MA/CSSE 474 Day 38 Summary

1) Review from recent sessions:
   a) Language L_1 (over alphabet Σ_1) is **mapping reducible** to language L_2 (over alphabet Σ_2) and we write $L_1 \leq L_2$ if there is a Turing-computable function $f : \Sigma_1^* \rightarrow \Sigma_2^*$ such that $\forall x \in \Sigma_1^*$, $x \in L_1$ if and only if $f(x) \in L_2$
   b) **Using reduction:** If P_1 is reducible to P_2,
      i) If P_2 is decidable, so is P_1.
      ii) If P_1 is not decidable, neither is P_2.
   c) **A framework for using reduction to show undecidability**.

      To show language L_2 undecidable:
      i) Choose a language L_1 that is already known not to be in D, and show that L_1 can be reduced to L_2.
         (1) Define the reduction *R and show that it can be implemented by a TM.*
      ii) Describe the composition *C* of *R* with *Oracle* (the purported TM that decides L_1).
         (1) Show that *C* correctly decides L_1 iff *Oracle* exists. We do this by showing that *C* is correct. I.e., If $x \in L_1$, then *C*(*x*) accepts, and If $x \notin L_1$, then *C*(*x*) rejects.

> Another way to say it: ***mapping reduction*** *R* from language L_1 to language L_2 is one or more Turing machines such that If there exists a Turing machine *Oracle* that decides (or semidecides) L_2, then the TMs in *R* can be composed with *Oracle* to build a deciding (or semideciding) TM for L_1.

2) H_{ANY} = {<*M*> : there exists at least one string on which TM *M* halts} is not in D
   i) Two different reductions from H: Details on slides. A place for your notes:

3) Undecidable problems and languages (there is a table of problems and languages in the previous day's class notes.
4) H_{ALL} = {<M> : TM M halts on all inputs} is not in D. Details on slides. A place for your notes:

5) A = {<M, w> : w ∈ L(M)} is not in D. Details on slides. A place for your notes:

6) EqTMs={<M_a, M_b>: L(M_a)=L(M_b)} is not in D. Details on slides. A place for your notes:
   a) "Reduction" from A_{ANY}

   b) Reduction from A_{ALL}

7) Practice: Show that these languages are not in D.

Note: Each can be shown to be undecidable using a reduction from H.

a) A_{ANY} = {<*M*> : TM *M* accepts at least one string}

b) A_{ALL} = {<*M*> : $L(M) = \Sigma^*$}

c) *REJ* = {<*M*, *w*> : Turing machine *M* rejects string *w*}