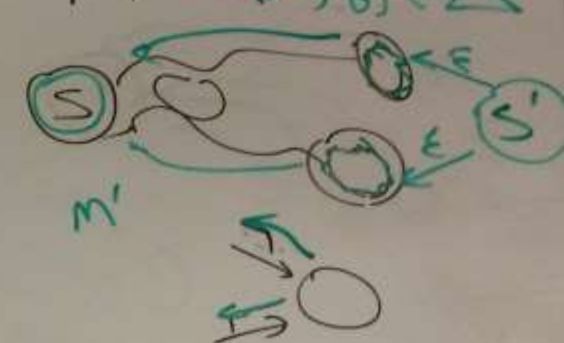
DFSM $M = (K, \Sigma, \delta, s, A)$ $\quad$ $M' = (K', \Sigma, \Delta, s', A')$ NDFSM

In M: $\delta(p,a) = q$

$(p,a,q) \in \Delta$

M



M'

Construct M' such that $L(M') = L(M)^R$

$s' =$ a new state, $K' = K \cup \{s'\}$, $A' = \{s\}$

$\Delta = \{(p,a,q) : \delta(q,a) = p\} \cup \{(s', \varepsilon, q) : q \in A\}$

Theorem $\boxed{\forall w \in \Sigma^* (M \text{ accepts } w \text{ iff } M' \text{ accepts } w^R)}$

Lemma $\forall p,q \in K (\forall w \in \Sigma^* ((p,w) \vdash_M^* (q,\varepsilon) \leftrightarrow (q,w^R) \vdash_{M'}^* (p,\varepsilon)))$

Proof of $\Rightarrow$ direction. Induction on $|w|$

Base case: $w = \varepsilon$. If $(p,\varepsilon) \vdash_M^* (q,\varepsilon)$, then $p = q$, so $(q,\varepsilon^R) \vdash_{M'}^* (p,\varepsilon)$

M is deterministic

in zero steps, configs are the same

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Induction step: $|w| \geq 1$, so $w = xa$ for some $x \in \Sigma^*$, $a \in \Sigma$

Assume $\rightarrow$ is ~~true~~ true for all strings that are shorter than w, and show it's true for w. (in particular, x)

for some $r \in K$, $(p, x) \vdash_M^* (r, \varepsilon)$, and $(r, a) \vdash_M (q, \varepsilon)$

Note that:

1. $(r, x^R) \vdash_{M'}^* (p, \varepsilon)$, by induction hypothesis.
2. $(q, a) \vdash_{M'} (r, \varepsilon)$, by def of Δ (1st part)

Put these together:

$(q, w^R) = (q, ax^R) \vdash_{M'} (r, x^R) \vdash_{M'}^* (p, \varepsilon)$

$(q, w^R) \vdash_{M'}^* (p, \varepsilon)$ transitivity of $\vdash^*$ ✓

Exercise: $\Leftarrow$

Use the lemma to prove the theorem

M accepts w iff $(s, w) \vdash_{M}^{*} (q, \varepsilon)$ for some $q \in A$ (def of acceptance by FSM)

iff $(q, w^R) \vdash_{M'}^{*} (s, \varepsilon)$ (Lemma)

iff $(q, w^R) \vdash_{M'}^{*} (s, \varepsilon) \wedge (s', \varepsilon) \vdash_{M'} (q, \varepsilon)$ (2nd part of def of Δ)

iff $(s', w^R) \vdash_{M'} (q, w^R) \vdash_{M'}^{*} (s, \varepsilon)$ put previous things together

iff $(s', w^R) \vdash_{M'}^{*} (s, \varepsilon)$ def of $\vdash^*$

iff M' accepts w^R def of acceptance

$$P \Longleftrightarrow P \wedge \text{True}$$