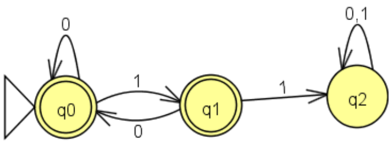
How I would write the Day 1 induction proof if I were turning in a 474 assignment:

Let M = 

, S = the language accepted by M (i.e., L(M))

T = {w ∈ {0,1}* : w does not have 11 as a substring}

Show that S = T.

--------- Solution ---------

First, show that S ⊆ T. I.e, if w accepted by M, then w does not contain 11. We do it by induction on |w|. But we show something more specific, namely:

P: For every string w∈{0,1}*, both of the following are true (and these cover both possibilities for strings in L(M)):

1. If δ(q0, w) = q0, then w has no consecutive 1's and does not end in 1.
2. If δ(q0, w) = q1, then w has no consecutive 1's and ends in 1.

Base case: |w| = 0, so w = ε. We show both parts:

1. δ(q0, w) = q0 and w has no consecutive 1's and does not end in 1. ✓
2. δ(q0, w) ≠ q1, so the statement is vacuously true (F → P is true no matter what P is) ✓

Induction step |w| > 0, so w = xa for some a∈{0,1}, x∈{0,1}*.
The induction hypothesis (IH) is that (1) and (2) are true for the shorter string x. We must show that this implies that they are true for w.

1. Suppose that δ(q0, w) = q0. Looking at the ways to get to q0 in the DFSM, we see that a=0, so w ends in 0 and that δ(q0, x) = q0 or q1. In either case, IH says that x does not contain 11, and thus w =x0 does not contain 11. ✓
2. Suppose that δ(q0, w) = q1. Looking at the way to get to q1 in the DFSM, we see that a=1, so w ends in 1 and that δ(q0, x) = q0. IH says that x does not end in 1 and does not contain 11 as a substring. Thus w=x1 ends in 1 and does not contain 11. ✓

Now, show that T ⊆ S. I.e, if w does not contain 11, it accepted by M. It is easier to show the (equyivalent) contrapositive – If w is not accepted by M, it contains 11 as a substring.
Base case: |w| = 0, so w = ε. Since this string is accepted by M, the statement is vacuously true.
Induction step |w| > 0, so w = xa for some a∈{0,1}, x∈{0,1}*. The induction hypothesis (IH) is that if x is not accepted by M, x contains 11. We must show that this implies that the same statement for w.
If M does not accept w, then δ(q0, w) = q3. Looking at the DFSM diagram, we can see that there are two possible ways this can happen.

- δ(q0, x) = q1 and a=1. By what we proved before, x ends with 1, so w ends with 11. ✓
- δ(q0, x) = q2. BY the IH, x contains 11 as a substring. Since w=xa, w also contains 11. ✓