that for H_{ALL}.

$EqTMs=\{<M_a, M_b>: L(M_a)=L(M_b)\}$



EqTMs={<M_a, M_b>: $L(M_a)=L(M_b)$}

A_{ANY} = {<*M*> : TM *M* accepts at least one string }

↓ *R*

(*Oracle*) EqTMs = {<M_a, M_b>: $L(M_a)=L(M_b)$}

R(<*M*>) =

1. Construct the description of *M#*(*x*):
   1.1. Accept.
2. Return <*M*, *M#*>.

If *Oracle* exists, then *C* = *Oracle*(*R*(<*M*>)) decides A_{ANY}:

- *C* is correct: *M#* accepts everything. So:
  - <*M*> $\in A_{ANY}$: *L*(*M*) =? *L*(*M#*). *Oracle* ? Oops.
  - <*M*> $\notin A_{ANY}$: *L*(*M*) ≠ *L*(*M#*). *Oracle* rejects.

EqTMs={<M_a, M_b>: $L(M_a)=L(M_b)$}

A_{ALL} = {<*M*> : $L(M) = \Sigma^*$}

↓ *R*

(*Oracle*) EqTMs = {<M_a, M_b>: $L(M_a)=L(M_b)$}

R(<*M*>) =

1. Construct the description of *M#*(*x*):
   1.1. Accept.
2. Return <*M*, *M#*>.

If *Oracle* exists, then *C* = *Oracle*(*R*(<*M*>)) decides A_{ALL}:

- *C* is correct: *M#* accepts everything. So if *L*(*M*) = *L*(*M#*), *M* must also accept everything. So:
  - <*M*> $\in A_{ALL}$: *L*(*M*) = *L*(*M#*). *Oracle* accepts.
  - <*M*> $\notin A_{ALL}$: *L*(*M*) ≠ *L*(*M#*). *Oracle* rejects.

But no machine to decide A_{ALL} can exist, so neither does *Oracle*.

A Practical Consequence

Consider the problem of virus detection. Suppose that a new virus *V* is discovered and its code is <*V*>.

- Is it sufficient for antivirus software to check solely for occurrences of <*V*>?
- Is it possible for it to check for equivalence to *V*?

How many Turing machines does it take to change a light bulb?

How many Turing machines does it take to change a light bulb?

One.

How can you tell whether your Turing machine is the one?

You can't!

Practice

- Practice: Show that these languages are not in D.
  - A_{ANY} = {<*M*> : TM *M* accepts at least one string}
  - A_{ALL} = {<*M*> : $L(M) = \Sigma^*$}
  - *REJ* = {<*M*, *w*> : Turing machine *M* rejects *w*}

Note: *Each can be shown by a reduction from H.*

Sometimes Mapping Reduction Doesn't Work

Recall that a mapping reduction from L_1 to L_2 is a computable function *f* where:

$\forall x \in \Sigma^* \, (x \in L_1 \leftrightarrow f(x) \in L_2)$.

When we use a mapping reduction, we return:

Oracle(*f*(*x*))

Sometimes we need to use *Oracle* as a subroutine and then do other computations after it returns.

{<*M*> : *M* accepts no even length strings}

H = {< *M*, *w*> : TM *M* halts on input string *w*}

↓ *R*

(?*Oracle*) L_2 = {<*M*> : *M* accepts no even length strings}

R(<*M*, *w*>) =

1. Construct the description <*M#*>, where *M#*(*x*) operates as follows:
   - 1.1. Erase the tape.
   - 1.2. Write *w* on the tape.
   - 1.3. Run *M* on *w*.
   - 1.4. Accept.
2. Return <*M#*>.

If *Oracle* exists, then *C* = *Oracle*(*R*(<*M*, *w*>)) decides H:

- *C* is correct: *M#* ignores its own input. It accepts everything or nothing, depending on whether it makes it to step 1.4. So:
  - <*M*, *w*> $\in$ H: *M* halts on *w*. *Oracle*:
  - <*M*, *w*> $\notin$ H: *M* does not halt on *w*. *Oracle*:

Problem:

{<*M*> : *M* accepts no even length strings}

H = {< *M*, *w*> : TM *M* halts on input string *w*}

↓ *R*

(?*Oracle*) L_2 = {<*M*> : *M* accepts no even length strings}

R(<*M*, *w*>) =

1. Construct the description <*M#*>, where *M#*(*x*) operates as follows:
   1.1. Erase the tape.
   1.2. Write *w* on the tape.
   1.3. Run *M* on *w*.
   1.4. Accept.
2. Return <*M#*>.

If *Oracle* exists, then *C* = ¬*Oracle*(*R*(<*M*, *w*>)) decides H:

- *R* and ¬ can be implemented as Turing machines.
- *C* is correct:
  - <*M*, *w*> ∈ H: *M* halts on *w. M#* accepts everything, including some even length strings. *Oracle* rejects so *C* accepts.
  - <*M*, *w*> ∉ H: *M* does not halt on *w*. *M#* gets stuck. So it accepts nothing, so no even length strings. *Oracle* accepts. So *C* rejects.

But no machine to decide H can exist, so neither does *Oracle*.

Are All Questions about TMs Undecidable?

Let *L* = {<*M*> : TM *M* contains an even number of states}

Let *L* = {<*M*, *w*> : *M* halts on *w* within 3 steps}.

Let L_q = {<*M*, *q*> : there is some configuration

(*p*, $u\underline{a}v$) of *M*, with $p \neq q$,

that yields a configuration whose state is *q* }.

Is L_q decidable?

Is There a Pattern?

- Does L contain some particular string w?
- Does L contain ε?
- Does L contain any strings at all?
- Does L contain all strings over some alphabet Σ?

- A = {<*M*, *w*> : TM *M* accepts *w*}.
- A_ε = {<*M*> : TM *M* accepts ε}.
- A_{ANY} = {<*M*> : there exists at least one string that TM *M* accepts}.
- A_{ALL} = {<*M*> : TM *M* accepts all inputs}.

Rice's Theorem

No nontrivial property of the SD languages is decidable.

or

Any language that can be described as:

{<*M*>: *P*(*L*(*M*)) = *True*}

for any nontrivial property *P*, is not in D.

A ***nontrivial property*** is one that is not simply:

- *True* for all languages, or
- *False* for all languages.

Because of time constraints, we will skip the proof of this theorem.

Applying Rice's Theorem

To use Rice's Theorem to show that a language *L* is not in D we must:

- Specify property *P*.
- Show that the domain of *P* is the SD languages.
- Show that *P* is nontrivial:
  - *P* is true of at least one language
  - *P* is false of at least one language

Applying Rice's Theorem

1. {<*M*> : *L*(*M*) contains only even length strings}.
2. {<*M*> : *L*(*M*) contains an odd number of strings}.
3. {<*M*> : *L*(*M*) contains all strings that start with a}.
4. {<*M*> : *L*(*M*) is infinite}.
5. {<*M*> : *L*(*M*) is regular}.
6. {<*M*> : *M* contains an even number of states}.
7. {<*M*> : *M* has an odd number of symbols in its tape alphabet}.
8. {<*M*> : *M* accepts ε within 100 steps}.
9. {<*M*>: *M* accepts ε}.
10. {<M_a, M_b> : $L(M_a) = L(M_b)$}.

Given a TM *M*, is *L*(*M*) Regular?

The problem: Is *L*(*M*) regular?

As a language: Is {<*M*> : *L*(*M*) is regular} in D?

No, by Rice's Theorem:

- *P* = *True* if *L* is regular and *False* otherwise.
- The domain of *P* is the set of SD languages since it is the set of languages accepted by some TM.
- *P* is nontrivial:
  - ♦ *P*(**a***) = *True*.
  - ♦ *P*(A^nB^n) = *False*.

We can also show it directly, using reduction. (Next slide)

Given a Turing Machine *M*, is *L*(*M*) Regular?

H = {<*M*, *w*> : TM *M* halts on input string *w*}

↓ *R*

(*Oracle*) L_2 = {<*M*> : *L*(*M*) is regular}

R(<*M*, *w*>) =

1. Construct *M#*(*x*):
   1.1. Copy its input *x* to another track for later.
   1.2. Erase the tape.
   1.3. Write *w* on the tape.
   1.4. Run *M* on *w.*
   1.5. Put *x* back on the tape.
   1.6. If $x \in A^nB^n$ then accept, else reject.
2. Return <*M#*>.

Problem:

But We Can Flip

R(<*M*, *w*>) =

1. Construct the description <*M#*>, where *M#*(*x*) operates as follows:
   1.1. Save *x* for later.
   1.2. Erase the tape.
   1.3. Write *w* on the tape.
   1.4. Run *M* on *w.*
   1.5. Put *x* back on the tape.
   1.6. If $x \in A^nB^n$ then accept, else reject.
2. Return <*M#*>.

If Oracle decides L_2, then *C* = ¬*Oracle*(*R*(<*M*, *w*>)) decides H:

- <*M*, *w*> ∈ H: *M#* makes it to step 1.5. Then it accepts *x* iff $x \in A^nB^n$. So *M#* accepts A^nB^n, which is not regular. *Oracle* rejects. *C* accepts.
- <*M*, *w*> ∉ H: *M* does not halt on *w*. *M#* gets stuck in step 1.4. It accepts nothing. *L*(*M#*) = ∅, which is regular. *Oracle* accepts. *C* rejects.

But no machine to decide H can exist, so neither does *Oracle*.

Or, Show it Without Flipping

R(<*M*, *w*>) =

1. Construct the description <*M#*>, where *M#*(*x*) operates as follows:
   1.1. If $x \in A^nB^n$ then accept, else:
   1.2. Erase the tape.
   1.3. Write *w* on the tape.
   1.4. Run *M* on *w.*
   1.5. Accept
2. Return <*M#*>.

If *Oracle* exists, *C* = *Oracle*(*R*(<*M*, *w*>)) decides H:

- *C* is correct: *M#* immediately accepts all strings in A^nB^n:
  - <*M*, *w*> ∈ H: *M#* accepts everything else in step 1.5. So *L*(*M#*) = Σ*, which is regular. *Oracle* accepts.
  - <*M*, *w*> ∉ H: *M#* gets stuck in step 1.4, so it accepts nothing else. *L*(*M#*) = A^nB^n, which is not regular. *Oracle* rejects.

But no machine to decide H can exist, so neither does *Oracle*.

Any Nonregular Language Will Work

R(<*M*, *w*>) =

1. Construct the description <*M#*>, where *M#*(*x*) operates as follows:
    1.1. If $x \in$ **WW** then accept, else:
    1.2. Erase the tape.
    1.3. Write *w* on the tape.
    1.4. Run *M* on *w.*
    1.5. Accept
2. Return <*M#*>.

If *Oracle* exists, *C* = *Oracle*(*R*(<*M*, *w*>)) decides H:

- *C* is correct: *M#* immediately accepts all strings ww:
    - <*M*, *w*> $\in$ H: *M#* accepts everything else in step 1.5. So *L*(*M#*) = Σ^*, which is regular. *Oracle* accepts.
    - <*M*, *w*> $\notin$ H: *M#* gets stuck in step 1.4, so it accepts nothing else. *L*(*M#*) = WW, which is not regular. *Oracle* rejects.

But no machine to decide H can exist, so neither does *Oracle*.

Is *L*(*M*) Context-free?

How about: L_3 = {<*M*> : *L*(*M*) is context-free}?

R(<*M*, *w*>) =

1. Construct the description <*M#*>, where *M#*(*x*) operates as follows:
    1.1. If $x \in A^nB^nC^n$ then accept, else:
    1.2. Erase the tape.
    1.3. Write *w* on the tape.
    1.4. Run *M* on *w.*
    1.5. Accept
2. Return <*M#*>.

Practical Impact of These Results

1. Does *P*, when running on *x*, halt?
2. Might *P* get into an infinite loop on some input?
3. Does *P*, when running on *x*, ever output a 0? Or anything at all?
4. Are P_1 and P_2 equivalent?
5. Does *P*, when running on *x*, ever assign a value to *n*?
6. Does *P* ever reach *S* on any input (in other words, can we chop it out?
7. Does *P* reach *S* on every input (in other words, can we guarantee that *S* happens)?

- Can the Patent Office check prior art?
- Can the CS department buy the definitive grading program?

Turing Machine Questions Can be Reduced to Program Questions

EqPrograms =

{$<P_a, P_b>$: P_a and P_b are *PL* programs and $L(P_a) = L(P_b)$}.

We can build, in any programming language *PL*, *SimUM*:

- that is a *PL* program
- that implements the Universal TM U and so can simulate an arbitrary TM.

{<*M*, *q*> : *M* reaches *q* on some input}

Hidden: M reaches q on some input

H_{ANY} = {<*M*> : there exists some string on which TM *M* halts}

↓ *R*

(?*Oracle*) L_2 = {<*M*, *q*> : *M* reaches *q* on some input}

R(<*M*>) =

1. Build <*M#*> so that *M#* is identical to *M* except that, if *M* has a transition $((q_1, c_1), (q_2, c_2, d))$ and q_2 is a halting state other than *h*, replace that transition with:
   $((q_1, c_1), (h, c_2, d))$.
2. Return <*M#*, *h*>.

If *Oracle* exists, then *C* = *Oracle*(*R*(<*M*>)) decides H_{ANY}:

- *R* can be implemented as a Turing machine.
- *C* is correct: *M#* will reach the halting state *h* iff *M* would reach some halting state. So:
  - <*M*> $\in H_{ANY}$: There is some string on which *M* halts. So there is some string on which *M#* reaches state *h*. *Oracle* accepts.
  - <*M*> $\notin H_{ANY}$: There is no string on which *M* halts. So there is no string on which *M#* reaches state *h*. *Oracle* rejects.

But no machine to decide H_{ANY} can exist, so neither does *Oracle*.

Side Road with a purpose: obtainSelf

From Section 25.3:

In section 25.3, the author proves the existence of a very useful computable function: *obtainSelf*. When called as a subroutine by any Turing machine M, *obtainSelf* writes <M> onto M's tape.

Related to quines:

A quine is a computer program which takes no input and produces a copy of its own source code as its only output.

Definition is from http://en.wikipedia.org/wiki/Quine_(computing)

Some quines

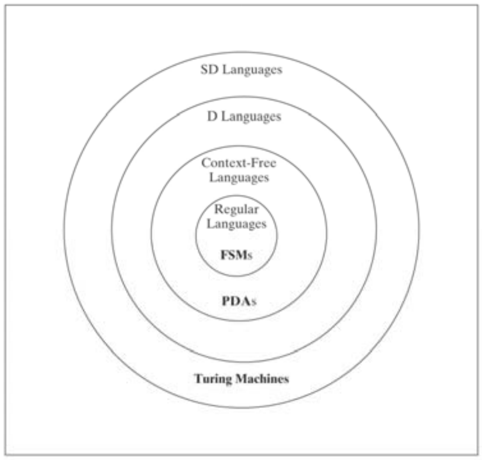
- ```
  main(){char q=34, n=10,*a="main() {char
  q=34,n=10,*a=%c%s%c;
  printf(a,q,a,q,n);}%c";printf(a,q,a,q,n);}
  ```
- ```
  ((lambda (x) (list x (list 'quote x)))
   (quote (lambda (x) (list x (list 'quote x)))))
  ```
- Quine's paradox and a related sentence:

  "Yields falsehood when preceded by its quotation" yields falsehood when preceded by its quotation.

  "quoted and followed by itself is a quine." quoted and followed by itself is a quine.

Non-SD Languages

There is an uncountable number of non-SD languages, but only a countably infinite number of TM's (hence SD languages). ∴ The class of non-SD languages is <u>much</u> bigger than that of SD languages!



Non-SD Languages

Intuition: Non-SD languages usually involve either infinite search (where testing each potential member could loop forever), or determining whether a TM will infinite loop.

Examples:

- ¬H = {<*M*, *w*> : TM *M* does *not* halt on *w*}.
- {<*M*> : *L*(*M*) = Σ*}.
- {<*M*> : TM *M* halts on nothing}.

Proving Languages are not SD

- Contradiction
- *L* is the complement of an SD/D Language.
- Reduction from a known non-SD language

Contradiction

Theorem: TM_{MIN} =
{<*M*>: Turing machine *M* is minimal} is not in SD.

Proof: If TM_{MIN} were in SD, then there would exist some Turing machine *ENUM* that enumerates its elements. Define the following Turing machine:

M#(*x*) =
1. Invoke *obtainSelf* to produce <*M#*>.
2. Run *ENUM* until it generates the description of some Turing machine *M′* whose description is longer than |<*M#*>|.
3. Invoke *U* on the string <*M′*, *x*>.

Since TM_{MIN} is infinite, *ENUM* must eventually generate a string that is longer than |<*M#*>|. So *M#* makes it to step 3 and thus M# is Equivalent to *M′* since it simulates *M′*. But, since |<*M#*>| < |<*M′*>|, *M′* cannot be minimal.
But M#'s description was generated by *ENUM*. Contradiction.

The Complement of *L* is in SD/D

Suppose we want to know whether *L* is in SD and we know:

- ¬*L* is in SD, and
- At least one of *L* or ¬*L* is not in D.

Then we can conclude that *L* is not in SD, because, if it were, it would force both itself and its complement into D, which we know cannot be true.

Example:

- ¬H (since ¬(¬H) = H is in SD and not in D)

$A_{anbn} = \{<M> : L(M) = A^nB^n\}$

A_{anbn} contains strings that look like:

```
(q00,a00,q01,a00,→),
(q00,a01,q00,a10,→),
(q00,a10,q01,a01,←),
(q00,a11,q01,a10,←),
(q01,a00,q00,a01,→),
(q01,a01,q01,a10,→),
(q01,a10,q01,a11,←),
(q01,a11,q11,a01,←)
```

It does not contain strings like `aaabbb`.

But A^nB^n does.

A_{anbn} = {<*M*> : *L*(*M*) = A^nB^n}

What's wrong with this proof that A_{anbn} is not in SD?

¬H = {<*M*, *w*> : TM *M* does not halt on *w*}

↓ *R*

(?*Oracle*) A_{anbn} = {<*M*> : *L*(*M*) = A^nB^n}

R(<*M*, *w*>) =

1. Construct the description <*M#*>, where *M#*(*x*) operates as follows:
   1.1. Erase the tape.
   1.2. Write *w* on the tape.
   1.3. Run *M* on *w*.
   1.4. Accept.
2. Return <*M#*>.

If *Oracle* exists, *C* = *Oracle*(*R*(<*M*, *w*>)) semidecides ¬H:

A_{anbn} = {<*M*> : *L*(*M*) = A^nB^n} is not SD

What about: ¬H = {<*M*, *w*> : TM *M* does not halt on *w*}

↓ *R*

(?*Oracle*) A_{anbn} = {<*M*> : *L*(*M*) = A^nB^n}

R(<*M*, *w*>) =

1. Construct the description <*M#*>, where *M#*(*x*) operates as follows:
   1.1 Copy the input *x* to another track for later.
   1.2. Erase the tape.
   1.3. Write *w* on the tape.
   1.4. Run *M* on *w*.
   1.5. Put *x* back on the tape.
   1.6. If $x \in A^nB^n$ then accept, else loop.
2. Return <*M#*>.

If *Oracle* exists, *C* = *Oracle*(*R*(<*M*, *w*>)) semidecides ¬H:

A_{anbn} = {<*M*> : *L*(*M*) = A^nB^n} is not SD

R(<*M*, *w*>) reduces ¬H to A_{anbn}:

1. Construct the description <*M#*>:
    1.1. If $x \in A^nB^n$ then accept. Else:
    1.2. Erase the tape.
    1.3. Write *w* on the tape.
    1.4. Run *M* on *w*.
    1.5. Accept.
2. Return <*M#*>.

If *Oracle* exists, then *C* = *Oracle*(*R*(<*M*, *w*>)) semidecides ¬H:
M# immediately accepts all strings in A^nB^n. If *M* does not halt on *w*, those are the only strings *M#* accepts. If *M* halts on *w*, *M#* accepts everything:

- <*M*, *w*> ∈ ¬H: *M* does not halt on *w*, so *M#* accepts strings in A^nB^n in step 1.1. Then it gets stuck in step 1.4, so it accepts nothing else. It is an A^nB^n acceptor**.** ***Oracle* accepts**.
- <*M*, *w*> ∉ ¬H: *M* halts on *w*, so *M#* accepts everything**.**
  ***Oracle* does not accept.**

But no machine to semidecide ¬H can exist, so neither does *Oracle*.